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On Higher Order Radiative Corrections in Muonic Atoms

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Abstract. Radiative corrections of order $\alpha^2(Z\alpha)$ corresponding to the fourth order Lamb shift for muonic atoms are calculated. Numerical results are presented for the $1s$, $2s$, and $2p$ states of Nd and Hg. All contributions other than the well-known contribution due to fourth order electron vacuum polarization are too small to be experimentally important.

Inspired by a persistent discrepancy between theory and experiment in certain muonic transitions [1, 2] a number of authors have calculated several small higher order radiative corrections in muonic atoms, including the effect of hadronic vacuum polarization [3] and the virtual Delbrück scattering contribution [4–7] of order $\alpha^2(Z\alpha)^2$. The purpose of the present note is to investigate the contributions of order $\alpha^2(Z\alpha)$ [1]), other than that due to fourth order vacuum polarization involving electron loops, which is well-known [8], since these contributions have not previously been evaluated for muonic atoms.

The additional corrections of order $\alpha^2(Z\alpha)$ come from the following sources:

(a) Fourth order muon vacuum polarization (Fig. 1). Following Källen and Sabry [9] and Baranger et al. [10], we have

$$\Delta E_{\mu VP} = -\frac{41}{162 m_\mu^2}\left(\frac{\alpha}{\pi}\right)^2 \langle \nabla^2 V \rangle \tag{1}$$

where V is the electrostatic potential due to the nucleus.

(b) The fourth order muon Lamb shift (Fig. 2) is obtained, following the work of Applequist and Brodsky [11], or of Barbieri et al. [12].

$$\Delta E_{LS}^{(4)} = \frac{1}{m_\mu^2}\langle \nabla^2 V \rangle \left(m_\mu^2 \frac{\partial F_1}{\partial q^2}\right)_{q^2=0} + \frac{(g-2)^{(4)}}{4m_\mu^2}\left[\langle \nabla^2 V \rangle + \left\langle \frac{2}{r}\frac{dV}{dr}\vec{\sigma}\cdot\vec{L}\right\rangle\right] \tag{2}$$

here

$$m_\mu^2 \frac{dF_1^{(4)}}{dq^2}\bigg|_{q^2=0} = 0.470\left(\frac{\alpha}{\pi}\right)^2 + (2.19 \pm 0.01)\left(\frac{\alpha}{\pi}\right)^2 \tag{3a}$$

1) The orders given here refer to the order of the operator whose expectation value is calculated to give the energy shift. Since the atomic number Z is not necessarily small, the quantities α and αZ are treated as separate expansion parameters.

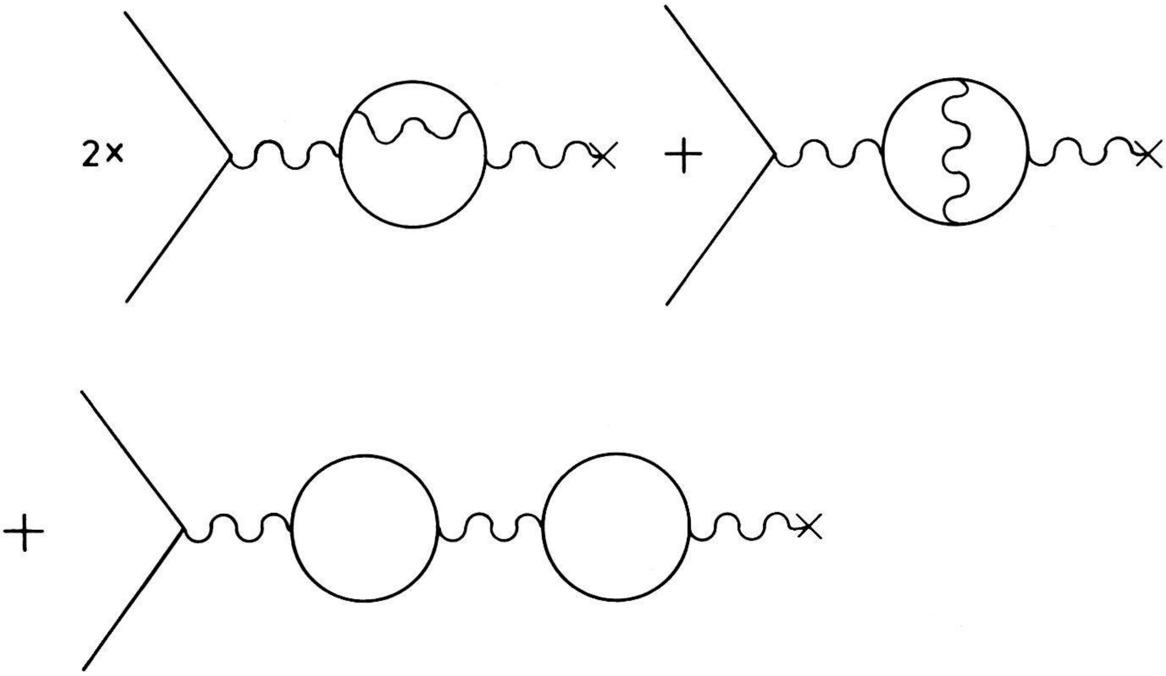

Figure 1
Fourth order muon vacuum polarization.

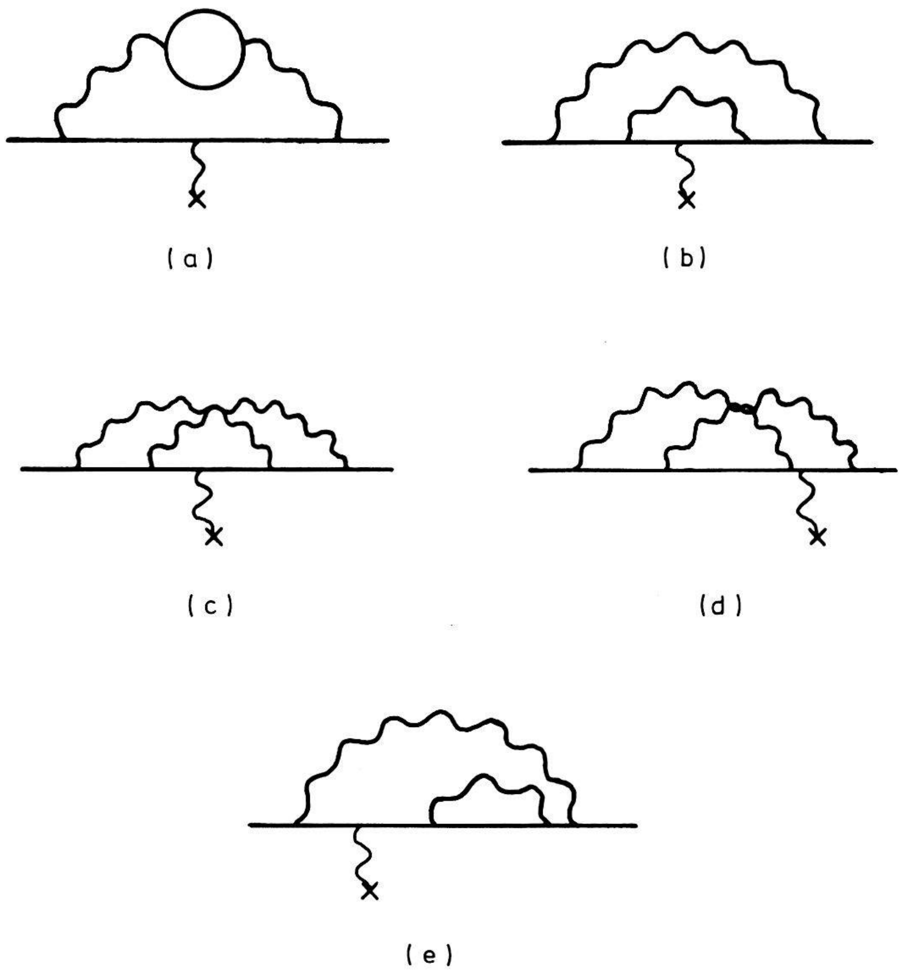

Figure 2
Graphs contributing to fourth order muon Lamb shift. Figure 2a occurs twice, once with an electron vacuum polarization insertion and once with a muon vacuum polarization insertion.

and

$$(g-2)^{(4)} = 0.766\left(\frac{\alpha}{\pi}\right)^2 \tag{3b}$$

is the fourth order contribution to the muon anomalous magnetic moment [13].

The contribution $\Delta E_{\mu e}$ of the two loop graph (Fig. 3) in which one loop represents an electron pair and the other a muon pair. The relevant formulas can be adapted from the work of Sundaresan and Watson [14], with the result

$$\Delta E_{\mu e} = \frac{2Z\alpha}{\pi}\int_0^\infty q\,dq\int_0^\infty (f^2+g^2)j_0(qr)\,dr\,U(q)\,\frac{F(q)}{q^2} \tag{4}$$

In equation (4), $F(q)$ is the nuclear form factor, f and g are the small and large components of the muon wave function and

$$\begin{aligned} U(q) &= 2\left(\frac{2\alpha}{\pi}\right)^2\int_0^1 dy\,y(1-y)\ln\frac{(1+q^2y(1-y))}{m_\mu^2} \\ &\quad\times\int_0^1 dz\,z(1-z)\ln\frac{(1+q^2z(1-z))}{m_e^2} \\ &\simeq -\left(\frac{\alpha}{\pi}\right)^2\frac{q^2}{15m_\mu^2}\left[(1-\tfrac{1}{3}\coth^2\phi)(1-\phi\coth\phi)-\tfrac{1}{9}\right] \end{aligned} \tag{5}$$

In equation (5), $\sinh^2\phi = q^2/4m_e^2$ and the approximation $q^2 \ll m_\mu^2$ has been made; this approximation is well-justified even for lowest order muon vacuum polarization, where it is good to one or two per cent.

We observe that with the exception of the spin-orbit term in equation (2) these corrections affect only muon orbits which lie within the nucleus and thus will have practically no effect on the transitions of interest in the tests of QED [1, 2]. The spin-orbit term arises from the muon anomalous magnetic moment and in fourth order contributes a shift of 0.025 eV for the $5g$–$4f$ transition in lead. It thus cannot have a bearing on the question of a possible discrepancy between theory and experiment for this transition, since the errors involved are some tens of eV. However it now seems that the discrepancy is disappearing from the experimental side [15].

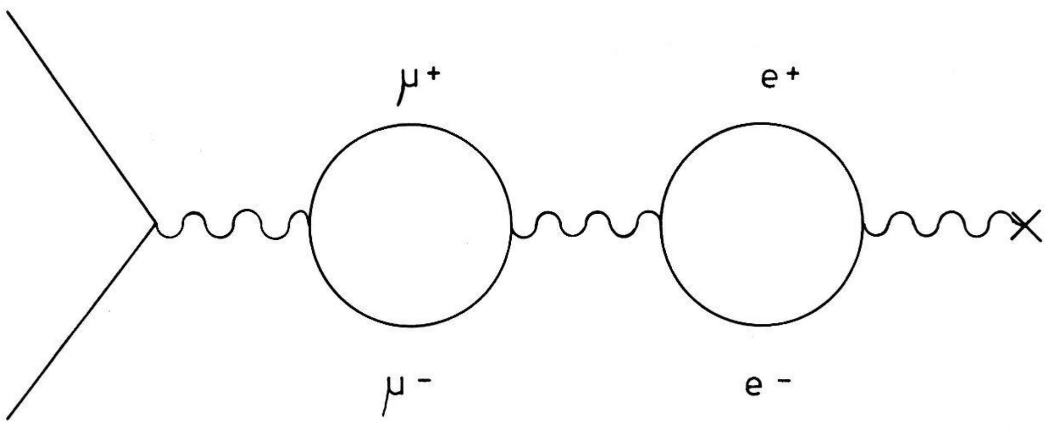

Figure 3
Graph contributing to $\Delta E_{\mu e}$.

To obtain an estimate of the importance of these fourth order corrections for lower-lying states, we have calculated energy level shifts for the $1s$, $2s$ and $2p$ states of Nd ($Z = 60$) and Hg ($Z = 80$). The results are shown in Table I. Shifts of higher levels are even smaller and are neglected. All shifts were calculated using muon wave functions for an extended nucleus with charge distribution of Fermi type. Charge density parameters were taken from Ref. [16]. For the calculation of the fourth order Lamb

Table I
Shifts in binding energy (in eV) due to various radiative corrections of order $\alpha^2(Z\alpha)$. Column 1: Fourth order muon Lamb shift, including fourth order muon vacuum polarization. Column 2: Mixed muon-electron vacuum polarization. Column 3: Fourth order electron vacuum polarization.

	$\Delta E^{(4)}_{LS}$	$\Delta E_{\mu e}$	$\Delta E^{(4)}_{eVP}$
$^{60}Nd_{146}$			
$1s$	−14.7	0.9	373
$2s$	−2.4	0.2	85
$2p_{1/2}$	−0.7	0.1	122
$2p_{3/2}$	−1.0	0.0	111
$^{80}Hg_{200}$			
$1s$	−21.4	1.4	536
$2s$	−3.7	0.2	143
$2p_{1/2}$	−2.6	0.2	239
$2p_{3/2}$	−3.2	0.1	219

shift, the required expectation values were calculated for the lowest order case [16, 17] and the fourth order shift computed from these with a desk calculator. The results are shown in the first column. This contribution is the largest of those calculated in this work. Values of $\Delta E_{\mu e}$ are given in the second column. For comparison, we show in the third column the contribution due to fourth order electron vacuum polarization as obtained from Ref. [16]. This is clearly the only experimentally important contribution of this order, since errors in the measurement of these transitions are of the order of several hundred eV, as are uncertainties in other corrections, such as the lowest order muon Lamb shift and nuclear polarization.

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