Introduction

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KNOTTING CODIMENSION 2 SUBMANIFOLDS LOCALLY

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INTRODUCTION

Let L be a connected oriented n-dimensional closed manifold smoothly embedded in a connected oriented $(n+2)$-dimensional closed manifold M, and let K be an oriented n-dimensional smooth knot in the oriented S^{n+2}. Then we consider the connected sum $(M, L) \sharp (S^{n+2}, K)$. In other words, we knot L locally using K. It yields another embedding of L in M; however, it does not always give a new embedding. In fact, the lightbulb theorem says that the connected sum of $(S^2 \times S^1, \{*\} \times S^1)$ with any knot in S^3 is always equivalent to the original embedding. Moreover, by the prime decomposition theorem for knots in 3-manifolds [My], $(S^2 \times S^1, \{*\} \times S^1)$ is essentially the only embedding of a circle with the above property. Litherland [Li] has generalized the lightbulb theorem to the higher dimensional cases. In the appendix of [V], Viro exhibits an example of a 2-knot whose connected sum with the standard projective plane in S^4 does not change the isotopy type of the projective plane. (See also [La].)

The purpose of this paper is to study under what conditions this phenomenon occurs (or does not occur). The first named author [Ms] studied this problem when the codimension is greater than 2.

Put it in another way. Let $\mathcal{K}_n$ be the set of isotopy classes of oriented n-knots diffeomorphic to S^n in the oriented S^{n+2}. The set forms an abelian monoid under connected sum for pairs. Analogously to the inertia group of a manifold, we define

$$I(M, L) = \{(S^{n+2}, K) \in \mathcal{K}_n \mid (M, L) \sharp (S^{n+2}, K) = (M, L)\}$$

where $=$ in the parenthesis indicates that there is an orientation preserving diffeomorphism of pairs. The set forms a submonoid of $\mathcal{K}_n$ and describes the effect of knotting L locally. We are also concerned with the following intermediate submonoid

$$I_0(M, L) = \{(S^{n+2}, K) \in I(M, L) \mid (M, L) \sharp (S^{n+2}, K) \equiv (M, L)\}$$

where $\equiv$ indicates that there is an orientation preserving diffeomorphism of pairs which is concordant to the identity map as a diffeomorphism of the ambient space M.

Our results suggest that $I(M, L)$ and $I_0(M, L)$ depend only on the order of a meridian of L in $\pi_1(M-L)$ or $H_1(M-L; \mathbf{Z})$. Roughly speaking, according as the order is infinite, 1, or $p\,(1<p<\infty)$, they can be distinguished by (at least) these three types:

Type 1 $I(M, L) = \{0\mathrm{N}\}$,

Type 2 $I(M, L) = \mathscr{K}_n$, $I_0(M, L) = \ker \sigma$,

Type 3 $\{0\} \subsetneq I(M, L) \subsetneq \mathscr{K}_n$, $\{0\} \subsetneq I_0(M, L) \subsetneq \ker \sigma$,

(see section 4 for $\sigma(S^{n+2}, K)$).

We refer the reader to 1.1, 2.6, 3.4, 5.1, 5.2, and 5.8 for the precise statement.

This paper consists of five sections. In Section 1, we deduce a necessary condition for $I_0(M, L)$, which is valid for any (M, L). We treat type 1 in Section 2. Type 2 is discussed in Sections 3, 4 and type 3 is discussed in Section 5. We will find that type 3 is closely related to the generalized Smith conjecture.

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§ 1. General remarks on $I_0(M, L)$

It is known (and it is easily verified) that the signature of a Seifert surface of an oriented n-knot K in S^{n+2} is independent of the choice of a Seifert surface; so it is an invariant of the oriented knot K. The invariant is called the signature of the knot K and denoted by $\mathrm{Sign}(S^{n+2}, K)$. We note that $\mathrm{Sign}(S^{n+2}, K)$ is trivially zero unless $n+1 \equiv 0\ (4)$.

As is seen in Section 3, there is a pair (M^{n+2}, L^n) such that $I(M, L) = \mathscr{K}_n$ for any $n \geqslant 3$. In contrast, we can deduce a necessary condition for $I_0(M, L)$ which holds for any pair (M, L).

THEOREM 1.1. *If* $(S^{n+2}, K) \in I_0(M, L)$, *then* $\mathrm{Sign}(S^{n+2}, K) = 0$.

Proof. Let V be a Seifert surface of K. Since $S^{n+2} = \partial D^{n+3}$, we can push the interior of V into the interior of D^{n+3} so that V is transverse to S^{n+2}. This yields an oriented pair (D^{n+3}, V) having (S^{n+2}, K) as the boundary.