An effective procedure for combining actions

An effective Procedure for Combining Actions

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Summary

The combinations which may be decisive for the dimensioning of cross-sections can directly be determined by vectorially adding the action effects within the *N/M*–diagram in the sequence of decreasing load eccentricity. The simplified combinations which are allowed for building structures are not easier to be applied. Besides they should be dropped because they may give more unfavourable as well as more favourable results. Computer programs should present the results in graphics which can easier be understood.

1. Introduction

A lot of criticism against the Eurocodes arises from the preconceived idea, that the verification of the general or fundamental combination rule is too complicated. Therefore simplications of the general rule as given in Eurocode 1 by equation (9.10) are deemed to be absolutely necessary. For this reason simplified rules for building structures are given by equations (9.13) and (9.14). These equations are the equations (2.7(a)) and (2.8(a)) and (2.8(b)) in Eurocode 2, "Design of Concrete Structures". In concrete structures normal forces may act favourable or unfavourable, especially with respect to the required amount of reinforcing steel. This fact also complicates the situation as it is.

From Table 1 it can be seen that the total number of possible combinations p really increases very much with the increasing number q of variable actions which are independent from each other. It can also be seen that this number p is significantly reduced by the simplification only in cases with 3 and more variable actions. The remaining number of possible actions still remains too great. It can be concluded that the reduction of the number of possible combitions is not yet an effective simplification. To determine the decisive combination for cross-section design with 3 variable actions from the totality of 16 simplified combinations is not yet more comfortable than to determine them from 26 combinations. For practical purposes a more pronounced reduction is aspected when speaking of a simplification or an effective procedure has to be applied, in order to concentrate on the decisive combinations.

For the dimensioning of reinforced cross-sections an effective procedure is to combine the combination of actions with the determination of the required reinforcement. This can be done with computer programs and in the same way by using design charts or other design tools. In both cases the actions are added as vectors within an *N/M*-diagram. The boundary with the most unfavourable and decisive combinations is directly obtained by adding the actions in the sequence of decreasing load eccentricity. This is shown and can easily be understood by giving an example and by explaining the results. For this aim the column shown in Fig. 1 with 3 variable actions is analysed.

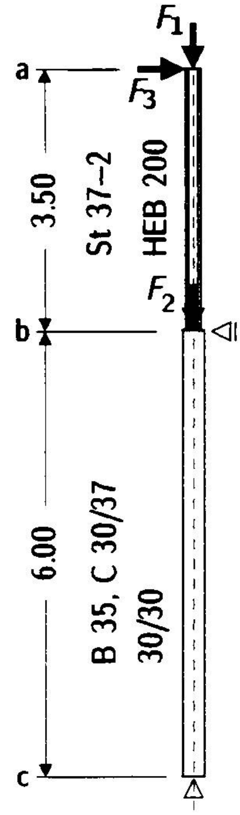
Combination of actions acc. to EC 1, ch. 9.4		q = 0	1	2	3	4	5
Ch. 9.4.2, eq. (9.10)							
$\Sigma\gamma_G \cdot G_k + 1.5\,(Q_{k,1} + \Sigma\psi_{0,i}\,Q_{k,i})$	$p = 2 + q \cdot 2^q$	2	4	10	26	66	162
$i > 1$	$r \leq q\,(q+3)/2$	–	2	5	9	14	20
Ch. 9.4.5, Simplified Verifications for Building Structures, eq. (9.13) or (9.14)							
(9.13) $\Sigma\gamma_G \cdot G_k + 1.5\,Q_{k,1}$	$p = 2^{q+1}$	(2)	(4)	8	16	32	64
(9.14) $\Sigma\gamma_G \cdot G_k + 1.35\,\Sigma Q_{k,i}$ $i > 1$	$r \leq 3\,q$	–	(2)	6	9	12	15

Table 1. Combination of actions for ultimate limit state design in persistent or transient design situations for q *different variable actions, independent from each other. Numbers* p *of all the possible combinations and number* r *of the reduced set of combinations which have to be considered for cross section dimensioning. The corresponding equations to the cited ones from Eurocode 1 are in Eurode 2 eq. (2.7(a)) and eq. (2.8(a) and (b)).*

2. An Extension of the Model Column Method from Eurocode 2

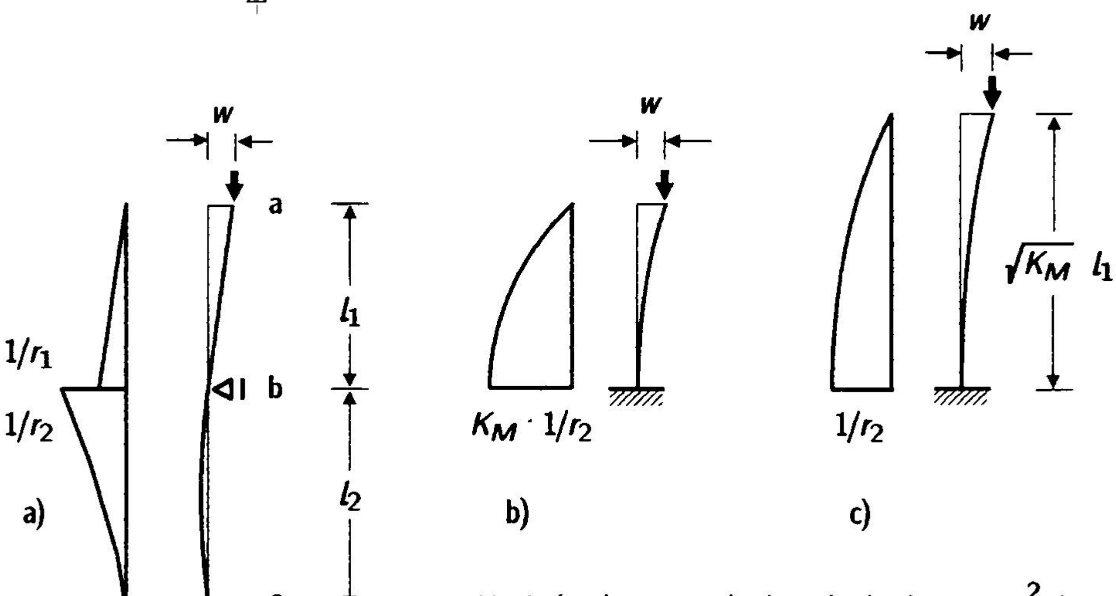
The well known effective column length for buckling design purposes of an isolated element is determined from the equivalence of the buckling load of the real system and of the isolated element. This fundamental idea can also be applied for using the model column method for other columns than real cantilever columns or pin ended columns with the corresponding effective buckling length. The equivalent model columns have to be determined with respect to equal effects of structural deformations.

For the chosen example in Fig. 1, consisting of a combination of a pin ended reinforced concrete column and a cantilever steel column, the two different model columns b) and c) in Fig. 2 can be derived for the real column system a). In all the three systems the same structural deflection w at the top has to occur so that the same second order effects result for dimensioning the reinforced cross-section b in span 2 of the concrete column.



F_i	action	$\psi_{0,i}$	$\gamma_{F,\ inf}$	$\gamma_{F,\ sup}$	$F_{k,i}$ [kN]
G_k	permanent				
1	dead load	–	1.0	1.35	70
2	dead load	–	1.0	1.35	260
Q_k	variable				
1	snow load	0.7	–	1.5	30
2	imposed load	0.8	–	1.5	115
3	wind load	0.6	–	1.5	14.6

Fig. 1. System and dimensions of a column, materials and combination factors $\psi_{0,i}$ as given in the german National Application Document, as well as partial safety factors $\gamma_{F,\ inf}$ and $\gamma_{F,\ sup}$ for favourable and unfavourable effects of the characteristic values of the permanent and variable actions G_k and Q_k.



$$w = K \cdot l_1\,(l_1 / r_1 + l_2 / r_2) = (4 / 10)\, K_M \cdot l_1^{\,2} / r_2$$

Fig. 2. Equivalent model columns for dimensioning the section b *in span 2,*
a) column as given,
b) model column with the same length l_1 and modified curvature $K_M \cdot 1/r_2$,
c) model column with the same curvature $1/r_2$ and modified column length $\sqrt{K_M}\ l_1$.

The expression for the top deflection w can be seen from Fig. 2. The model column b) in Fig. 2 with the same column length l_1 and the modified curvature $K_M \cdot 1/r_2$ is taken here for the application of a computer program, the results of which are shown in Fig. 3 to 6. The model column c) with the same curvature $1/r_2$ and the modified column length $\sqrt{K_M}\ l_1$ allows to use standard design charts.

Instability of statically determined slender columns occurs when yielding in the most stressed cross-section happens, which in Fig. 2 is section b. The top deflection w can directly be calculated from the curvatures $1/r_1$ and $1/r_2$ at yielding.

For the steel column the influence of the longitudinal force N is unimportant and by not considering it the overestimation of the curvarture at yielding is very small.

$$\begin{aligned} r_1 &= (h/2) / \epsilon_{ay} = 0.5\ h \cdot E_a / f_{ay} \\ &= 0.5 \cdot 0.2 \cdot 210\,000 / 240 && = 87.5 \text{ m.} \end{aligned}$$

For the reinforced concrete column r_2 can be determined as given by eq. (4.72) in Eurocode 2 with the coefficient $K_2 = f(N_d, A_s) = 1$ because of $|N_d| < N_{bal}$.

$$\begin{aligned} r_2 &= 0.9 \cdot d / (2 \cdot \epsilon_{yd}) \\ &= 0.45\ d / (0.0025/1.15) = 207\ d \\ &= 207 \cdot 0.255 && = 52.8 \text{ m.} \end{aligned}$$

Assuming triangular diagrams for the curvatures, which in this example is on the safe side for the concrete column because of the limited moment magnification, the top deflection is obtained acc. to the corresponding expression in Fig. 2 with the coefficient $K = 1/3$,

$$\begin{aligned} w &= (1/3) \cdot 3.50\ (3.50/87.5 + 6.00/52.8) \\ &= 0.179 \text{ m} = (4/10) \cdot K_M \cdot 3.50^2/52.8, \end{aligned}$$

which then gives the model column coefficient K_M for this example as

$$K_M = 1.93.$$

3. Notes to the Combination of Actions and Dimensioning

For the combination of actions together with the dimensioning of reinforced cross sections the computer code EKoB was written. It allows to consider all the possible combinations acc. to eq. (2.7(a)) and all the simplified combinations acc. to eq. (2.8(a)) and (2.8(b)). The results as given in Fig. 3 can be limited to the most important combinations. The second order analysis of a column is transformed to cross section design acc. to ch. 4.3.5.6.3 (b) of Eurocode 2. The total design moment $M_{Sd,tot}$ is the sum of the first order moment $M_{Sd,0}$ augmented by $M_{Sd,a}$ allowing for the effect of imperfections and by $M_{Sd,2}$ allowing for the effect of structural deformations, the socalled second order effect,

$$M_{Sd,tot} = M_{Sd,0} + M_{Sd,a} + K_2(N_{Sd}, A_s) \cdot M_{Sd,2}\,.$$

The characteristic values of $M_{Sk,0}$, $M_{Sk,a}$ and $M_{Sk,2}$ for $K_2(N_{Sd}, A_s) = 1$ can be seen in Fig. 3. With respect to the cross section b there are no effects of imperfections and of structural deformations within the action effects from G_2, Q_2 and Q_3.

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EKoB (C)94 Quast - Einwirkungen, Kombinationen, Bemessung nach EC 2-1-1
                    Actions, Combinations, Dimensioning acc. to
---------- Worked example: Concrete column with steel column ----------
      K.2 G.k,1   G.k,2   Q.k,1   Q.k,2   Q.k,3 cross-section : R2 - 15
-----------------------------------------------------------------------
N   kN   -70.00 -260.00  -30.00 -115.00    0        conrete    : C 30/37
M.0 kNm    0       0       0       0     51.10   reinforcing : BSt 500
x    m    3.50     0      3.50     0       0        model column method
M.a kNm   1.23     0      0.52     0       0        alfa.a     :  1/200
M.2 kNm  12.54     0      5.37     0       0      EC 2-1-1, 4.3.5.6.3 b)
 = 1.930*|N|*x*x / (517.5*d) ; without creep effects.
e.tot/h    -      0.14    0.66     0      >1E6
psi.0      -       -      0.70    0.80    0.60           b    : 0.300 m
gam.F,sup 1.35    1.35    1.50    1.50    1.50           h    : 0.300 m
gam.F,inf 1.00    1.00     -       -       -             d    : 0.255 m
========== Only decisive combinations of all possible ones ===========
26 fundamental combinations, EC 2-1-1, Gl.(2.7(a)) N.Sd     M.Sd     A.s
 1  1.000 1.00    1.00    1.05     -      1.50 -361.50   96.61   12.43
 2  1.000 1.00    1.00     -       -      1.50 -330.00   90.42   11.63
 3  1.000 1.35    1.35    1.05     -      1.50 -477.00  101.43   11.60
16 simplified combinations,  EC 2-1-1, Gl.(2.8(a) oder (b))
 1  1.000 1.00    1.00     -       -      1.50 -330.00   90.42   11.63
 2  1.000 1.00    1.00    1.35     -      1.35 -370.50   90.72   10.96
 3  1.000 1.35    1.35     -       -      1.50 -445.50   95.23   10.71
=======================================================================
req A.s =  12.43 cm2,    min A.s (0.3% / 0.15 nue) =   2.70 /   2.24 cm2
M.tot   =  1.232 M.1,    eps.c / eps.s = -3.50 /  5.73 mm/m, x/d = 0.379
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Fig. 3. Display of the dimensioning of the reinforced concrete column from Fig. 1. According to Fig. 2 the column analysis has been transformed to cross section design by adopting the model column method with the coefficient $K_M = 1.93$.

In this example longitudinal forces N act favourably. The decisive fundamental combination is therefore $1.00\ G_k + 1.5\ Q_{k,3} + 1.5 \cdot 0.7\ Q_{k,1}$. The dominant variable action is Q_3. The variable action Q_2 acts favourably and is therefore not included.

The decisive simplified combination is $1.00\ G_k + 1.5\ Q_{k,3}$. It is more unfavourable than the simplified combination $1.00\ G_k + 1.35\ Q_{k,1} + 1.35\ Q_{k,3}$. It requires 11.63 cm^2 reinforcing steel, which are only 94% of the reinforcing steel of 12.43 cm^2, which is required for the fundamental combination.

The results are graphically shown within a detail of the N_d / M_d-diagram. For better clarity all the 26 fundamental combinations are shown in Fig. 4 whereas all the 16 simplified combinations are shown in Fig. 5. The figures show the design values $\gamma_G\ G_k$ and $\gamma_Q\ Q_k$ of the action effects as vectors. On the corresponding vectors of the variable action effects also the design values of the combination values $\gamma_Q\ \psi_{0,i}\ Q_{k,i}$ are marked by smaller quadrats. The starting point of all the vectors of the variable action effects is the value $1.00\ G_k$ of the permanent action. The part $0.35\ G_k$, which has to be added in cases when being unfavourable, appears like a variable action.

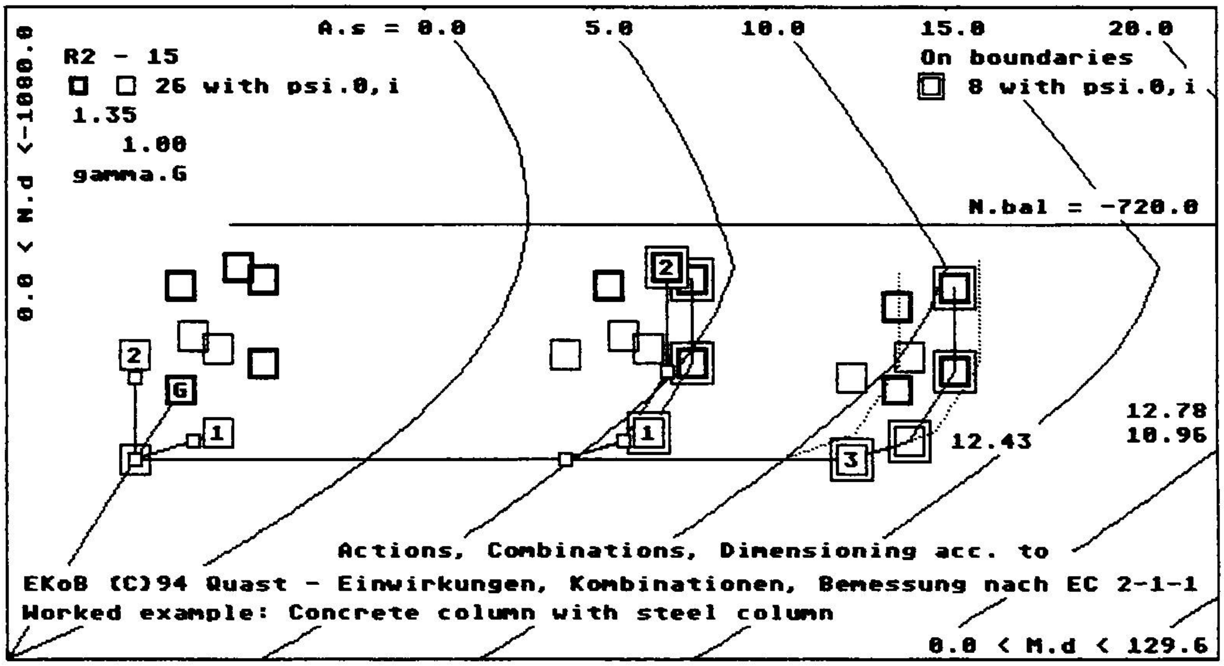

Fig. 4. Representation of the 26 combinations acc. to EC 1, eq. (9.10), within a detail of the N_d / M_d –diagram. For the vectors of the variable actions 1 to 3 the design values $\gamma_Q \cdot Q_{k,i}$ and the design values of the combination values $\gamma_Q \cdot \psi_{0,i} \cdot Q_{k,i}$ are marked. The 8 combinations on the boundaries, which have to be considered for dimensioning the cross section, are marked by double quadrats.

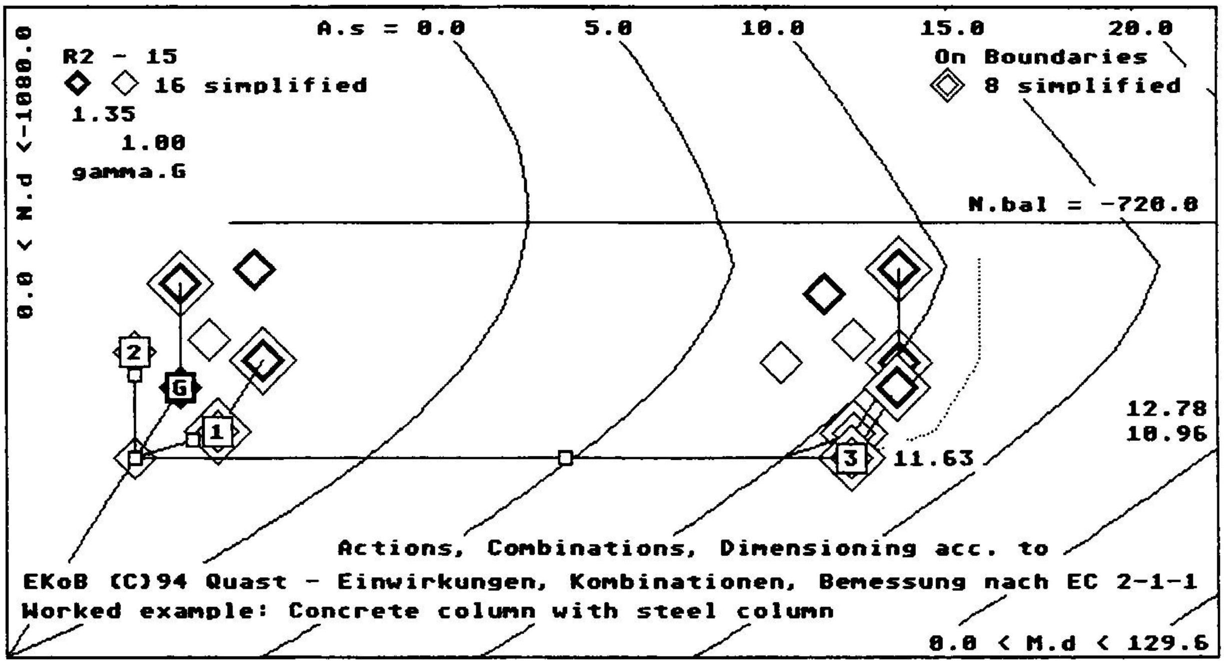

Fig. 5. Representation of the 16 simplified combinations acc. to EC 1, eq. (9.13) and (9.14), within a detail of the N_d / M_d –diagram. The 8 combinations on the boundaries, which have to be considered for dimensioning the cross section, are marked by double diamonds.

The decisive combination in Fig. 4, giving the maximum required reinforcement area, is a point on the boundary which is formed by the polygone

$$1.0\ G_k + \mathbf{1.5\ Q_{k,3} + 1.5\ \psi_{0,1}\ Q_{k,1} + 0.35\ G_k + 1.5\ \psi_{0,2}\ Q_{k,2}}\ .$$

This boundary is formed by the permanent action effect, the dominant variable action effect, the other variable action effects and the 0.35fold permanent action effect in the sequence of decreasing load eccentricity $e_d = M_d\,/\,|N_d|$. Which point on this boundary gives the greatest required reinforcement depends from the greater or smaller inclination of the $N_d\,/\,M_d$-line, as it is the case for different arrangements of the reinforcement in the cross section, for example at four sides instead of only two sides as in Fig. 4 to 6. Especially this point needs not be the point with the greatest axial force, nor the point with the greatest bending moment, nor the point with the greatest load eccentricity, as can be seen from Fig. 4.

Within the two other polygones the corresponding dominant actions are Q_1 and Q_2. Adding the action effects in the sequence of decreasing load eccentricity results in the polygones:

$$1.0\ G_k + 1.5\ \psi_{0,3} \cdot Q_{k,3} + \mathbf{1.5\ Q_{k,1} + 0.35\ G_k + 1.5\ \psi_{0,2}\ Q_{k,2}}$$

and

$$1.0\ G_k + 1.5\ \psi_{0,3} \cdot Q_{k,3} + 1.5\ \psi_{0,1}\ Q_{k,1} + 0.35\ G_k + \mathbf{1.5\ Q_{k,2}}\ .$$

The points which are possible for dimensioning are marked by double quadrats. The first of these points is the point of the corresponding dominant action and then all the following ones. These alltogether 8 points are emphasized in the above given expressions.

In this example Q_3 is not the dominant action because it yields the most unfavourable action effect, as it can clearly be seen from Fig. 4. Q_3 is in this example the dominant action because its reduction $\gamma_Q\,(1 - \psi_{0,i})\ Q_{k,i}$ when not being the dominant action is the most unfavourable one compared with the possible reductions of the other variable actions Q_1 and Q_2. These possible reductions of the variable action effects are the distances between the smaller mark and the end of the vectors of these variable action effects. When graphically adding the action effects the dominant action i can clearly be detected as that action which has the most unfavourable part $\gamma_Q\,(1 - \psi_{0,i})\ Q_{k,i}$.

From the 16 simplified combinations in Fig. 5 the decisive one is within the polygone $1.0\ G_k + \mathbf{1.5\ Q_{k,3} + 0.35\ G_k}$, which is one possible acc. to eq. (2.8(a)). The two remaining polygones for combinations acc. to eq. (2.8(a)) $1.0\ G_k + \mathbf{1.5\ Q_{k,1} + 0.35\ G_k}$ and $1.0\ G_k + 0.35\ G_k + \mathbf{1.5\ Q_{k,2}}$ are not decisive. Also the simplified combination acc. to eq (2.8(b)) $1.0\ G_k + 1.35\ Q_{k,3} + \boldsymbol{1.35\ Q_{k,1} + 0.35\ G_k + 1.35\ Q_{k,2}}$ does in this example not give the maximum amount of reinforcement. All the 8 possible points are in Fig. 5 marked by double diamonds and emphasized in the expressions in top.

The number r of the reduced set of combinations which form the possible polygones and which are in general sufficient to be considered and which have to be considered only then, if the dominant variable action is not known in before, are given in Table 1. There is nearly no difference between the fundamental combinations and the simplified ones. Knowing that Q_3 is the dominant variable action in this example, the corresponding polygone in Fig. 4 needs to consider 4 combinations only from the totality of 26, whereas the corresponding polygones in Fig. 5 have to consider 2 plus 3 combinations from the totality of 16. It can be concluded that the simplified verification of the combination of actions for building structures acc. to eq. (9.13) and (9.14) in Eurocode 1, which are the eq. (2.8(a)) and (2.8(b)) in Eurocode 2, is not really simpler. It should therefore be taken away. The advantage would be, that equivocal and contradictory dimensionings are avoided and that it becomes very obvious, that a comprehensible procedure has to be applied.

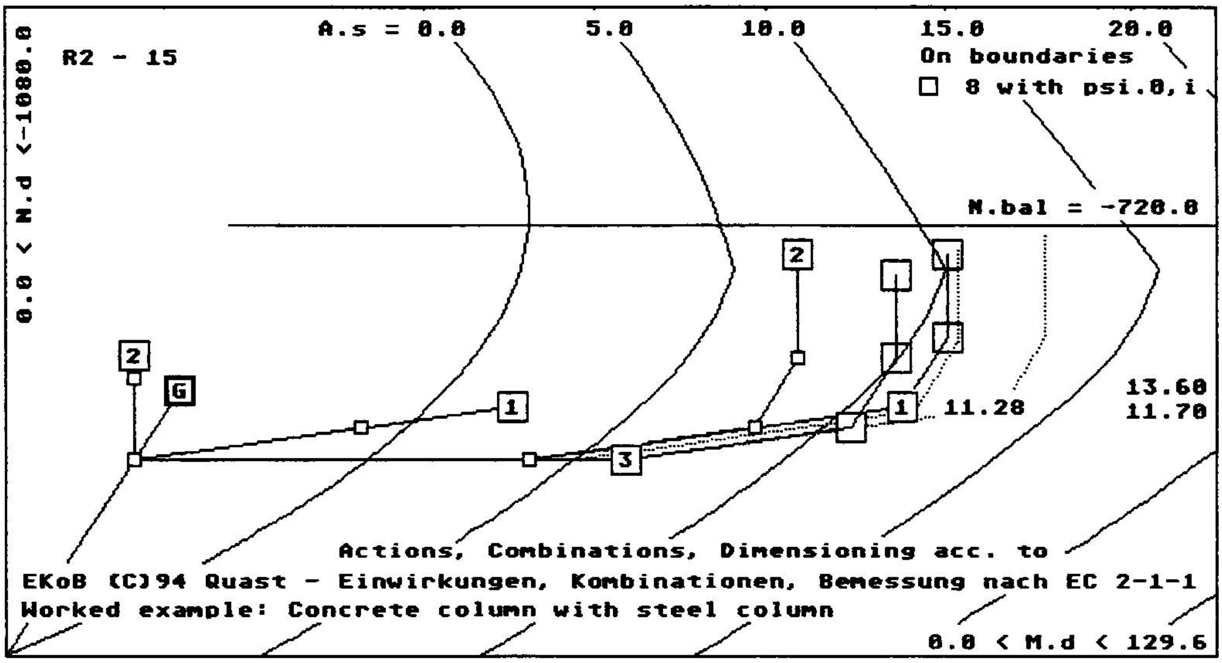

Fig. 6. Representation of the required 8 combinations on boundaries acc. to EC 1, eq. (9.10), within a detail of the N_d / M_d *–diagram. Only these combinations have to be considered for dimensioning of the cross section out of a totality of 26. For the vectors of the variable actions 1 to 3 the design values* $\gamma_Q \cdot Q_{k,i}$ *and the design values of the combination values* $\gamma_Q \cdot \psi_{0,i} \cdot Q_{k,i}$ *are marked.*

The last Fig. 6 deals with a modification of the action effects such that the possible reduction $\gamma_Q (1 - \psi_{0,i})\ Q_{k,i}$ when not being the dominant variable action effect is most unfavourable for the variable action Q_1, which is not the first one in the sequence of decreasing load eccentricities. It is obvious that this most unfavourable distance between the smaller mark and the end of the action effect vector belongs to Q_1. In this case only the polygone

$$1.0\ G_k + 1.5\ \psi_{0,3} \cdot Q_{k,3} + \mathbf{1.5}\ \boldsymbol{Q_{k,1}} + \mathbf{0.35}\ \boldsymbol{G_k} + \mathbf{1.5}\ \boldsymbol{\psi_{0,2}\ Q_{k,2}}$$

needs to be considered, which give the 3 possible combinations which have to be looked at for determining the required amount of reinforcement out of a totality of 26.