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A Note on the Alpha-Decay Half-Lives of Heavy and Superheavy Elements[1])

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Abstract. The known alpha-decay half-lives of transuranium elements can be reproduced using an interaction potential having a repulsive core. Results are insensitive to the magnitude of the core height but the derived nuclear half-density radius $= 1.1A^{1/3}$ F. is consistent with those obtained from the analysis of mu-mesic and electron scattering experiments. Using this model and theoretical Q-values from Green's mass formula, the upper limits of alpha-decay half-lives of $(112)^{292,294,296,298,300}$, $(114)^{294,296,298,300,302}$, and $(126)^{310,312,314,316,318}$ are computed. For isotopes of elements 112, our calculations barely overlap with those of Nilsson et al. For isotopes of element 114, our estimated half-lives are considerably shorter than those of Nilsson et al. but agree with those of Grumann et al. For isotopes of 126, our calculated half-lives are longer than those estimated by Muzychka.

1. Purpose

The purpose of this note is a) to investigate the alpha-decay probabilities of the superheavy elements 112, 114 and 126 using a dynamical model, b) to understand the alpha-decay probabilities of elements 112 and 114 which are tabulated by Nilsson et al. [1] but the model used by them to compute these is not at all discussed, c) to demonstrate that contrary to the usual comment, the nuclear half-density radius obtained from the analysis of alpha decays could be $1.1A^{1/3}$F., i.e., in agreement with our knowledge of this parameter obtained from other experiments such as the μ-mesic data or the electron scattering data, and d) to investigate the variation of the alpha-decay probabilities of elements 112, 114 and 126 with the Q-values of the reaction. It will be shown (and this is also a well-known experimental fact) that the half-lives depend rather critically on Q-values which are at best known within one MeV. Because of this, we shall provide here upper and lower limits of alpha decay probabilities of elements 112, 114, and 126.

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Apart from Nilsson et al., Grumann et al. [2] and Muzychka [3] have also made estimates of the alpha-decay probabilities of the superheavies. However, they essentially use an extrapolation formula whose parameters are empirically determined by fitting to known alpha-decay lifetimes. This extrapolation formula has been used by Taagepera and Nurmia [4], Viola and Seaborg [5]. Gallagher and Rasmussen [6] have clearly shown this formula to be a simple approximation to the decay expression obtained in the JWKB approximation. Instead of using an approximation to the JWKB method, we, here, propose to use the JWKB approximation itself.

2. The Model

A sophisticated theory of alpha-decay has been formulated by Mang [7] and further developed by Mang, Rasmussen and Poggenburg [7]. Since there is considerable uncertainty in predicting theoretically the Q-values and our main purpose is to estimate the order of magnitude of the decay lifetime, we do not propose to apply their sophisticated model. Instead, we take recourse to the simple model, proposed by Winslow [8].

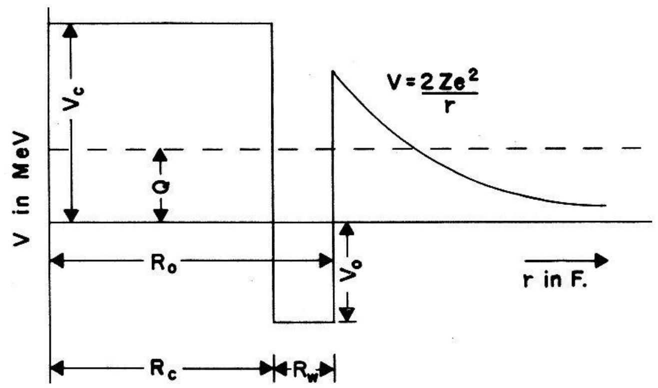

Figure 1
A schematic representation of interaction potential used here. R_c, the core radius $= r_0(A_1^{1/3} + A_2^{1/3})$ F. $R_0 = R_c + R_w$, where $R_w = \frac{1}{2}(t_1 + t_2)$. t_1 and t_2 are surface thickness parameters. In Winslow's model $V_c = \infty$.

As pointed out originally by Bethe [9] the key difference between a one-body decay process and a many-body decay process represented by an interaction potential in relative coordinates is that the penetrability must be multiplied by a 'preformation probability'. This has further been investigated by Winslow [8] and Scherk and Vogt [10]. Estimates indicate [8, 9, 11] that this preformation probability for alpha-decay is 10^{-2} to 10^{-3}. Consequently we adopt here a value of 10^{-3}. This is consistent with the original estimate of Bethe. He also showed that an uncertainty of two orders in preformation probability introduces an uncertainty of only 10% in the derived nuclear radius.

For the radial form of the interaction potential we essentially adopt the form suggested by Winslow except that we replace the infinite repulsive core by a finite one. Thus our alpha-decay potential is (Fig. 1)

$$V(R) = \begin{cases} V_c & R < R_c \\ V_0 < V_c & R_c \leqslant R < R_0 \\ Z_1 Z_2 e^2/R & R_0 \leqslant R \end{cases} \tag{1}$$

Setting V_c to ∞ reduces this to the Winslow model. This simple form of the interaction is justified on the ground that the alpha-decay probability is insensitive to the details of the inner part of the potential—a fact already noted by Blatt and Weisskopf [12] and analysed in a recent paper [13]. In addition to the discussion already presented by Winslow, any α-nucleus interaction calculated on the basis of statistical nuclear many-body theory warrants the presence of a core. The α–α interaction obtained empirically and from the resonating group structure supports also the idea of a core. However, it should clearly be emphasized that the presence of a core is not imperative to yield proper alpha half-lives, and must simply be taken as a feature of our model. Within the context of the statistical nuclear many-body theory, the core radius $\simeq$ the sum of the half-density radii of the alpha particle and the residual nucleus, as it is shown in Figure 2. This is $\simeq 1.1(A_1^{1/3} + A_2^{1/3})$ F. $\simeq 7$ F. The entire computation is totally insensitive to the choice of R_c, so long as $(R_0 - R_c) \lesssim 1.5$ F. Similarly, the calculation is insensitive to the choice of V_0 and V_c so long as V_0 and V_c are chosen, respectively, to be less than and greater than E (i.e., Q_α). They will be kept fixed as 300 MeV and (E–60 MeV), respectively.

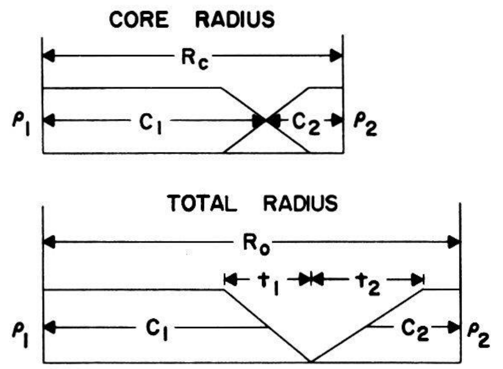

Figure 2
Rough estimate of R_0 and R_w in terms of the statistical nuclear matter theory. For a trapezoidal density distribution R_c is the sum of two half-density radii c_1 and c_2, i.e., $R_c = c_1 + c_2 = 1.1$ $(A_1^{1/3} + A_2^{1/3})$ F. R_0 is the distance when two nuclei are just touching each other i.e., $R = R_1 + R_2 = C_1 + C_2 + (\frac{1}{2})(t_1 + t_2)$ fm. ρ_1 and ρ_2 in the figure are nuclear densities of two nuclei.

Thus we have two essential parameters controlling the decay process a) the total reaction radius R_0 and b) Q-values. Our purpose here is to show that a choice of $R_0 = 1.1(A_1^{1/3} + A_2^{1/3}) + (1/2)(t_1 + t_2)$ (A_1, t_1, A_2, and t_2 are, respectively, the mass number and the surface thickness of the daughter and those of the alpha particle) can yield the observed data. This choice of R_0 is, entirely, consistent with our knowledge of the nuclear radius from the analysis of the μ-mesic and electron scattering data.

To obtain, theoretically, Q-values, we follow the prescription

$$Q = M(A, Z) - M(A-4, Z-2) - M(4, 2) \tag{2}$$

where M and Z are, respectively, mass and charge of a nucleus. We shall use the mass formula of Green [14] to obtain $M(A, Z)$ and $M(A-4, Z-2)$ but the experimental mass for $M(4, 2)$ (the use of Meyers-Swiatecki mass formula [15] without shell correction to calculate $M(A, Z)$ and $M(A-4, Z-2)$ does not produce any appreciably different result).

The solution of the wave equation for the potential (1) with the appropriate boundary condition required for a decay process is obtained by matching a linear combination of modified spherical Bessel function solutions in the region $0 \leqslant R < R_c$, to linear combination of a set of spherical Bessel function solutions, defined in the region $R_c \leqslant R < R_0$ and this in turn to an outgoing spherical wave solution constructed from the regular and singular Coulomb wave function F and G. The latter functions are then approximated by

$$G_l = |T_l(kr)|^{-1/4} e^{\omega(kr)}; \qquad F_l = \tfrac{1}{2}|T_l(kr)|^{-1/2} e^{-\omega(kr)} \tag{3}$$

where

$$T_{l(kr)} \equiv 2\eta/kr + (l+\tfrac{1}{2})^2/k^2 r^2 - 1 \tag{4}$$

and

$$\omega(kr) \equiv \int_1^2 T_l[(kr)]^{\frac{1}{2}}\, dr \tag{5}$$

(η and k are, respectively, $Z_1 Z_2 e^2/\hbar$ [Reduced Mass/complex energy E] and the wave number). The decay constant λ and hence the barrier penetrability and corresponding half-live are subsequently evaluated using Winslow's method. The integration limits in (5) are two turning points.

3. Results and Discussion

A. The Q-value:

In Table 1, we compare the computed Q-values with those observed for the transuranic even–even elements. We conclude that the experimental Q-values could be reproduced by our prescription with an uncertainty of about ± 0.5 MeV.

Although we could obtain the observed Q-values of the alpha-decay from the transuranic even–even nuclei, the uncertainty in the theoretical calculation of the Q-value is somewhat greater for the decay of a closed shell nucleus, e.g., the computed Q-values for the decay of Po^{204}, Po^{206}, Po^{208}, and Po^{210} are, respectively, 5.05, 4.68, 4.31 and 3.93 MeV, whereas the respective experimental values are 5.48, 5.33, 5.21, and 5.41 MeV. From this and other comparisons, we find that our theoretical computation will *underestimate* the Q-value at a shell closure by as much as 1.5 MeV. Thus our estimated half-lives using computed (Q-value $\pm$ 0.5 MeV) form an *upper bound* to the actual half-lives. With this type of uncertainties in mind we shall also provide computed half-lives corresponding to (computed Q-values + 1.5 MeV) in the case of elements 112 and 114 and (computed Q-values + 2.0 MeV) for the element 126. This also exemplifies the strong dependence of half-lives on Q-values.

B. Known alpha half-lives:

In Table 2, we compute the alpha half-lives using our model and taking once observed Q-values (Column 4) and again the computed Q-values (last column). Clearly, the model can reproduce all these known half-lives using the experimental Q-values. In the same table we also present a calculation of the alpha life times within the context of Winslow model, i.e., setting $V_c = \infty$, using the observed Q-value. We note that there is no discernible difference between ours and his model. Consequently, we shall restrict ourselves to a finite $V_c \simeq 300$ MeV in computing the half-lives of elements

Table 1
Comparison between the experimental (Column 3) and the theoretical (last column) alpha-decay energies. The latter are calculated by equation (2) using Green's mass formula. Experimental data are taken from standard tables.

Z	A	Q(Exp) MeV	Q(th) MeV
$_{92}$U	228	6.68	6.23
	230	5.89	5.88
	232	5.32	5.53
	234	4.77	5.18
	236	4.49	4.82
	238	4.20	4.46
$_{94}$U	232	6.59	6.87
	234	6.20	6.53
	236	5.77	6.19
	238	5.50	5.85
	240	5.17	5.50
	242	4.90	5.15
	244	4.58	4.79
$_{96}$Cm	240	6.29	6.83
	242	6.12	6.51
	244	5.81	6.17
	246	5.39	5.82
	248	5.08	5.48
$_{98}$Cf	244	7.18	7.48
	246	6.76	7.16
	248	6.27	6.82
	250	6.03	6.49
	252	6.12	6.15
$_{100}$Fm	248	7.85	8.12
	250	7.44	7.80
	252	7.05	7.47
	254	7.16	7.14

112, 114, and 126. The interesting point to note is that the nuclear half-density radius parameter, $1.1A^{1/3}$ F., is consistent with the alpha-decay half-lives. However, the important thing is that the simple prescription that R_0 be the sum of two radii of the daughter pair can reproduce the trend of the experimental data for 30 or so isotopes.

C. *Element* 112:

In Table 3, we present the alpha-decay half-lives for isotopes 292, 294, 296, 298, and 300 of the element 112. Q-values obtained from Green's formula have been used. This should be uncertain within ± 0.5 MeV if the element 112 is not a good closed shell. If the element 112 is a good closed proton shell nucleus, the actual Q-values could be as much as 1.5 MeV higher than the calculated one. This table, therefore, presents calculations with Q-values computed using Green's formula along with those obtained by adding and subtracting ± 0.5 MeV and ± 1.5 MeV, respectively to it. If 112 forms a good closed shell, half-lives corresponding to $(Q - 0.5$ MeV) are good upper limits. Thus the approximate upper limit of alpha-decay half-lives of isotopes 292, 294, 296, 298, and 300 are respectively, about 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1} and 10 years.

Table 2
Comparison between experimental (Column 3) and theoretical alpha-decay half-lives in years. Column 4 and the last column record half-lives obtained using, respectively, the observed and the computed Q-value and the model of a finite repulsive core. Column 5 reproduces results of Winslow model (i.e., an infinite hard core) using observed Q-values.

Z	A	Measured $T_{1/2}$ (years)	$T_{1/2}$(th) with Q(exp)	$T_{1/2}$ (Winslow) with Q (exp)	$T_{1/2}$(th) with Q(th)
$_{92}$U	228	2.0×10^{-6}	0.2×10^{-6}	0.2×10^{-6}	2.1×10^{-3}
	230	5.7×10^{-2}	8.8×10^{-2}	7.8×10^{-2}	9.6×10^{-2}
	232	7.2×10^{1}	11.4×10^{1}	10.1×10^{1}	9.7×10^{-2}
	234	2.5×10^{5}	4.1×10^{5}	3.6×10^{5}	7.9×10^{2}
	236	2.4×10^{7}	4.6×10^{7}	4.0×10^{7}	1.7×10^{5}
	238	4.5×10^{9}	10.7×10^{9}	9.1×10^{9}	7.6×10^{7}
$_{94}$Pu	232	6.9×10^{-5}	34.7×10^{-5}	30.5×10^{-5}	2.3×10^{-5}
	234	1.0×10^{-3}	19.7×10^{-3}	17.3×10^{-3}	5.7×10^{-4}
	236	2.9×10^{0}	2.8×10^{0}	2.5×10^{0}	2.0×10^{-2}
	238	8.7×10^{1}	8.6×10^{1}	7.5×10^{1}	1.0×10^{0}
	240	6.6×10^{3}	8.3×10^{3}	7.3×10^{3}	8.3×10^{1}
	242	3.9×10^{5}	4.9×10^{5}	4.3×10^{5}	1.1×10^{4}
	244	8.3×10^{7}	10.2×10^{7}	9.0×10^{7}	2.7×10^{6}
$_{96}$Cm	240	7.4×10^{-2}	5.0×10^{-2}	4.4×10^{-2}	1.6×10^{-4}
	242	4.5×10^{-1}	3.2×10^{-1}	2.8×10^{-1}	4.6×10^{-3}
	244	1.8×10^{1}	1.3×10^{1}	1.1×10^{1}	1.8×10^{-1}
	246	4.7×10^{3}	3.1×10^{3}	2.8×10^{3}	1.0×10^{1}
	248	3.5×10^{5}	2.9×10^{5}	2.5×10^{5}	9.1×10^{2}
$_{98}$Cf	244	3.8×10^{-5}	4.3×10^{-5}	2.6×10^{-5}	2.9×10^{-6}
	246	4.1×10^{-3}	2.3×10^{-3}	2.0×10^{-3}	5.1×10^{-5}
	248	9.6×10^{-1}	4.1×10^{-1}	3.5×10^{-1}	1.1×10^{-3}
	250	1.3×10^{1}	0.6×10^{1}	0.6×10^{1}	3.5×10^{-2}
	252	2.6×10^{0}	2.0×10^{0}	1.8×10^{0}	1.4×10^{0}
$_{100}$Fm	248	1.2×10^{-6}	0.8×10^{-6}	0.6×10^{-6}	1.1×10^{-8}
	250	5.7×10^{-5}	2.3×10^{-5}	2.0×10^{-5}	1.6×10^{-6}
	252	2.6×10^{-3}	0.8×10^{-3}	0.7×10^{-3}	9.4×10^{-5}
	254	3.7×10^{-4}	2.5×10^{-4}	2.3×10^{-4}	3.1×10^{-4}

Table 3
Computed alpha-decay half-lives of isotopes of element 112. A denotes the mass numbers. Rows 1, 2, 4, 5 and 6 record corresponding half-lives using, respectively, (Q(th) + 1.5 MeV), (Q(th) + 0.5 MeV), Q(th), (Q(th) − 0.5 MeV) and (Q(th) − 1.5 MeV). Row 3 records the theoretical Q-values, Q(th), obtained from equation (2). Rows 7 and 8 give, respectively, Q-values and corresponding half-lives of Ref. 1. All half-lives are in years and energies are in MeV.

A =	292	294	296	298	300
1. Green +1.5 MeV	1.0×10^{-11}	1.5×10^{-10}	3.9×10^{-9}	2.9×10^{-8}	2.4×10^{-7}
2. Green +0.5 MeV	6.0×10^{-8}	4.9×10^{-7}	7.7×10^{-6}	8.6×10^{-5}	1.1×10^{-3}
3. Q(th) (MeV)	8.80	8.49	8.19	7.88	7.57
4. Green	3.5×10^{-6}	3.7×10^{-5}	4.5×10^{-4}	6.5×10^{-3}	1.2×10^{-1}
5. Green −0.5 MeV	6.2×10^{-4}	2.6×10^{-3}	4.1×10^{-2}	8.0×10^{-1}	2.0×10^{1}
6. Green −1.5 MeV	2.2×10^{0}	5.2×10^{1}	1.7×10^{3}	6.9×10^{4}	4.1×10^{6}
7. Q (Nilsson) MeV	7.46	6.83	6.54	7.50	7.24
8. $T_{1/2}$ (Nilsson)	4.4×10^{-1}	3.5×10^{2}	1.1×10^{4}	2.4×10^{-1}	3.3×10^{0}

In the same table the published Q-values and half-lives of Nilsson et al. [1] are documented. This difference in Q-values stems from the fact that Nilsson et al. have considered element 112 to be a closed shell. Using our model with our preformation probability and their Q-values, their quoted half-lives could be nearly reproduced. Since their Q-values are considerably lower than ours, their half-lives are longer. Our results definitely disagree with theirs and Muzychka's [3] for isotopes 294 and 296. However, any theoretical calculation of Q-values has an uncertainty of at least 0.5 MeV. If this is added to their quoted Q-value, their half-lives would be shorter by about two orders of magnitude and overlap with ours.

D. Element 114:

The computed half-lives of its isotopes 294, 296, 298, 300, and 302 are recorded in Table 4. Assuming no large shell effects, reasonable upper limits are about 10^{-7}, 10^{-6}, 10^{-5}, 10^{-4} and 10^{-3} years, respectively. If we make an allowance of an additional one MeV for the shell effect, the upper limits are given in the sixth row. Our computations of half-lives noted in the fourth row of this table (marked Green) for isotopes 294, 298, and 302 are in agreement with those estimated by Grumann et al. [2] who used the extrapolation formulas of Refs. 4 and 5.

Table 4
Computed alpha-decay half-lives of isotopes of element 114. A denotes mass numbers. Rows 1, 2, 4, 5 and 6 record corresponding half-lives using, respectively, (Q(th) + 1.5 MeV), (Q(th) + 0.5 MeV), (Q(th)), (Q(th) − 0.5 MeV) and (Q(th) − 1.5 MeV). Row 3 records the theoretical Q-values, Q(th), obtained from equation (2). Rows 7 and 8 give, respectively, Q-values and corresponding half-lives of Ref. 1. All half-lives are in years and energies in MeV.

A =	294	296	298	300	302
1. Green +1.5 MeV	2.2×10^{-12}	1.0×10^{-11}	5.1×10^{-11}	2.7×10^{-10}	1.7×10^{-7}
2. Green +0.5 MeV	5.9×10^{-10}	3.6×10^{-9}	2.3×10^{-8}	1.8×10^{-7}	1.6×10^{-6}
3. Q(th) (MeV)	9.74	9.44	9.14	8.84	8.54
4. Green	1.4×10^{-8}	9.6×10^{-8}	7.7×10^{-7}	6.9×10^{-6}	1.2×10^{-4}
5. Green −0.5 MeV	4.21×10^{-7}	3.6×10^{-6}	5.8×10^{-5}	6.6×10^{-4}	9.4×10^{-3}
6. Green −1.5 MeV	1.8×10^{-3}	2.5×10^{-2}	4.1×10^{-1}	8.0×10^{0}	2.2×10^{2}
7. Q (Nilsson) MeV	7.97	7.55	7.40	7.50	7.24
8. $T_{1/2}$ (Nilsson)	2.0×10^{-2}	1.0×10^{0}	4.5×10^{0}	1.5×10^{0}	2.1×10^{1}

The quoted half-lives of Nilsson et al. [1] and estimates of Muzychka [3] definitely lie outside our range. The reason is that their estimated Q-values are almost 2 MeV lower than ours. If we add an additional 0.5 MeV to their Q-values, the computed half-lives would barely overlap with our results.

E. Element 126:

In Table 5, we compute alpha half-lives for isotopes 310, 312, 314, 316 and 318 of element 126. Our estimated upper limits are, respectively, 10^{-14}, 10^{-13}, 10^{-12}, 10^{-12}, and 10^{-11} years. (These are half-lives corresponding to Q-values from Green's formula + 1 MeV obtained by extrapolating between third and fourth rows of Table 5.) These are longer than Muzychka's estimates.

Table 5

Computed alpha-decay half-lives of isotopes of element 126. A denotes mass numbers. Rows 1, 3 and 4 record their half-lives using, respectively (Q(th) + 2.0 MeV), Q(th) and (Q(th) − 2.0 MeV). Row 3 notes the theoretical Q-values, Q(th), obtained from equation (2). All half-lives are in years and energies in MeV.

A =	310	312	314	316	318
1. Green +2 MeV	2.3×10^{-18}	4.7×10^{-18}	1.0×10^{-17}	2.2×10^{-17}	5.0×10^{-17}
2. Q(th) (MeV)	14.42	14.16	13.90	13.64	13.36
3. Green	1.8×10^{-15}	4.5×10^{-15}	1.2×10^{-14}	3.3×10^{-14}	9.6×10^{-14}
4. Green −2 MeV	9.6×10^{-21}	3.3×10^{-11}	1.2×10^{-10}	4.6×10^{-10}	1.9×10^{-9}

4. Conclusion

This paper demonstrates that a) the nuclear radius derived from the analysis of known alpha-decay half-lives could be made compatible with those obtained from the μ-mesic and electron scattering experiments. Following Bethe's estimate, the uncertainty in the derived nuclear radius is about 10%, and b) the previous estimates of alpha-decay half-lives for elements 112 and 114 (except for Grumann et al.'s estimates of element 114) are, somewhat, optimistic. We present the dependence of half-lives on Q-values. This latter point is important in any kind of estimates, since any theoretical computations of Q-values are likely to be uncertain at least by 0.5 MeV (or even more). This means that estimates would have an inherent uncertainty of 3 orders of magnitudes. Our estimates clearly indicate that alpha-decay probabilities of elements 112 and 114 are long enough to be measured if they are produced in the laboratory. However, alpha-decay probabilities of these elements are too short for them to exist naturally or in the remnant of super novae. The alpha-decay probabilities of isotopes of 126 could barely be within our present detection capacity.

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