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Resonant states of Nucleon and Bilocal Field Theory

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1. Introduction

Recently, several resonant states have been discovered and interpreted as particles with a very short life time. In particular, there exist resonant states of nucleon with spins and parities $3^-/2$, $5^+/2$, and masses 1512 MeV, and 1688 MeV, called the 2-nd and 3-rd nucleon resonant state respectively. Being characterized by the same value of isospin and strangeness as the nucleon they must be very closely related to the latter. It seems plausible to suppose that these discoveries are not the end but rather the beginning of the story: probably in the future there will be discovered further resonant states with higher and higher spins and masses and with alternating parities.

Some physicists felt a little uneasy in the past because they could not understand why only the lowest spins 0, 1/2, and 1 appear in Nature. According to the theory of group representations all irreducible representations are equally important and there is no reason why some of them should be privileged. This difficulty would disappear if there existed infinite sets of resonant states with higher and higher spins.

If it is so, the problem arises to formulate a theory accounting, in a natural way, for the existence of infinite families of particles with all (integral or half-integral) spin values. Obviously, the naive procedure of adding more and more fields describing particles with higher spins and choosing arbitrarily the types of interactions between them could not be considered as satisfactory. The traditional local types of interaction lead to hopeless divergences and yield a non-renormalizable theory in the case of higher spins.

The situation is similar to that in the past when one tried to take account of states with an arbitrary number of particles: it was not sufficient to increase more and more the number of dimensions of the configuration space and assume some interactions between the different particles. The necessary tool to deal with the case of an arbitrary number of particles appeared to be a new theory; the theory of quantized fields. Hereby not only states with an arbitrary number of particles appeared quite automatically but also new features of interaction came forth, i. e. the self action of particles leading to such observable consequences as e. g. the change of the Landé factor of electron.

Similarly in the present case, to deal satisfactorily with infinite families of particle types, a new theory is needed in which the particles with higher spins would appear quite automatically, and possibly also some new features of interaction would appear so that the difficulties with infinities and non-renormalizability could be avoided.

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A promising starting point for such a theory has existed for about a dozen of years, namely the non-local (bilocal) theory of YUKAWA[1]). This theory is based on BORN's[2]) idea of reciprocity which promotes the analogy between the operators of position $\boldsymbol{x}_\mu$ and the infinitesimal displacement operators $\boldsymbol{d}_\mu$.

$$[\boldsymbol{x}_\mu, \boldsymbol{d}_\nu] = \delta_{\mu\nu} . \tag{1}$$

According to BORN, the analogy between $\boldsymbol{x}_\mu$ and $\boldsymbol{d}_\mu$ has not been exploited so far and one should look for a theory in which the roles played by $\boldsymbol{x}_\mu$ and $\boldsymbol{d}_\mu$ would be as similar as possible.

A straightforward application of this idea is a generalization of the traditional field concept by assuming field quantities to be not only functions of coordinates but also of the infinitesimal displacement operators

$$\boldsymbol{\Psi} = \boldsymbol{\Psi}(\boldsymbol{x}, \boldsymbol{d}) . \tag{2}$$

The traditional local fields are contained as a special case in this general framework, namely if the following four supplementary conditions are satisfied

$$[\boldsymbol{x}_\mu, \boldsymbol{\Psi}] = 0 . \tag{3}$$

However, these supplementary conditions just violate the idea of reciprocity and the point is to replace them by some other conditions in which $\boldsymbol{x}_\mu$ and $\boldsymbol{d}_\mu$ appear on equal footing. Therefore YUKAWA assumed the following (two) supplementary conditions

$$[[\boldsymbol{\Psi}, \boldsymbol{d}_\mu], \boldsymbol{x}_\mu] = 0 , \quad [[\boldsymbol{\Psi}, \boldsymbol{x}_\mu], \boldsymbol{x}_\mu] = \lambda^2 \boldsymbol{\Psi} \tag{4}$$

which, together with the Klein-Gordon equation

$$[[\boldsymbol{\Psi}, \boldsymbol{d}_\mu], \boldsymbol{d}_\mu] = M^2 \boldsymbol{\Psi} \tag{5}$$

exhibit a symmetry between $\boldsymbol{x}$ and $\boldsymbol{d}$ (provided the constants λ and M are suitably related).

FIERZ[3]) has shown that the generalized field (2) satisfying (4) and (5) is equivalent to an infinite set of ordinary (i.e. local) tensor fields describing particles with higher and higher spins. This fact calls our attention in connection with the discoveries of resonant states. For reasons of simplicity we shall investigate in this paper only a bilocal scalar field as a model of "nucleon" (and its resonant states) in interaction with a local scalar field as a model of "pion". A generalization to the case of a bilocal spinor field is straightforward.

2. Precisation of the Concept of Reciprocity

YUKAWA's conditions (4), (5) may be subjected to a criticism. First of all, they exhibit a symmetry between $\boldsymbol{x}_\mu$ and $\boldsymbol{d}_\mu$ whereas, for reasons of hermiticity, one should rather expect a symmetry between $\boldsymbol{x}_\mu$ and $i\,\boldsymbol{d}_\mu$. Rewriting (4) and (5) in terms of $i\,\boldsymbol{d}_\mu$ it is seen that the symmetry is lost because of the difference in sign of the right hand sides. Secondly, when going over to the case of spinor fields the Equation (5) should be replaced by a Dirac equation whereby the symmetry of the whole set would be spoiled again. Moreover, it is not clear how to introduce an interaction without spoiling the symmetry of the set (4) and (5). Therefore we proposed a formulation in which the supplementary conditions are reciprocal by themselves (without advocating the Klein-Gordon equation) and are assumed to be valid even in the case of interaction with other fields[4]). Our conditions are

$$[[\boldsymbol{\Psi}, \boldsymbol{\pi}_\mu], \boldsymbol{x}_\mu] \equiv [[\boldsymbol{\Psi}, \boldsymbol{x}_\mu], \boldsymbol{\pi}_\mu] = 0 \tag{6'}$$

$$[[\boldsymbol{\Psi}, \boldsymbol{\pi}_\mu], \boldsymbol{\pi}_\mu] + [[\boldsymbol{\Psi}, \boldsymbol{x}_\mu], \boldsymbol{x}_\mu] = 0 \tag{6''}$$

with $\boldsymbol{\pi}_\mu = i\, l^2\, \boldsymbol{d}_\mu$ satisfying the commutation relations

$$[\boldsymbol{x}_\mu, \boldsymbol{\pi}_\nu] = i\, l^2\, \delta_{\mu\nu} \tag{7}$$

where l is a constant with dimension of a length. (6) together with (7) are invariant under the following group of transformations (being a subgroup of canonical transformations)

$$\boldsymbol{x}'_\mu = \alpha\, \boldsymbol{x}_\mu + \beta\, \boldsymbol{\pi}_\mu, \quad \boldsymbol{\pi}'_\mu = \gamma\, \boldsymbol{x}_\mu + \delta\, \boldsymbol{\pi}_\mu \tag{8}$$

where the transformation matrices $\left(\begin{smallmatrix}\alpha & \beta\\ \gamma & \delta\end{smallmatrix}\right)$ form a group consisting on four elements $\pm \left(\begin{smallmatrix}1 & \\ & 1\end{smallmatrix}\right)$, $\pm \left(\begin{smallmatrix} & 1\\ -1 & \end{smallmatrix}\right)$. This precises the meaning of reciprocity.

Since no specific features of the field (like the mass M) appear in the supplementary conditions (6) they represent some very general properties of the field in contradistinction to the field equation (stricte sensu) which must exhibit more specific features of the phenomena described in terms of a field theory. Hence, field equations are not on equal footing with supplementary conditions and do not need to possess all the symmetry properties exhibited by the supplementary conditions. Nevertheless, assuming a Klein-Gordon equation (with an interaction term which does not involve derivatives of field quantities) the theory is reciprocally invariant if the transformation $\boldsymbol{x} \to \boldsymbol{\pi}$, $\boldsymbol{\pi} \to -\boldsymbol{x}$ is supplemented by $M \to i\, M$ and $g \to -g$.

3. A Bilocal Free Field

In the x-representation the field quantities become matrices $\langle x' \,|\, \psi \,|\, x'' \rangle$, i.e. functions of a pair of points $\psi\,(x', x'')$ (but satisfying the matrix multiplication law!). For physical interpretation it is convenient to introduce the variables

$$x = \frac{x' + x''}{2}, \quad r = x' - x'' \tag{9}$$

representing the coordinates of the particle centre and the internal structure variables respectively, and to consider the field quantity $\psi(x, r)$. In consequence of the constraints (6) the Fourier transform $\psi(p, r)$ is of the form

$$\psi(p, r) = \delta(p\, r)\ \ \delta(r^2\, l^{-2} + p^2\, l^2)\ \ \varphi(p, r) \tag{10}$$

(where p, r are abbreviations for p_μ, r_μ and pr, r^2, p^2 are abbreviations for scalar products of these fourvectors) which means that the quantity $\psi(p, r)$ vanishes unless $pr = 0$, $r^2 = -p^2\, l^4$. The quantity $\varphi(p, r)$ appearing to the right hand side of (10) can be assumed to be independent of the projection of r upon the direction of p and of the square of the fourvector r. We shall call ψ a Yukawa field quantity and φ a Fierz field quantity. In the interaction-free case we assume that φ (as well as ψ) satisfies the Klein-Gordon equation

$$(p^2 + M^2)\ \ \varphi(p, r) = 0\,. \tag{11}$$

The Fierz quantity can be developed as follows

$$\varphi(p, r) = \sqrt{N(p)}\, \left(\varphi(p) + \frac{1}{r}\, r_\mu\, \varphi_\mu(p) + \frac{1}{r^2}\, r_\mu\, r_\nu\, \varphi_{\mu\nu}(p) + \ldots \right) \tag{12}$$

where the local tensor fields appearing to the right hand side are normalized in the usual way well known from the local field theory whereas the additional normalizing factor $N(p)$ is chosen as follows

$$N(p) = \left(\int d^4 r \, \delta(p\, r) \, \delta(r^2 l^{-2} + p^2 l^2)\right)^{-1} = \frac{1}{2 \pi l^4} \tag{13}$$

where the last equality is valid only for time-like p and shows that, as long as p is time-like, $N(p)$ does not actually depend on p.

Going back to the x-variables the Fierz quantity is

$$\varphi(x, r) = \sqrt{N}\left(\varphi(x) + \frac{1}{r} r_\mu \varphi_\mu(x) + \frac{1}{r^2} r_\mu r_\nu \varphi_{\mu\nu}(x) + \ldots\right) \tag{12'}$$

and the Yukawa field quantity $\psi(x, r)$ is connected to $\varphi(x, r)$ by the connection of their Fourier transforms (10)*). The local tensor fields appearing to the right hand side of (12) or (12′) are symmetric, have vanishing traces, and vanishing divergences. Thus, the bilocal field can be decomposed into an infinite set of irreducible local tensor fields describing particles with arbitrarily high spins and with alternating parities. If $\psi(x, r)$ is scalar we get a set $0^+, 1^-, 2^+, \ldots$ and if it is pseudoscalar we get $0^-, 1^+, 2^-, \ldots$.

In this sense the theory of a free bilocal field is equivalent to the theory of (an infinite set of) local fields with all spin values. Its Lagrangian must be equivalent to the sum of Lagrangians for the separate local fields and the expressions for energy-momentum and charge must be composed additively of the corresponding expressions for the separate local tensor fields. Quantization of the bilocal free field obviously consists in quantizing the separate local constituents appearing in the decomposition (12′). The commutation relations are to be found in the above quoted paper of Fierz.

In spite of the equivalence of the theory of a bilocal free field with the theory of an infinite set of local fields with higher and higher spin values it is worth while to set up expressions for the action integral W**) and for the densities of energy-momentum $T_{\mu\nu}$ and charge-current j_μ in a compact form in terms of the bilocal field quantities. These expressions are

$$W = \frac{1}{2}\int d^4x \int d^4r \left[\frac{\partial \psi^*}{\partial x_\mu} \cdot \frac{\partial \varphi}{\partial x_\mu} + \frac{\partial \varphi^*}{\partial x_\mu} \cdot \frac{\partial \psi}{\partial x_\mu} + M^2 (\psi^* \varphi + \varphi^* \psi)\right] \tag{14}$$

$$j_\mu(x) = \frac{i e}{2}\int d^4r \left[\psi^* \left(\overrightarrow{\frac{\partial}{\partial x_\mu}} - \overleftarrow{\frac{\partial}{\partial x_\mu}}\right) \varphi + \varphi^* \left(\overrightarrow{\frac{\partial}{\partial x_\mu}} - \overleftarrow{\frac{\partial}{\partial x_\mu}}\right) \psi\right] \tag{15}$$

and a similar expression for $T_{\mu\nu}$. These expressions are quite analogous to the corresponding expressions for the local scalar field except for the facts that they contain symmetrized products of a Yukawa quantity by a Fierz quantity and are integrated over the internal structure variables. The densities satisfy the continuity equations

$$\partial_\mu j_\mu(x) = 0\,, \quad \partial_\mu T_{\mu\nu}(x) = 0\,. \tag{16}$$

The integrated quantities

$$Q = \int d^3x \, j_0(x)\,, \quad P_\mu = \int d^3x \, T_{\mu 0}(x) \tag{17}$$

*) It should be noticed that the integral over the internal structure variables of the Yukawa field yields the lowest term of the decomposition (12′) $\sqrt{N} \int d^4r \;\; \psi(x, r) = \varphi(x)$.

**) The quantities to be varied independently are the components of the local tensor fields appearing in the decomposition (12).

are nothing else but an infinite sum of the corresponding quantities for the separate local tensor fields[5]).

Thus, in the interaction-free case, the bilocal field theory can be regarded to be only a convenient shorthand for infinite sets of local fields with higher and higher spins. However, in the next Section we shall show that, by taking into account interaction, the bilocal theory will be no more equivalent to the traditional local theory of particles with arbitrarily high spins but will exhibit new features of a specifically non-local character.

4. The Problem of Interaction

Assuming a bilocal scalar field as a model of "nucleon" and introducing a local scalar field $B(x)$ as a model of "pion" we are aiming at a formulation of a theory of interaction derivable from a variational principle with an action functional

$$W = W^{(0)} + W' \tag{18}$$

where $W^{(0)}$ is identical with W given by (14) while W' should correspond to the expression of Yukawa type

$$g \int d^4x \, \varphi^*(x) \, B(x) \, \varphi(x) \, . \tag{19}$$

While setting up a bilocal expression corresponding to (19) one should take into account that the bilocal field quantities are matrices satisfying the matrix multiplication law. Thus, a natural generalization of (19) will be the trace of the corresponding matrix

$$W' = \frac{g}{2} \int d^4x' \, \langle x' \mid \psi^* \, B \, \varphi + \varphi^* \, B \, \psi \mid x' \rangle \tag{20}$$

where, in analogy with the expressions (14) and (15), symmetrized products of Yukawa field quantities by Fierz field quantities have been introduced. Since B is assumed to be a local field, it is represented by a diagonal matrix

$$\langle x' \mid B \mid x'' \rangle = B(x') \, \delta^{(4)}(x' - x'') \tag{21}$$

hence

$$W' = \frac{g}{2} \int d^4x' \int d^4x'' \, (\langle x' \mid \psi^* \mid x'' \rangle \, B(x'') \, \langle x'' \mid \varphi \mid x' \rangle + \\ + \langle x' \mid \varphi^* \mid x'' \rangle \, B\,(x'') \, \langle x'' \mid \psi \mid x' \rangle) \, . \tag{20'}$$

Rewriting this expression in terms of the (x, r)-variables we get

$$W' = \frac{g}{2} \int d^4x \int d^4r \, B\Big(x - \frac{r}{2}\Big) \, \big(\psi^*(x, r) \, \varphi(x, r) + \varphi^*(x, r) \, \psi(x, r)\big) \, . \tag{20''}$$

It is just the circumstance that the field B appearing in (20″) is not taken at the central point x but at a shifted point $x - r/2$ that means a non-local character of interaction distinguishing qualitatively the bilocal interaction from the local one*). The non-local character of interaction is a direct consequence of the fact that the bilocal field quantities are matrices satisfying the matrix multiplication law. A modification of (20″) consisting in putting $B(x)$ instead of $B(x \pm r/2)$ would mean going over to a local interaction but such a modification would be quite unnatural and would violate the spirit of bilocality.

The bilocal field quantities being matrices (or operators) even before the usual field quantization is performed means that the transition from local to bilocal theory is already a quantization, although it is a quantization of a quite different type. The usual field quantization (second quantization) converts the theory of a single particle into the theory of an infinite number of particles whereas the "bilocalization" converts the theory of a single field into a theory of an infinite number of fields. The usual quantization is characterized by HEISENBERG's uncertainty relations (which may be interpreted as a non-localizability in the phase space) whereas the "bilocalization" is characterized by a non-localizable interaction (in the ordinary space). Both types of quantization lead to some resignations: ordinary quantization leads to a resignation of a simultaneous, exact knowledge of canonically conjugated quantities whereas "bilocalization" leads to a resignation of microcausality.

5. Equivalence with a Formfactor Theory

Going over to the Fourier transforms of the field quantities we get

$$W' = \frac{g}{8\pi^2}\iiint d^4p\, d^4q\, d^4r\, \big(\delta(p\,r)\,\delta(r^2 l^{-2} + p^2 l^2)\, e^{(i/2)qr} + \\ + \delta(q\,r)\,\delta(r^2 l^{-2} + q^2 l^2)\, e^{-(i/2)pr}\big) \times \big(B(p-q) + B^*(q-p)\big)\,\varphi^*(p,r)\,\varphi(p,r)\,. \tag{22}$$

Developing the bilocal field quantities according to (12) we obtain

$$W' = \frac{g}{4\pi^2}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\iint d^4p\, d^4q\, F(p,q)_{\mu_1\ldots\mu_n\nu_1\ldots\nu_m}\big(B(p-q) + B^*(q-p)\big) \times \\ \times\, \varphi^*_{\mu_1\ldots\mu_n}(p)\,\varphi_{\nu_1\ldots\nu_m}(q) \tag{23}$$

where

$$F(p,q)_{\mu_1\ldots\nu_m} = \frac{1}{2}\,\big(N(p)\,N(q)\big)^{(1/2)}\,\big(\Phi(p,q)_{\mu_1\ldots\nu_m} + \Phi(q,-p)_{\mu_1\ldots\nu_m}\big) \tag{24}$$

where

$$\Phi(p,q)_{\mu_1\ldots\mu_n\nu_1\ldots\nu_m} = \int d^4r\,\delta(p\,r)\,\delta(r^2 l^{-2} + p^2 l^2)\,e^{(i/2)qr}\, r_{\mu_1}\ldots r_{\mu_n}\, r_{\nu_1}\ldots r_{\nu_m}\, r^{-(n+m)} \tag{25}$$

or

$$\Phi(p, q_\mu)_{1\ldots\nu_m} = (-p^2 l^4)^{-(n+m)/2}\left(\frac{2}{i}\right)^{n+m}\frac{\partial^{n+m}\Phi(p,q)}{\partial q_{\mu_1}\ldots\partial q_{\mu_n}\,\partial q_{\nu_1}\ldots\partial q_{\nu_m}} \tag{26}$$

with the basic scalar function

$$\Phi(p,q) = \int d^4r\,\delta(p\,r)\,\delta(r^2 l^{-2} + p^2 l^2)\,e^{(i/2)qr}\,. \tag{27}$$

Going over to the x-representation of the tensor fields appearing in (23) the interaction term may be written also in the form

$$W' = g\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\iiint d^4x'\, d^4x''\, d^4x'''\, F(x'-x'', x''-x''')_{\mu_1\ldots\nu_m} \times \\ \times\, \varphi^*(x')_{\mu\ldots\mu_n}\, B(x'')\,\varphi(x''')_{\nu_1\ldots\nu_m} \tag{28}$$

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*) This shifting is brought about by the circumstance that we assumed the field quantities in the order $\varphi^*\, B\, \varphi$. The other possibility would be $(B\,\psi^*\,\varphi + \varphi^*\,\psi\, B)/2$ which yields again the expression $(20'')$ with the only difference that B would be taken at the point $x + r/2$ instead of $x - r/2$ which yields completely equivalent results.

where

$$F(x, y)_{\mu_1 \dots \mu_n \nu_1 \dots \nu_m} = \frac{2}{(2\pi)^8} \iint d^4p\, d^4q\, F(p, q)_{\mu_1 \dots \nu_m} e^{ipx} e^{iqy} . \tag{29}$$

In this way the bilocal scalar field theory with a bilocal interaction has been reduced to the theory of local tensor fields with a non-local interaction. This interaction couples all tensor fields with each other through the "pion" field and involves a set of tensorial formfactors (29).

6. A Discussion of the Formfactors

Let us evaluate the basic scalar function (27) involved in all tensorial formfactors. The integral in (27) may be represented as

$$\Phi(p, q) = \frac{1}{(2\pi)^2} \int d\alpha \int d\beta \int d^4r\, e^{i\alpha pr}\, e^{i\beta(r^2 l^{-2} + p^2 l^2)}\, e^{(i/2)\, qr} \tag{27'}$$

and computed by standard methods. The result is

$$\Phi(p, q) = -2\pi l^2 \frac{\sin l^2 (\Omega)^{1/2}}{(\Omega)^{1/2}} \operatorname{sgn} p^2 \quad \text{for} \quad \Omega > 0 \tag{30'}$$

$$\Phi(p, q) = 2\pi l^2 \frac{e^{-l^2(-\Omega)^{1/2}}}{(-\Omega)^{1/2}} \operatorname{sgn} p^2 \quad \text{for} \quad \Omega < 0 \tag{30''}$$

where Ω denotes the following invariant

$$\Omega = -\frac{1}{8} (p_\mu q_\mu - p_\nu q_\mu)^2 = \frac{1}{4} \left[(p\, q)^2 - p^2 q^2\right] . \tag{31}$$

The other tensors $\Phi(p, q)_{\mu 1 \dots \nu m}$ are easily obtained from the scalar $\Phi(p, q)$ by the process of differentiation according to (26).

Besides these functions the formfactors involve the normalizing factor

$$N(p, q) = (N(p)\, N(q))^{1/2} \tag{32}$$

which has to be considered with some care. This factor arises from the fact that we have normalized the bilocal field quantities according to (12) with $N(p)$ given by (13) in order to get correctly normalized expressions for such quantities as charge, and momentum for interaction-free fields (the local tensor fields appearing in the decompostion (12) are normalized in the usual way). The evaluation of the integral over d^4r n (13) yields a constant value $2\pi l^4$ only for time-like p (which is the case for free ields) but for space-like or light-like p this integral diverges. Hence, the formula (13) fs to be extended to

$$N(p) = \begin{cases} \dfrac{1}{2\pi l^4} & \text{for} \quad p^2 < 0 \\ 0 & \text{for} \quad p^2 \geqslant 0 . \end{cases} \tag{33}$$

Consequently, the basic formfactor is

$$F(p, q) = \begin{cases} \dfrac{\sin l^2 \Omega^{1/2}}{l^2 \Omega^{1/2}} & \text{if} \quad p^2 < 0 \quad \text{and} \quad q^2 < 0 \\ 0 & \end{cases} \tag{34a}$$

otherwise.

All tensorial formfactors $F(p, q)_{\mu 1 \ldots \nu m}$ vanish as well unless both p and q are time-like. In other words, the virtual states of the "nucleon" and its virtual resonant states with space-like or light-like momenta drop out from the interaction! This result is of so far reaching consequences that we have to examine carefuly whether it follows unavoidably from the fundamental concepts underlying the present formulation or not. In the next Section it will be shown that it is possible to extend the formalism so as to avoid a complete ruling out of space-like momenta in virtual states.

7. An Alternative Formulation

Hitherto we assumed tacitly that field quantities constitute a tool containing all the information about the properties of the physical system. But one may assume another viewpoint according to which not field quantities but observable densities are all important whereas field quantities play only an auxiliary role in construction of densities. If it is so, we have not to bother about a proper normalization of field quantities but should be concerned only with the problem of a proper normalization of the densities. Thus, the formula (12) can be used without the factor $(N(p))^{1/2}$ but, instead, when defining densities (being bilinear in the field quantities) a suitable normalizing factor can be attached directly to them. But, if a normalizing factor applies directly to densities and not to the field quantities separately, it does not need to be of the form of a product (32) but may be a more general function $N(p, q)$ of the two momenta.

Indeed, $N(p, q)$ can be chosen so that the interaction is extended analytically beyond the domain of time-like p and q unless the invariant Ω changes its sign to negative. In this case the basic formfactor is

$$F(p, q) = \begin{cases} \dfrac{\sin l^2 \Omega^{1/2}}{l^2 \Omega^{1/2}} & \text{for} \quad \Omega > 0 \quad (\text{including } \Omega \to +0) \\ 0 & \text{for} \quad \Omega < 0 \quad (\text{including } \Omega \to -0)\,. \end{cases} \tag{34b}$$

The two versions of the theory characterized by (34a) and (34b) will be called "version a" and "version b". Version b is equivalent to the following normalization: each term in the densities proportional to the product of field quantities $\varphi^*(p)\,\varphi(q)$ is to be normalized so that, instead of (24), we have

$$F(p, q)_{\mu_1 \ldots \nu_m} = \frac{1}{2}\left(N(p, q)\, \Phi(p, q)_{\mu_1 \ldots \nu_m} + N(q, -p)\, \Phi(q, -p)_{\mu_1 \ldots \nu_m}\right) \tag{35}$$

with

$$N(p, q) = \begin{cases} -\dfrac{\operatorname{sgn} p^2}{2\pi l^4} & \text{for} \quad \Omega > 0 \quad (\text{and } \Omega \to +0) \\ 0 & \text{for} \quad \Omega < 0 \quad (\text{and } \Omega \to -0)\,. \end{cases} \tag{36}$$

This prescription of normalization is still in formal agreement with the former definition (13) because the integral appearing in (13) can be regarded as a limes

$$N^{-1} = \int d^4 r \;\; \delta(p\, r) \;\; \delta(r^2\, l^{-2} + p^2\, l^2) = \lim_{q \to 0} \Phi(p, q) \tag{37}$$

according to (27). But, as is seen from (30), this limes transition is ambiguous: if we perform a transition $q \to 0$ from the domain $\Omega > 0$ the result is $-2\pi l^4 \operatorname{sgn} p^2$, but if from the domain $\Omega < 0$ the result is infinite. We have taken advantage of this ambiguity and assumed that the denominator in the formal expression

$$N\,\Phi(p, q) = \frac{\int d^4r\, \delta(p\, r)\, \delta(r^2\, l^{-2} + p^2\, l^2)\, e^{(i/2)\, q\, r}}{\int d^4r\, \delta(p\, r)\, \delta(r^2\, l^{-2} + p^2\, l^2)} \tag{38}$$

is to be regarded as limes of the nominator from this domain of Ω which is characteristic for the nominator, i.e. if p and q in the nominator yield $\Omega > 0$ the denominator is defined as

$$\lim_{\left(\substack{q \to 0 \\ \Omega \to +0}\right)} \Phi(p, q) = -2\pi\, l^4 \operatorname{sqn} p^2 \tag{39'}$$

but if p and q in the nominator yield $\Omega < 0$ the denominator is defined as

$$\lim_{\left(\substack{q \to 0 \\ \Omega \to -0}\right)} \Phi(p, q) = \infty \tag{39''}$$

which justifies the choice (36).

We believe that the two alternatives (version a and b) and only these two fit naturally into the framework of a bilocal and reciprocal theory. The third possibility consisting in extending the formfactors analytically to any values Ω would lead to hopeless divergences for $\Omega \to -\infty$.

8. Correspondence and Macrocausality

When discussing the correspondence between the bilocal theory and the traditional local theory of nuclear interactions one has to bare in mind that the local theory is divergent and therefore the correspondence, if it exists, cannot have a very precise meaning. The bilocal theory involves two arbitrary dimensional constants: the characteristic length l appearing in the formfactors and the bare mass of the "nucleon" M. Going over to the limit $l \to 0$ the basic scalar formfactor goes over into

$$F(p, q) \to \begin{cases} 1 & \text{for} \quad p^2 < 0 \quad \text{and} \quad q^2 < 0 \\ 0 & \text{otherwise,} \end{cases} \tag{40a}$$

or

$$F(p, q) \to \begin{cases} 1 & \text{for} \quad \Omega > 0 \quad \text{and} \quad \Omega \to +0 \\ 0 & \text{for} \quad \Omega < 0 \quad \text{and} \quad \Omega \to -0 \end{cases} \tag{40b}$$

in the versions a or b of the theory respectively. All the remaining tensor formfactors $F(p, q)_{\mu_1 \ldots \mu_n\, \nu_1 \ldots \nu_m}$ (obtained, according to (26), through the process of differentiation) vanish in both versions of the theory. In other words, in the limit $l \to 0$ the interaction between the "nucleon" and its resonant states as well as the interaction between any of the resonant states vanishes altogether so that they can be completely left out of consideration. This is an interesting result showing that there is no place for resonant states within the local field theory*). Resonant states appear to be intimately connected with the existence of the characteristic length and with the essentially non-local character of the theory.

Thus, in the limit $l = 0$ there exist only processes in which particles with the lowest spin value are involved. But even, these processes do not correspond very closely to those described by the traditional theory since, according to (40a) or (40b), the formfactor vanishes for a certain domain of the variables p and q. Consequently, the formfactor $F(x' - x'', x'' - x''')$ appearing in (28) is not a product of Dirac functions and the interaction remains non-local even in the limit $l = 0$. Even this fact does not destroy altogether a correspondence with the local theory: the interaction remains still practically local in the low energy limit where the nucleon may be regarded, with sufficient accuracy, as infinitely heavy. Indeed, in the limit $M \to \infty$ the momenta of virtual nucleons cannot become space-like in any order of the perturbation theory so that the domain for which $F(p, q)$ vanishes can be neglected. As the traditional local theory of nuclear interactions has proved to be successful only in the low energy limit ($M \to \infty$) we can be satisfied with such a restricted correspondence.

Of course, version b corresponds closer to the traditional theory since the domain in which $F(p, q) = 1$ surpasses the corresponding domain of the version a. In particular, in the version b the case $F(p, q) = 0$ begins to play a role in the order g^3 whereas in the version a it plays a role already in the order g^2 of the perturbation theory.

The correspondence breaks down in the case of energetic processes for which the Q value is at least of the order M (and in higher order corrections to processes with lower Q values). However, it is not at all obvious that it means a break down for $Q \simeq 900$ MeV since M means a bare mass of the nucleon whose value is not yet known. We suspect it to be considerably bigger than the mass of the physical (dressed) nucleon.

Let us discuss now the problem of causality. For theories with relativistic formfactors this problem was discussed by some authors[6])[7]) who found some criteria for securing macrocausality. In particular, the conditions of Chrétien and Peierls require, roughly speaking, the Fourier transforms of the formfactors to be slowly varying functions of the invariants like p^2 or Ω. Our formfactors do not satisfy such criteria since they are just discontinuous functions of p^2 and q^2 in the case a and discontinuous functions of Ω in the case b. Nevertheless, we claim that our formfactors do not violate the principle of macrocausality while the discrepancy between our results and those of Peierls et al. is partly due to the difference in the assumption of what is a "reasonable" wave packet, and partly in what sense the word "macrocausality" is to be used.

As it is just the discontinuity of the formfactor which seems to cause the trouble with causality, we may discuss, in the first instance, the case $l = 0$ and see whether the formfactors (40) give rise to violations of causality for macroscopic distances or not.

Consider the case a. Since in this case the Fourier transforms of the nucleonic field quantities with non-time-like momenta completely drop out of the interaction, the theory may be regarded to be a priori limited to the space of functions $[\varphi(x)]$ of the form

$$\varphi(x) = \int\limits_{(p^2 < 0)} d^4p \; \varphi(p) \; e^{ipx} \; . \tag{41}$$

Therefore, any reasonable wave packet within such a theory is that composed of

*) Indeed, local interactions of particles with higher spins are nonrenormalizable.

waves with time-like wave vectors. Obviously, it is possible to construct wave packets composed exclusively of such waves and limited in space and time separately. Let us denote by $\varphi(x, \xi)$ a packet which is essentially different from zero only for small $|\vec{x} - \vec{\xi}|$ and $|x_0 - \xi_0|$ separately. Putting such wave packets into the interaction term (in the limit $l = 0$) we get

$$\iint d^4x' \ d''x'' \ F(x' - x, x - x'') \ B(x) \ \varphi^*(x', \xi') \ \varphi(x'', \xi'') = 0 \tag{42}$$

unless ξ' and ξ'' are very close to x spatially and temporally because the formfactor of the form (40a) acts exactly as a product of Dirac delta functions upon wave packets composed merely of waves with time-like wave vectors. Indeed, in the case (40a) and (41) we have

$$F(x' - x, x - x'') = f(x' - x) \ f(x - x'') \tag{43}$$

and

$$\int dx' \ \varphi(x', \xi') \ f(x - x') = \varphi(x, \xi') \, . \tag{44}$$

This proves a strict causality of the version *a* in the limit $l = 0$.

In the case *b* the situation is not so simple since the formfactor (40b) does not restrict the space of functions $\varphi(x)$ in any way and we have to consider a more general class of wave packets. However, if we limit our considerations to low energy processes (which may be secured by limiting suitably the class of initial states) then the class of "reasonable" wave packets will be characterized by the fact that waves with wave vectors of magnitude $p^2 \simeq -m^2$ (where m is the mass of dressed nucleon) will be predominant in the Fourier analysis of the packet (otherwise violent processes would become probable contrary to our assumption). It means also that, if we have a reasonable wave packet limited in space and time separately, a cut off of its wings (with the values of p^2 very far from $-m^2$) will not smear out the packet considerably. But it is only for $p^2 > 0$ and $q^2 > 0$, i.e. very far from $-m^2$, that the case $\Omega < 0$ may occur, hence the formfactor (40b) does not produce a considerable smearing out of the packet. This secures a macrocausal character of processes whose Q value is small in comparison with m. For processes with higher and higher energy exchange an acausal behaviour will become more and more probable. But this is still compatible with a principle of causality in a less restrictive sense: it is quite conceivable that the magnitude of the region of space and time for which acausal behaviour comes into play increases together with the Q-value of the process in question. Such a view upon causality may be supported by the following argument: it is obviously impossible to approach with our measuring apparatus too closely to the very energetic events without exposing them to the danger of being destroyed by such violent processes. Thus, e.g. the following dependence of the region of acausality Δx upon the Q-value

$$\Delta x = \frac{Q}{m^2} \tag{45}$$

is still physically acceptable.

Hitherto we have considered the case $l = 0$. A transition to the case of a finite l does not change essentially the situation since hereby a smooth function $\sin l^2 \Omega^1|^2/ l^2 \Omega^1|^2$ is introduced and it is known that a modulation of wave packets by formfactors whose Fourier transforms decrease smoothly does not lead to any inadmissible smearing out. In this case the spatial and temporal magnitude of the packets will be increased over a region of the order l and l/c respectively.

Recapitulating let us state that the bilocal theory corresponds to the local one in the limits $l \to 0$, $M \to \infty$ whereby the resonant states come into play only for $l \neq 0$ i.e. constitute an essentially non-local effect. The bilocal theory does not violate the principle of causality. In the version *a* of the theory the acausal effects are limited to spatial regions of the order l and temporal regions of the order l/c (in the centre of mass system) no matter how energetic the processes are. In the version *b* the spatial and temporal regions of acausality increase with the energy involved in the process but also this alternative is physically acceptable. We cannot see decisive arguments against either of the two alternatives *a* or *b*. It remains to be seen which one fits better the experimental data.

9. Quantization of the Bilocal Field

Since the bilocal theory is reducible to a theory of (an infinite set of) local fields with a non-local interaction, the problem of field quantization may be also reduced to that of quantization of local fields with non-local interaction. This problem was solved by the present author[8]) several years ago so that only a brief sketch of the procedure will be presented at this place.

Contrary to the widely spread opinion we showed that not only a pure S-matrix formalism is possible in the case of a non-local interaction but it is also possible to consider states defined at any space-like hypersurface. The change of the state from σ_1 to σ_2 is to be described by the formalism starting from the action integral

$$W_{12} = \int_{\sigma_1}^{\sigma_2} d^4x\, L^{(0)}(x) + \int_{\sigma_1}^{\sigma_2} d^4x' \int_{\sigma_1}^{\sigma_2} d^4x'' \int_{\sigma_1}^{\sigma_2} d^4x'''\, L'(x', x'', x''') \tag{46}$$

where L^0. is the sum of the Lagrangians for the separate local fields $\varphi, \varphi_\mu, \varphi_{\mu\nu}, \ldots,$ and B of the same form as in the interaction-free case, while L' is identical with the integrand appearing in (28). Since the theory is derivable from a variational principle, the procedure of quantization is quite similar to that well known from the traditional local theory. Varying the separate local fields one gets the Lagrange equations. In consequence of these equations the variation of W_{12} is of a form of a difference of two integrals $F_1 - F_2$ over the hypersurfaces σ_1 and σ_2. These are linear in $\delta\varphi$. The coefficients of $\delta\varphi$ are canonically conjugated momenta. The surface integrals F are interpretable as infinitesimal operators generating a change of basis in the Hilbert space induced by the change of field quantities $\delta\varphi$. Herefrom follow the usual commutation relations between the field quantities and their canonically conjugated momenta on σ_1 and σ_2 (but not on any other intermediate hypersurface σ_3!). Since the commutation relations on both σ_1 and σ_2 are the same, there exists a unitary matrix U_{12} transforming the quantities on σ_1 into those on σ_2. The elements of this matrix are interpretable as probability amplitudes. In the limit $\sigma_1 \to -\infty$, $\sigma_2 \to +\infty$ the matrix U_{12} goes over into HEISENBERG's S matrix.

Thus, the procedure of quantization by means of variational methods is exactly the same as in the theory with local interaction. In particular, all quantities commute with each other if taken at different points on σ_1 (or σ_2), and the only non-vanishing commutators are those between field quantities and their conjugate momenta taken at the same point on σ_1 (or σ_2). Thus, there is no question of violation of causality on the hypersurfaces where measurements actually take place.

The differences in comparison with the local theory are: (i) the field equations are not independent of the domain restricted by the hypersurfaces σ_1 and σ_2, and (ii) since there is no additivity of the action integrals $W_{13} + W_{32} \neq W_{12}$, there is no multiplicative law for the transformation matrices $U_{12} \neq U_{13} \cdot U_{32}$ and there does not exist a time dependent Schrödinger equation.

The possibility of describing the development of the state vector in the course of time by means of a Schrödinger equation in the local theory was closely connected with the fact in the local theory there are possible measurements which do not disturb the measured system at all but only confirm in which state the system was just before the measurement took place (such are measurements of observables commuting with those whose eigenvalues characterized the state just before the measurement). In the non-local theory the situation is different: there exist no measurements which would not disturbe the state of the measured system. Any measurement taken at an intermediate hypersurface σ_3 situated between σ_1 and σ_2 would change the field equations and disturb the state. This is understandable because such a measurement necessarily brings about a localization of the fields on σ_3 (as they must commute exactly like local fields on any hypersurface where measurements actually take place). Therefore, by switching on additional measurement on σ_3 we change principally the situation and we are no more enrightened to use W_{12} but W_{13} and W_{32} one after the other.

The above considerations show that the influence of the conditions of measurement upon the measured system is more pronounced in the bilocal theory than it was in the local theory. This is understandable in view of the fact that transition from local to bilocal theory may be also regarded as quantization (in a special sense), and quantization always brings about new refinements in the theory of measurements.

Nevertheless, in view of the discussions of formfactors in Section 8, the difference between W_{12} and $W_{13} + W_{32}$ is practically small, at least if limiting our considerations to processes in which the energy exchange is not very high (in the version *b*), and in this sense the bilocal and quantized theory fulfills the requirements of macrocausality and correspondence with the local quantized field theory.

10. Outlook

The above considerations show that the bilocal field theory offers a promising possibility for a correct description of nuclear interactions. It accounts in a natural way for the existence of resonant states (the number of which is supposed to be infinite), it constitutes a progress in the problem of convergence (compare ref. 7) and does not contradict the postulate of macrocausality.

We assumed that the bare mass of nucleon and its resonant states is the same. It is to be expected that the non-local interaction will remove this degeneracy. It remains to be seen whether the bilocal interactions will give correct rations of the masses of the resonant states to the nucleonic mass. However, in order to get results comparable with experiment one has to take into consideration all strong interactions including other mesons and hyperons. In order to get a mass spectrum with increasing masses it seems to be necessary the self mass corrections to be negative $(-\delta m)$, and their absolute values to decrease with increasing spin:

$$m_{1/2} = M - \delta m_{1/2} < M - \delta m_{3/2} = m_{3/2} < \cdots < M.^{*})$$

Another interesting problem is whether the other strongly interacting particles form similar families as the nucleon and its resonant states. The existence Y_0^* with mass 1520 MeV and spin-parity assignements $3^-/2$ indicates that Λ_0 is a lowest member of a family all members of which are characterized by isospin and strangeness equal to these of Λ_0. The existence of Y_1^* may be regarded as a hint that there exists also a Σ-family, though nothing is known yet about spin and parity of Y_1^*. The existence of the fameous resonance (3/2, 3/2) presents a special difficulty: we cannot understand why there does not appear a resonant state with isospin 3/2 and spin 1/2 to provide a starting point for a family with isospin 3/2? Maybe, we encounter here an anomaly of the mass spectrum so that the mass of (3/2, 3/2) is lower than that of (3/2, 1/2)? If it were so then the new discovered resonant state with mass 1650 MeV could be the member of this family with the lowest spin value 1/2.

The problem of existence of bilocal meson families is quite open. At any rate, neither ω is the second member of the η-family nor ϱ is the second member of the π-family because of wrong parity assignments.

Let us conclude with the following remark: there are indications that for high energy processes nucleon becomes bigger and more transparent. This seems to fit well with the bilocal theory. The increase of nucleon size may be understood because more and more resonant states come into play while the transparency may be accounted for by the bilocal formfactors.

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*) *Note added in the proofs.* The other alternative is to assume the rest mass M to be not a parameter but a function of the spin S, e.g. $M^2 = l^{-2} S^{1/2}$ while the field masses δm should be regarded as small corrections.