L'Enseignement Mathématique, t. 36 (1990), p. 115-141

TOPOLOGICAL SERIES OF ISOLATED PLANE CURVE SINGULARITIES

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ABSTRACT. For plane curve singularities, a topological definition of series of isolated singularities, based on the Milnor fibration, is given. Several topological invariants, including the spectrum, are computed.

1. INTRODUCTION

Let $f: (\mathbf{C}^2, 0) \to (\mathbf{C}, 0)$ be a plane curve singularity, in other words, let f be an element of the ring of convergent power series $\mathbf{C}\{x, y\}$. Assume $f \neq 0$. Because $\mathbf{C}\{x, y\}$ is factorial, we can write $f = f_1^{m_1} \cdots f_r^{m_r}$ with all f_i irreducible and whenever $i \neq j$, there is no unit u with $f_i = uf_j$. The *branches* of f are the curves $f_i(x, y) = 0$.

It is well-known that for $\varepsilon > 0$ small, the intersection $L = f^{-1}(0) \cap S_\varepsilon^3$ of the curve $X: f = 0$ and a small 3-sphere of radius ε is a *link*, consisting of r components corresponding to the branches of f, and that this link determines the topological type of f (or of X). Moreover, the map $f/|f|: S_\varepsilon^3 \backslash L \to S^1$ is a fibration, called the *Milnor fibration*.

It is natural to consider L as a *multilink*, i.e. a link with integral multiplicities assigned to each component. We use the notation $L = m_1 S_1 + \cdots + m_r S_r$, where $S_i = f_i^{-1}(0) \cap S_\varepsilon^3$. These multiplicities reflect in the behaviour of the Milnor fibre F (i.e. a typical fibre of the Milnor fibration, which is a Seifert surface bounded by L) near S_i: F approaches S_i from m_i directions (see [EN]).

The Milnor fibration is important in our discussion of *topological series of isolated singularities*. A striking feature of Arnol'd's series A, D, E, J, etc. (see [AGV]), is that they are somehow related to a non-isolated singularity. For example: $D_k: xy^2 + x^{k-1}$ is related to $D_\infty: xy^2$ and $Y_{r,s}: x^2y^2 + x^{r+4} + y^{s+4}$ to $Y_{\infty,\infty}: x^2y^2$. This relationship is still not completely understood.

In this paper we give (for plane curve singularities) a topological definition of series (definition 3.1), as follows. A singularity belongs to the topological