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Preload Analysis of Reinforced Concrete Flexural Members

Influences des précharges sur les éléments en béton armé sollicités à la flexion

Vorbelastungseinflüsse auf biegebeanspruchte Stahlbetonbauteile

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SUMMARY

An analysis is made of the stress and strain in reinforced concrete flexural members at the preload stage under combined effects of temperature variations and shrinkage. Typical results of such analysis are presented in this article. A «prestressing analogy» is proposed and used by the author to provide a simplified concept of approach to the evaluations of the time dependent stresses in the members up to specified stages of durations of the external influence. The severe effects of temperature and shrinkage can also be opposed or «balanced» by imposing reverse prestressing effects determined from this analogy.

RESUME

L'article analyse les contraintes et les déformations induites dans les éléments porteurs en béton armé sollicités à la flexion, à l'état de mise en charge initiale par l'action combinée des variations de température et du retrait. L'auteur fournit quelques exemples typiques relatifs à cette analyse. Il propose une analogie avec la précontrainte et en déduit un concept simplifié, en vue de déterminer les contraintes en fonction du temps ainsi que les durées spécifiques des influences externes. A partir de l'analogie proposée, il est possible de compenser les effets considérables de la température et du retrait par une précontrainte agissant en sens contraire.

ZUSAMMENFASSUNG

Spannungen und Dehnungen in biegebeanspruchten Stahlbetontraggliedern werden im Stadium der Vorbelastung durch kombinierte Einwirkung von Temperaturschwankungen und Schwinden analysiert. Dafür werden einige typische Beispiele gegeben. Der Autor schlägt eine Analogie zur Vorspannung und ein vereinfachtes Konzept vor, um zeitabhängige Spannungen bis zu spezifizierten Einwirkungsdauern zu bestimmen. Insbesondere können aufgrund dieser Analogie erhebliche Temperatur- und Schwindeffekte durch entgegengesetzt wirkende Vorspannung ausgeglichen werden.

INTRODUCTION

Preload serviceability performance of reinforced concrete has seldom been considered in the design. The initial state of stresses caused by drying shrinkage and temperature variations have been frequently considered as secondary and hence negligible. In a large number of cases it is reasonable to ignore this initial state of stresses and performance since shrinkage may or may not act along with the effect of temperature variations. However in many not unusual cases, for example, in deep raft foundations or reinforced concrete walls under rigid end or side constraints, the preload effect of shrinkage and temperature can cause excessive cracking or warping. Hence affecting the serviceability performance of the structural members in the subsequent, loading, stage. The inadequacy of an analytical treatment on the time varying behaviour of reinforced concrete structure with particular consideration given to preload shrinkage and temperature effects has been mentioned in a few separate literatural surveys viz., by the ACI Committee 435(1) and by the unified Code Committee (2,3).

BASIC CONSIDERATIONS

The worst effect in preload analysis is to consider both shrinkage and the variation in temperature acting in unisón, i.e. causing longitudinal compressive strains in an unloaded reinforced concrete member. The longitudinal steel reinforcement provides some restraint to shortening so that internal stresses are induced in the concrete and steel. In a reinforced concrete flexural member which contains a preponderance of 'tensile' steel reinforcement near one face, the unsymmetric restraint provided by the longitudinal steel results in a non-uniform distribution of concrete stress and strain. The concrete tensile stresses vary from a maximum value at the face near the tensile steel to a minimum value at the opposite face, whereas concrete compressive strains vary from a minimum in the face near the tension steel to a maximum in the opposite face. The phenomena produce not only longitudinal shortening but also warping to the members. Such warping affects the long term performance of reinforced concrete flexural members particularly slender beams and slabs. Furthermore, when deformation is restrained, stresses developed and whenever concrete is stressed, creep occurs. Similarly, whenever stress is maintained over a constant deformation, it relaxes in the time course. All these bring about a continuous adjustment within such a member for self-equilibrium against the exterior influence.

PRELOAD ANALYSIS OF STRESSES AND STRAINS IN REINFORCED CONCRETE BEAMS

Consider a doubly reinforced concrete beam of rectangular section of width x depth = bxa and with compressive and tensile steel areas of As' and As which are located respectively at d' and d from the topmost fibre. Assuming that at this stage there is no external load acting on the member except that the beam is being subjected to the combined effects of temperature and shrinkage acting in unity (i.e. causing contracting to the member dimensions). At this stage it is further assumed that due to such combined action, the section remains uncracked and that the total strains and stresses are distributed linearly across. Such beam and section together with the strain and stress distributions and the resultants of forces are shown in Figure 1. The equilibrium of horizontal forces and bending moments (taken w.r.t fibre 1) in the section at time t leads to the formation of the following equations (with $\sigma 1$, $\sigma 4$, $\epsilon 1$, $\epsilon 2$, $\epsilon 3$, $\epsilon 4$ all time dependent qualities)

$$(\sigma_1+\sigma_4) + pE_s\,(2\beta\epsilon_2 + 2\beta r\epsilon_3) = 0 \qquad [1]$$

$$\sigma_1+2\sigma_4 + pE_s\,(6\beta^2\epsilon_2 + 6\beta\delta r\epsilon_3) = 0 \qquad [2]$$

where $p = As/bd$, $r = As'/As$, $\beta = d/a$ and $\delta = d'/a$

The linear distribution of strains allows $\epsilon 2$ and $\epsilon 1$ to be expressed in terms of $\epsilon 1$ and $\epsilon 4$ i.e.

$$\epsilon_2 = (1-\beta)\,\epsilon_1 + \beta\epsilon_4 \text{ and } \epsilon_3 = (1-\delta)\epsilon_1 + \delta\,\epsilon_4$$

By substituting the above expression into Eqs.[1] [2] $\sigma 1$ and $\sigma 4$ can be expressed as,

$$\sigma_1 = pE_s\,(Q_1\epsilon_1 + Q_2\,\epsilon_4) \qquad [3]$$

$$\sigma_4 = pE_s\,(Q_3\epsilon_1 + Q_4\,\epsilon_4) \qquad [4]$$

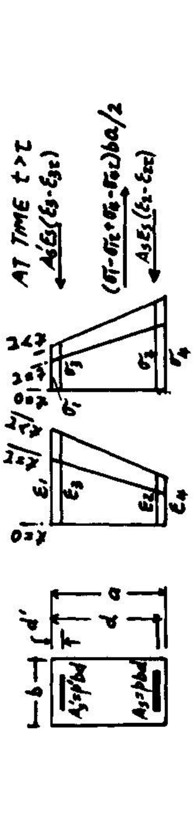

FIG.1: DEVELOPMENT OF TIME DEPENDENT STRAINS AND STRESSES DUE TO TEMPERATURE AND SHRINKAGE EFFECTS

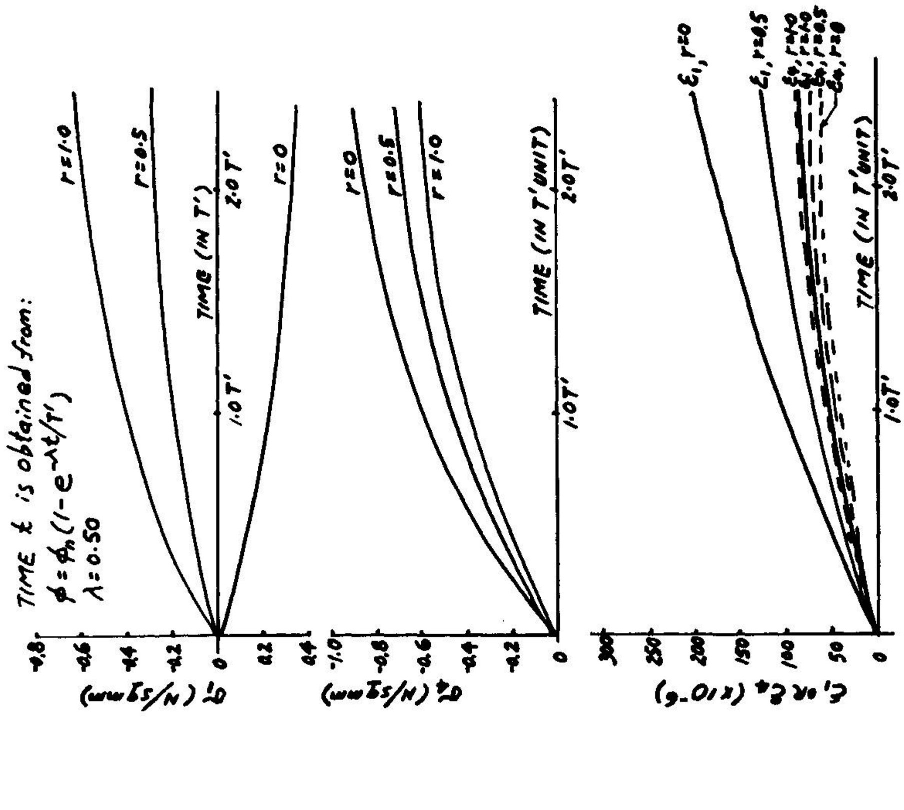

FIG.2: STRESSES AND STRAINS DUE TO SHRINKAGE ONLY

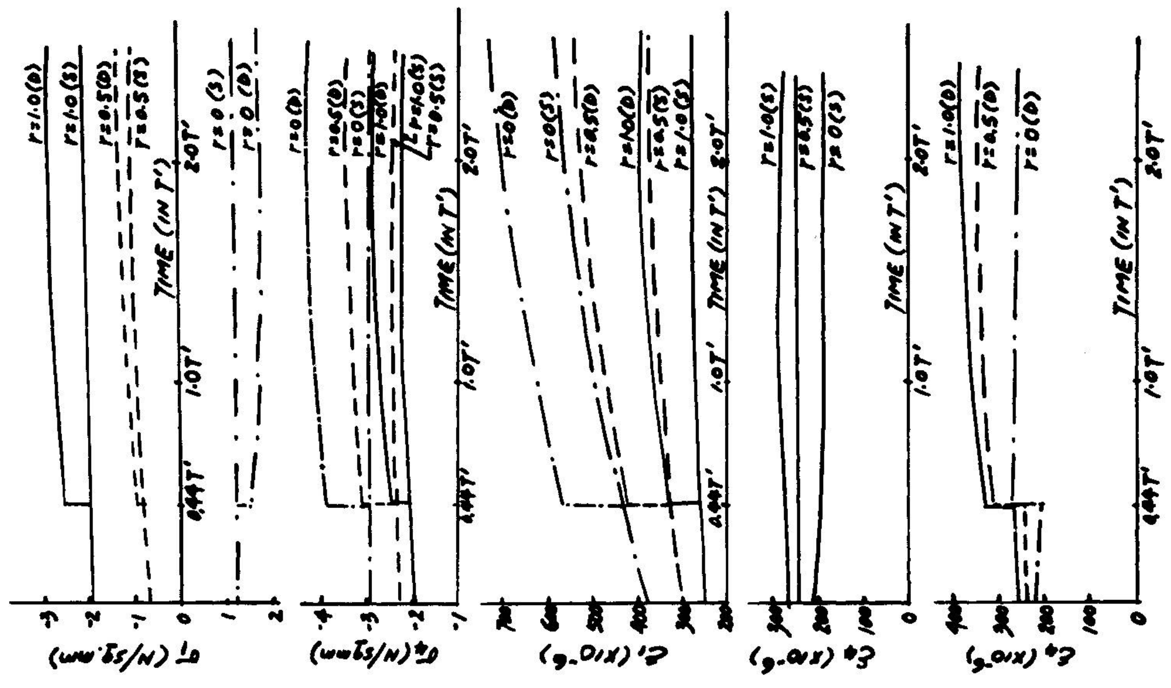

FIG.3: STRESSES AND STRAINS IN SINGLE(S) AND DOUBLE(D) STAGE PRELOAD ANALYSIS DUE TO COMBINED TEMPERATURE AND SHRINKAGE INFLUENCE

where $Q_1 = -2\beta[(1-\beta)(2-3\beta) + (1-\delta)r\,(2-3\delta)]$

$Q_2 = -2\beta[(2-3\beta)\beta + (2-3\delta)r\delta]$

$Q_3 = 2\beta[(1-\beta)(1-3\beta) + (1-\delta)(1-3\delta)r]$

$Q_4 = 2\beta[(1-3\beta)\,\beta + \delta\,)1-3\delta)r]$

The magnitudes of the time dependent strains $\epsilon 1$ and $\epsilon 4$ depends not only on the shortening caused by temperature and shrinkage but also on the sustained stressed developed at the same levels due to restraints provided by the reinforcing bars. The relationship between strain and stress in the concrete has been studied by Trost (3) and Bazant (4,5).

On basis of the principle of superposition in which an ageing coefficient has been incorporated for a range of situations involving stress relaxation histories the general form of this relation can be expressed as,

$$\epsilon = \sum_{i=1}^{n} \frac{\sigma_i}{E_i} [1+X_i\,\phi_i(t)\,] + X_i\,\phi_i\,(t)\,\frac{\epsilon_{sn}}{\phi_n} + \alpha\,\Delta T_i \qquad [5]$$

in which ϵ, σ, and X are all functions of age τ and time t. In the above expression X is the ageing coefficient which is a function of the relaxation of concrete and ϕ is the creep function. ϕ is related to ϕn which is the long term creep value.

In a one stage development the shrinkage and temperature variation are assumed commencing from one age and applying Eq.[5] for concrete strains at levels 1 and 4 in the section, the following equations are obtained,

$$\epsilon_1 = \delta_1\,[1+X\,\phi(t)\,]\,/\,E_o + X\,\phi\,(t)\,\epsilon_{sn}\,/\,\phi_n + \alpha\,\Delta T \qquad [6]$$

$$\epsilon_4 = \delta_4\,[1+X\,\phi(t)\,]\,/\,E_o + X\,\phi\,(t)\,\epsilon_{sn}\,/\,\phi_n + \alpha\,\Delta T \qquad [7]$$

By substituting the above equations into Eqs.[3],[4] and by rearrangement, a pair of simultaneous equations expressing $\sigma 1$ and $\sigma 4$ are obtained, i.e.

$$\sigma_1 = np[X\phi(t)E_o\epsilon_{sn}/\ \phi_n+E_o\alpha\Delta T][(Q_1+Q_2)\{1-npQ_4(1+X\phi(t))\}+(Q_3+Q_4)npQ_2(1+X\phi(t))]/R \qquad [8]$$

$$\sigma_4 = np[X\phi(t)E_o\epsilon_{sn}/\ \phi_n+E_o\alpha\Delta T][(Q_1+Q_2)npQ_3\ \{1+X\phi(t)\}+(Q_3+Q_4)\{1-npQ_1(1+X\phi(t))]/R \qquad [9]$$

in which $R = [1-npQ_1\{1+X\phi(t)\}][1-npQ_4\{1+X\phi(t)\}]-n^2p^2Q_2Q_3\{1+X\phi(t)\}^2$

In the above expressions the creep function $\phi(t)$ represents the time development of creep in concrete. $\phi(t)$ can thus be used to replace time in an one to one correspondence type of evaluations for the tensile stresses $\sigma 1$ and $\sigma 4$ in the concrete section. The signs for $\sigma 1$ and $\sigma 4$ are generally evaluated in negative values (representing tension). By substituting $\sigma 1$ and $\sigma 4$ values obtained from Eqs.8,9 into 6,7 the time dependent concrete strains $\epsilon 1$, $\epsilon 4$ caused by the effects of temperature and shrinkage can thus be obtained. The time varying curvature can thus be evaluated ie. $\rho=(\epsilon_1-\epsilon_4)\,/\,a$

For a simply supported beam if the self weight is negligible and if the curvatures in all other sections are developed in identical manner, the warping deflection at mid-span at time t is thus $\Delta = \rho\ell^2/8 = (\epsilon_1-\epsilon_4)\,\ell^2/$(8a)

If further stages of changes in the shrinkage and temperature values take place e.g. at a second stage commencing at an age τ (which also marks the end of the first stage), the time variations in $\sigma 1$ and $\sigma 4$ (ie. $\sigma_1(t)-\sigma_{1\tau}$ and $\sigma_4(t)-\sigma_{4\tau}$ see Figure 1) can be derived in a similar process as shown above. To obtain $\sigma_1(t)-\sigma_{1\tau}$ and $\sigma_4(t)-\sigma_{4\tau}$ which replace $\sigma 1$ and $\sigma 4$ in Eqs.8,9 it is only necessary to replace the various variables in these two equations by corresponding modified values ie.

variable in Eqs. 8, 9 :	n	X	E_o	ϵ_{sn}	ΔT
replaced by :	n_τ	X_τ	E_τ	$\epsilon_{sn}-\epsilon_{s\tau}$	$\Delta T \pm \Delta T_\tau$

The subscript (τ) above is used to denote either changes in the parameter values initiated at age τ or known parameter values at end of the first stage of analysis. In a likewise manner the above derivation and modification processes can be used to obtain the time varying stresses and strains (i.e. using Eqs. [8],[9] and [6][7]) involving multi-stage changes in shrinkage and temperature values.

TYPICAL PRELOAD PERFORMANCE OF REINFORCED CONCRETE BEAMS

The results of several typical calculations are given in Figs. 2,3 to illustrate numerically the effects of shrinkage and temperature on the stress and deformational behaviour of reinforced concrete beams and slabs. The time dependent stresses and strains in concrete due to shrinkage are as shown in Fig. 2. The parameters used to obtain this set of data are p=0.02; r=0, 0.5, 1.0; β=0.9; δ=0.1, Eo=25 x 10^3 N/sq.mm; ϕn=3.0; ϵsn=0.003; X=0.67. In the calculations, creep and relaxation of concrete actually alleviate the effects of shrinkage. The effect of compressive reinforcing steel assists greatly in the redistributions of stresses and strains in the sections. These can be seen in the set of figures i.e. in between σ_1 and σ_4 as well as ϵ1 and ϵ4.

The parameters used to obtain Fig. 3 are basically the same as in Fig. 2 except that a temperature difference of 30° C is imposed in addition to the shrinkage effects in the single stage analysis and in the two-stage analysis from τ=0.6ϕ (corresponding to a time t=0.44 T) onward such temperature variation has been increased to an accumulated value = 40° C. Results of the single stage analysis is shown in the set of curves marked with (S) whereas those for the two-stage analysis is shown in the set of curves marked with (D). In the second stage of the two-stage analysis the ageing coefficient used i.e. X = 0.79 instead of 0.67. The effects of X can be seen in Fig.3.

UNDESIRABLE PERFORMANCE AT PRELOAD STAGE

The modulus of elasticity used in the above analysis corresponds to a concrete grade strength of only 20 N/sq.mm. and the free long term shrinkage value (ϵ_{sn}) is assumed 0.0003 which is considered moderate. It can be seen in Fig. 2 that the long term value of σ_4 corresponding to r=0 may well achieve 1.2 N/mm² in tension. Had ϵ_{sn} been assigned higher values, σ_4 values could had been higher than the tensile strength of concrete. The result as shown in Fig. 3 illustrate how an additional variation in temperature of 30° C to 40° C can actually cause undesirable tensile stresses and cracking (concrete cracks when its actual tensile stress exceeds its tensile strength in the set of reinforced concrete members. The tensile strength for the concrete used in this set of analysis is assumed 2 N/sqmm in numerical value.

A 30° C to 40° C temperature variation seems relatively high. However it can occur in mass concrete pour when a combined variation is caused by the hydration temperature and the ambient temperature.

PRESTRESSING ANALOGY

The developments of stresses and strains in reinforced concrete in the preload stage can also be visualized in a slightly different way. For each beam analysed the effects of shrinkage and temperature can be considered to be replaced by a natural prestressing force (P) and maintaining at certain eccentricity from the centroid of the section. The magnitude and the eccentricity of this prestressing force of nature will eventually produce the same long term (ie. at time ∞) stresses and strains in the concrete. If the section is uncracked the magnitude and eccentricity of this prestressing force (ie. Peff and e) can be obtained by equating σ_1, σ_4 obtained from Eqs.[8][9] with the expressions, $\sigma_1 = -P_{eff}/A + P_{eff}e/Z_t$ and $\sigma_4 = -P_{eff}/A - P_{eff}e/Z_b$.

To ensure the section remains uncrack the critical values of external influence from shrinkage and temperature must be evaluated. The long term value of σ_4 in Equation 9 (and similar expression in the multi-stage approach) must not exceed f_t which is the tensile strength of the concrete. By setting the long term σ_4 values to f_t and by estimating one of the two influencing variables (ie. between ϵ_{sn} and

ΔT) the limiting value of the other variable can be computed. For example, by equating σ_4 at $\phi = \phi_n = 3$ to $f_t = -2$ N/sq mm in the set of numerical evaluations used in the single stage analysis (with the results as shown in Fig.3) and assuming ϵ_{sn} is maintained at 0,0003, the limiting temperature variations (ie.ΔT) for the flexural sections with r=0, 0.5 and 1.0 are respectively 14.5°C, 22.25°C and 27.37°C.

With σ_4 being assigned the value of f_t, the corresponding σ_1 can be obtained from the ratio σ_1/σ_4 obtained from Eqs. 8, 9. Thus σ_1 equals 0.82, -0.92 and -2 N/sqmm corresponding to beam sections containing r=0, 0.5 and 1.0 and subject the combined external influence of respectively $\epsilon_{sn}=0.0003$, $\Delta T=14.5$°C; $\epsilon_{sn}=0.0003$, $\Delta T=22.25$°C and $\epsilon_{sn}=0.0003$ and $\Delta T=27.37$°C.

The prestressing effects producing the respective σ_1 and σ_4 values in each member can then be computed according to the above prestressing equations. In a rectangular beam of 200 mm x 500 mm (width x total depth) the following results are obtained,

r	r=0	0.5	1.0
P_{eff}(KN)	159	146	200
e(mm)	199	30.8	0

The initial values of P (ie. P_{ini}) which produce P_{eff} (in the long term) can thus be estimated on basis of the materials deformational properties. Reversing the direction of the applications of P_{ini} in these members can effectively provide counterbalancing towards undesirable preload structural performance in the reinforced concrete beams.

CONCLUDING REMARKS

The analysis with details given in this article allows the stresses, strains and deflections of reinforced concrete flexural members to be evaluated under preload, temperature and shrinkage effects. The evaluated long term stresses, strains and warping deflections can be compared to those permissible values to assess whether the effect of preload influence has already caused undesirable serviceability problems to the proposed structures. In a reverse process, by restricting the maximum tensile stress in the concrete (to value equals to its tensile strength) the limiting range of combined influence of shrinkage and temperature corresponding to a condition of serviceability limit of cracking in the preload stage can also be determined. This allows preventive measures to be taken (such as providing curing and in the control of temperature) to alleviate the anticipated structural performance. The prestressing analogy enables a reproduction of the long term concrete stresses induced by the external influence of temperature and shrinkage. Analogous prestressing effects in opposite sign can therefore be applied to the members to counterbalance (or even defuse) the undesirable preload stresses.

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