

# Modelling of members with transverse reinforcement

Autor(en): **Reineck, Karl-Heinz**

Objektyp: **Article**

Zeitschrift: **IABSE reports = Rapports AIPC = IVBH Berichte**

Band (Jahr): **62 (1991)**

PDF erstellt am: **13.05.2024**

Persistenter Link: <https://doi.org/10.5169/seals-47674>

## **Nutzungsbedingungen**

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern. Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden. Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

## **Haftungsausschluss**

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

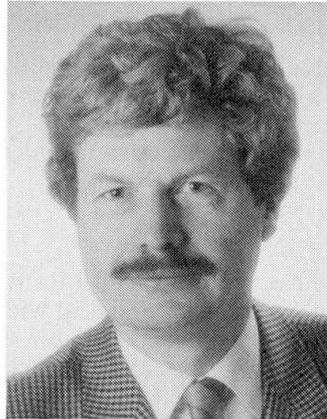
## Modelling of Members with Transverse Reinforcement

Modélisation d'un élément en béton pourvu d'armatures transversales

Modellieren von Konstruktionsbauteilen mit Stegbewehrung

### Karl-Heinz REINECK

Dr. Eng.  
Univ. of Stuttgart  
Stuttgart, Germany



Karl-Heinz Reineck received his Dipl.-Ing. and Dr.-Ing. degrees from the University of Stuttgart. He is involved in both research and teaching at the Institute for Structural Design. His research covers theoretical work and several experiment projects on the shear-behaviour of structural concrete members as well as large-scale tests on reinforced concrete-shells for offshore platforms. He is a member of the CEB-Commission «Member Design».

### SUMMARY

The biaxial state of stress in the webs of structural concrete members is described, which is presented by a truss model with the combined action of the stirrups and concrete tensile ties. The magnitude of the load carried by concrete in tension is determined by the friction of the crack faces and therefore the state of strain in the web and crack width have to be determined. However, this allows the calculation of the behaviour from cracking until failure. The influence of axial forces and of the prestress on the ultimate resistance can be consistently described as well as the effective concrete strength explained.

### RÉSUMÉ

L'état de contrainte biaxial au sein d'éléments en béton est traduit par l'analogie du treillis, qui modélise les efforts de traction apparaissant soit dans les étiers, soit dans le béton. L'amplitude de la charge reprise par le béton en traction est déterminée par le frottement des surfaces des fissures, ce qui impose la détermination de la largeur des fissures ainsi que de l'état de contrainte au sein de l'élément considéré. Or cette démarche permet le calcul du comportement allant de la fissuration à la rupture; par conséquent, on pourra décrire l'influence des forces axiales et de la précontrainte sur la résistance ultime aussi bien que la résistance effective du béton de l'élément étudié.

### ZUSAMMENFASSUNG

In Stegen von Konstruktionsbetonteilen herrscht ein zweiachsiger Spannungszustand, der durch ein Fachwerk mit Zugkräften in den Bügeln sowie im Beton modelliert wird. Der vom Beton auf Zug getragene Lastanteil wird aus den durch die Reibung in den Rissen übertragbaren Spannungen bestimmt, und deshalb müssen der Dehnungszustand im Steg und die Rissbreiten bestimmt werden. Dies erlaubt jedoch die Berechnung des Tragverhaltens von der Rissbildung bis zum Bruch. Der Einfluss von Längs Kräften und der Vorspannung auf die Tragfähigkeit kann konsistent angegeben und die «effektive Betonfestigkeit» erklärt werden.



## 1. INTRODUCTION

The well-known truss for members with transverse reinforcement is a basic model for structural concrete as shown by MacGregor, Marti and Schlaich in /1/. However, it is also a simple model with only two variables to cover all design cases: the strut inclination  $\Theta$  and the strength  $\sigma_{cw}$  of the compression struts. This simplicity may lead to contradictions with the real behaviour of members and even to inconsistencies with other models used in the design concept. An important case are the members with moderate shear, which only require light transverse reinforcement: until now most codes provide an empirically derived  $V_c$ -term; yet for a truss model unrealistic low values for  $V_c$  have to be assumed, which allow almost no staggering of the tension chord reinforcement. A further example is the "effective strength" of the compression fields or struts, where either simply different values depending on the stress situation are proposed or refined strain-considerations are made. Finally, the truss with an uniaxial compression field is an insufficient model for the limit state of serviceability and therefore in codes mostly detailing rules are given (MacGregor /1/).

The aim of this article is to contribute to a clear understanding of the structural behaviour of B-regions with shear forces by presenting a model for the stresses and strains of cracked webs with transverse reinforcement. It will be shown that tensile stresses occur in the web due to the friction of the crack-faces, and that by modelling this the above mentioned inconsistencies are avoided as explained in /2/.

## 2. EQUILIBRIUM

The equilibrium in the B-region of a r.c.- or p.c.-member is investigated according to the well-known method by Mörsch, to cut the member along the cracks and to deal with the resulting elements in free-body diagrams. For a typical B-region at an end-support the elements are the solid concrete struts between the cracks (Fig.2). For simplicity straight cracks are assumed, whereby their inclinations depend on the degree of prestress and on the magnitude of the axial compressive or tensile force  $N$ . Mörsch was satisfied with the capacity of the truss formed by the stirrups and the struts between the cracks (Fig.2a), but now generally the contribution of the friction of the crack faces (Fig.1) is taken into account by assuming flatter strut- than crack-inclinations. (The dowel force of the longitudinal reinforcement is neglected in the following for simplicity.) The term "friction" is used here as a general term covering all types of concrete, e.g. also lightweight concrete with no aggregate interlock. Such frictional stresses were so far mostly regarded as components of an uniaxial compression field with an inclination flatter than that of the cracks, like by Kupfer/Mang/Karavesyrogrou /3/, Kirmair/Mang /4/, as well as dei Poli/Gambarova/Karakoc /5/. In the following the biaxial state of stress in the web is derived.

From the vertical equilibrium in Fig.1 follows:

$$V = \frac{A_{sw}}{s_w} \sigma_{sw} \cdot z \cdot \cot \beta_{str} + V_f + V_p \quad (1)$$

whereby  $V_p$  is the vertical component of the total force in an inclined prestressed reinforcement. Prestressing is here considered internally in the strains, which is here of advantage since later the strains and relative displacements of the crack-faces have to be determined. The strain in the tension-chord follows from the force, which can be calculated from the equilibrium of moments of the end-support region (Fig.1). Similarly the force  $C$  and the corresponding strain in the compression chord are determined from the horizontal equilibrium.

The vertical component  $V_f$  of the friction forces can be expressed in terms of the friction stresses in the middle of the web

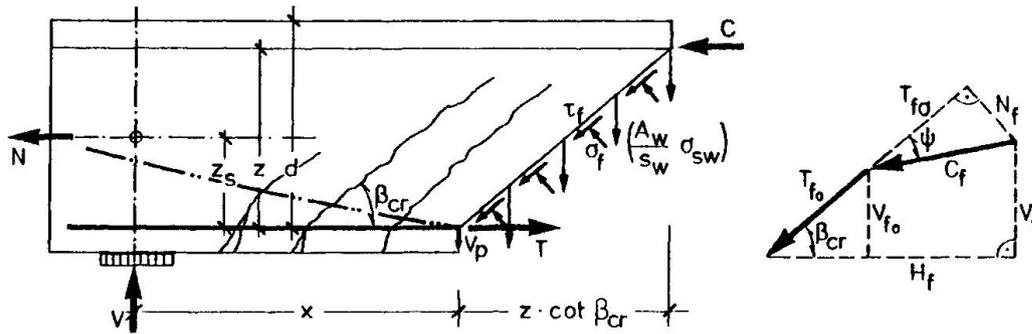
$$V_f = b_w z [\tau_{f0} + \tau_{f\sigma} (1 - \cot \beta_{cr} / \mu_f)] \quad (2)$$

Thereby the well-known relation for friction was assumed

$$\tau_f = \tau_{f0} + \mu_f \cdot \sigma_f = \tau_{f0} + \tau_{f\sigma} \quad \text{with} \quad \mu_f = \cot \psi = 1,7 \quad (3)$$

which, however, contrary to the usual applications depends on the crack displacements. The relatively high value for  $\mu_f$  was derived from Walraven's constitutive laws for crack widths up to 0,5 mm (see /2, 15/).

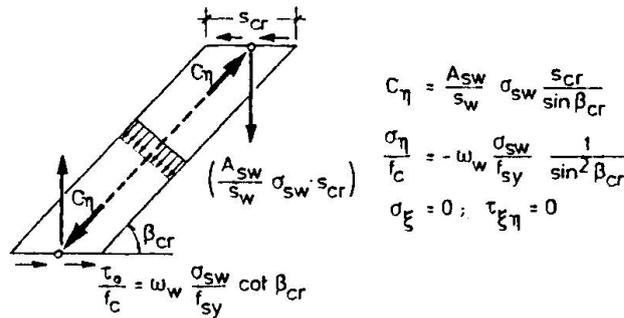
Now all the forces and stresses along the crack are defined, and the stress field in the solid concrete strut between the cracks can be determined. In Fig.2 all stresses are given with respect to the crack direction. The truss-action (Fig.2a) is made up by the stirrups and the uniaxial compression field between the cracks. The stress fields due to the friction (Fig.2b) are looked at separately for a better understanding: the shear stresses  $\tau_f$  result in a biaxial tension-compression field with an inclination of  $\beta_{cr}/2$  of the compression field; the normal stresses  $\sigma_f$  result in principal compressive stresses  $\sigma_2$  and tensile stresses  $\sigma_1$  parallel to the crack. Altogether a biaxial stress field exists, whereby the principal stresses and the principal inclination  $\psi_1$  (Fig.2b) may be determined acc. to the linear elastic theory. This is valid since the principal tensile stresses remain smaller than the concrete tensile strength once the crack pattern has formed.



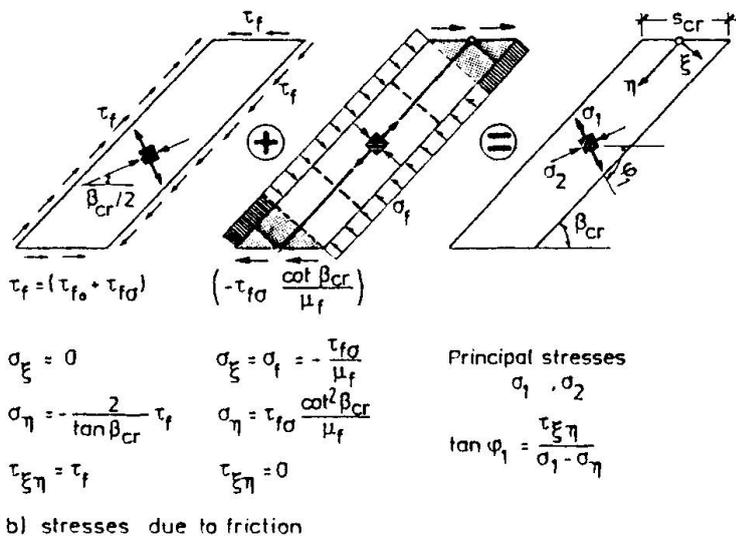
a) end-support region and forces

b) forces due to friction

Fig. 1: Free-body diagram for an end-support region of a structural concrete member



a) forces and concrete stresses due to truss-action



b) stresses due to friction

Fig. 2: Forces and stresses for the concrete strut between the cracks

The stress field resulting in the web from all the actions in Fig. 2 is that of a principal compression  $\sigma_2$  inclined at the angle  $\theta$  and a principal tension (for higher shear also small compression) perpendicular to that. This was already described by Reineck in /6/, and even earlier by Lipski /7/ who, however, gave a different explanation for the tension field. This state of stress results in the two models shown in Fig. 3: the well-known truss model formed by an uniaxial compression field and the stirrups (Fig. 3a), as well as a truss-model with concrete tensile ties (Fig. 3b). These are the two load paths referred to by Schlaich/Schäfer/Jennewein /8/. The models in Fig. 3 are statically equivalent to the model shown in Fig. 1, and this also means that there is no principal contradiction between the two well-known approaches in the shear design: the "shear-friction theory" leading e.g. to a  $V_c$ -term on one side, and the truss-analogy on the other side. In these truss-models the discrete cracks are not modelled but these must be looked at in order to determine the magnitude of the tensile stress  $\sigma_1$ , which only depends on the friction stresses along the crack. However, these trusses visualize the flow of the forces in a member more clearly and simpler. So the overall model (Bruggeling /1/) and the sectional approach are both necessary.

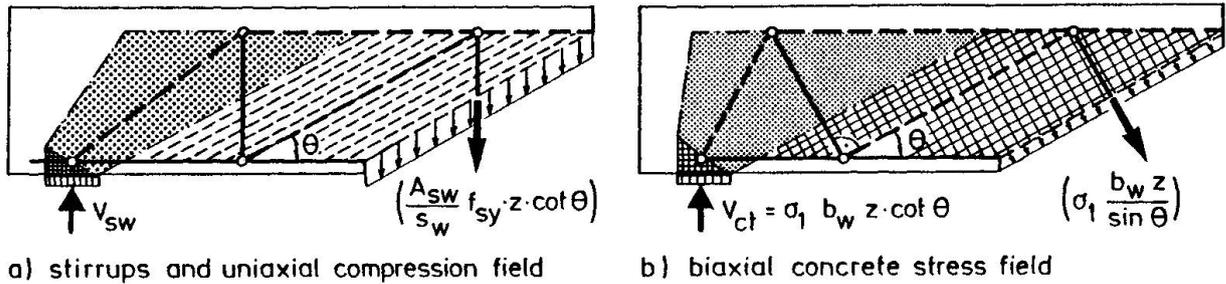


Fig.3: Truss models following from the principal stresses between the cracks

It must be mentioned that further tensile stresses occur in the struts due to the bond of the stirrups (see /3, 4, 9, 10/). These tensile stresses reduce the stirrup stresses and strains between the cracks (tension stiffening effect), but they do not contribute to the load transfer.

With this model for the biaxial state of stress in the web of structural concrete members with transverse reinforcement a clear transition to the model for members without transverse reinforcement is achieved as also explained in /2, 15/. So apart from being transparent, this model enables a consistent treatment from members with transverse reinforcement to unreinforced members.

### 3. KINEMATICS AND CONSTITUTIVE RELATIONS

#### 3.1 Kinematics

The strains of an element in the B-region can be calculated from the strains of the truss formed by the stirrups and the solid concrete struts between the cracks (Fig.2a). The stresses due to friction (Fig.2b) are then considered by the deformations of the concrete struts:

$$\epsilon_{cw} = \epsilon_{\eta} = (\sigma_{\eta} - 0,2 \cdot \sigma_{\xi}) / E_c \quad (4)$$

The complete state of strain of a beam-element in a B-region with shear forces is as follows:

$$\text{- longitudinal strain in the middle of the web:} \quad \epsilon_x = (\epsilon_s - \epsilon_c) / 2 \quad (5a)$$

$$\text{- curvature:} \quad \bar{\kappa} = \kappa \cdot z = (\epsilon_s + \epsilon_c) / 2 \quad (5b)$$

$$\text{- vertical strain:} \quad \epsilon_z = \epsilon_{sw} \quad (5c)$$

$$\text{- shear strain:} \quad \gamma_{xz} = \epsilon_x / \tan \beta_{cr} + \epsilon_{cw} / \sin \beta_{cr} \cdot \cos \beta_{cr} + \epsilon_{sw} \cdot \tan \beta_{cr} \quad (5d)$$

For the vertical strain the beneficial tension stiffening effect between cracks was neglected, because the anchorage slip of the stirrups has a controversial effect; more refined considerations were e.g. made by Kupfer et al. /3, 4/.

With these equations the bending- and shear-stiffnesses of the beam-element are principally given and may be used either for a non-linear analysis or for calculating deformations, since the equations are not limited to the ultimate limit state. The difference to many well-known works, e.g. also by Collins/Mitchell /11/ is, that the crack-inclination is considered and that the direction of the principal strain does not coincide with that of the compression field; further explanations were given by Hardjasaputra /12/ and Reineck/Hardjasaputra /13/.

Since the strains are known also the crack width  $n$  and the slip  $s$  in the middle of the web can be calculated for a given crack-spacing  $s_{cr}$  (measured horizontally):

$$\frac{\Delta n}{s_{cr}} = (\epsilon_x + \epsilon_{sw} + \epsilon_{cw}) \cdot \sin \beta_{cr} + \bar{\kappa} \frac{s_{cr}}{z} \cos \beta_{cr} \quad (6a)$$

$$\frac{\Delta s}{s_{cr}} = -\epsilon_x \cdot \cos \beta_{cr} + (\epsilon_{sw} + \epsilon_{cw}) \sin^2 \beta_{cr} / \cos \beta_{cr} + \bar{\kappa} \frac{s_{cr}}{z} \sin \beta_{cr} - 2,4 \frac{\tau_1}{E_c} \sin \beta_{cr} \quad (6b)$$

These crack displacements determine the magnitude of the friction transferring the biaxial stress field over the cracks.

#### 3.2 Constitutive Relations

For the concrete and the steel bi-linear stress-strain curves can be used /2/. The strength of the solid concrete struts between the cracks (see also /14/ and section 6) is not lower than:

$$f_{cw} = 0,85 \cdot f_c \quad \text{or} \quad f_{cw} = 0,80 \cdot f_c \quad (7)$$

The constitutive equations for the friction of the crack-faces were already explained in /15/ and so here only the result is given:

$$\frac{\tau_f}{f_c} = \frac{\tau_{f0}}{f_c} \cdot \frac{\Delta s - 0,24\Delta n}{0,096 \cdot \Delta n + 0,01} \quad \text{with } \Delta n, \Delta s \text{ [mm]} \quad (8)$$

The stress  $\sigma_f$  or  $\tau_{f0}$  follows then from Eq.(3). The friction stress  $\tau_{f0}$  is the limiting value without normal stresses  $\sigma_f$  on the crack face and was set to

$$\frac{\tau_{f0}}{f_c} = 0,45 \frac{f_{ct}}{f_c} \left(1 - \frac{\Delta n}{0,9}\right) \quad \text{with } \Delta n \text{ [mm]} \quad (9)$$

This is a much lower value than that given by Vecchio/Collins/Bhide /9, 10/. With these formulations the whole response of the B-region in a member may principally be determined from cracking until failure, and this is demonstrated in the following by two examples.

#### 4. STIRRUP STRAINS AT SERVICE LOAD

For determining the crack width of the inclined cracks in a web all strains must be known acc. to Eq. (6a), but the stirrup strains play of course a dominant role. Presently a semi-empirical approach is used for predicting the stresses and strains for in the stirrups under service load conditions: the stresses are calculated for a truss model with struts at 45° and the actual load V is reduced by a  $V_c$ -term. The presented combination of truss models with additional tensile struts in the web can be used for checking the requirements at the serviceability limit state. As an example for this the well-known test series of identically reinforced beams with varying web thicknesses (varying  $b/b_w$ -ratio) by Leonhardt/Walther /16/ was calculated, and Fig.4 shows the comparison of the measured and calculated stirrup stresses with increasing load. Contrary to a truss-model, the presented model yields the typical characteristic of the measured curves. It can also be seen that the usually used  $V_c$ -term is not equal to the cracking load as often pretended. However, the calculated values are partly very conservative because the tension stiffening effect was not considered for the stirrup strains. If this is improved the model may serve as a relatively simple tool for determining crack widths. Of course, additionally to the strains also the crack spacing must be determined.

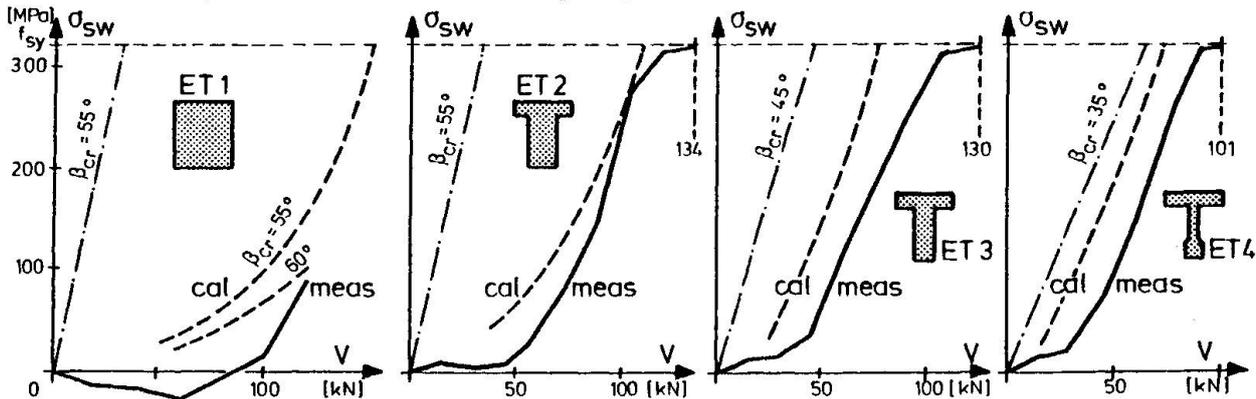


Fig.4: Development of stirrup stresses with the load for test-beams with varying web-thicknesses of Leonhardt/Walther /16/

These results may be interpreted in terms of the load proportions taken by the models with the stirrups or the tensile struts (Fig.3), and such values are given in Fig.5 for the load step 60kN (service load) and the load at yielding of the stirrups. For the two beams with the thick webs the truss with the concrete tensile struts carry quite a considerable part of the load, although the tensile stresses remain low; for the thin-webbed beams the load is almost totally carried by the truss with stirrups. From this it is obvious that the model with the concrete tensile struts is especially relevant for members with moderate shear like in buildings and for foundations.

beam	$\beta_{cr}$	Service load $V \sim 60 \text{ kN}$					Yielding $\epsilon_{sw} = 1,6 \text{ ‰}$				
		$\theta$	$\sigma_1/f_c$	$v_{ct}$	$\sigma_{sw}/f_{sy}$	$v_{sw}$	$v$	$\theta$	$\sigma_1/f_c$	$v_{ct}$	$v_{sw}$
ET 1	55°	31,7	0,0180	52,6	0,106	7,4	151,9	24,3	0,0140	56,2	95,7
ET 2	55°	26,0	0,0201	37,2	0,256	22,6	113,9	26,0	0,0137	25,5	88,4
ET 3	45°	25,2	0,0014	1,9	0,634	58,2	75,9	27,4	-0,0062	-7,1	83,0
ET 4	35°	30,5	0,0028	1,5	0,794	58,2	72,1	30,8	-0,0002	-0,1	72,2

Fig.5: Calculated values for two load stages of the test-beams in Fig.4



## 5. DIMENSIONING AT THE ULTIMATE LIMIT STATE

The main results for the ULS are summarized in the well-known dimensioning diagram (Fig.6) which is simplified by assuming a constant shear force component  $q_f$  with increasing ultimate shear force. However, this is quite a good approximation as comparisons with similarly derived diagrams by Kupfer et al /3, 4/ and Gambarova /5/ show. Also Hardjasaputra/Reineck /12, 13/ came to similar results based on a simple kinematic condition proposed by Hardjasaputra /12/, which includes the slip in the cracks and thereby the direction of the principal strain deviation from the direction of the principal compression.

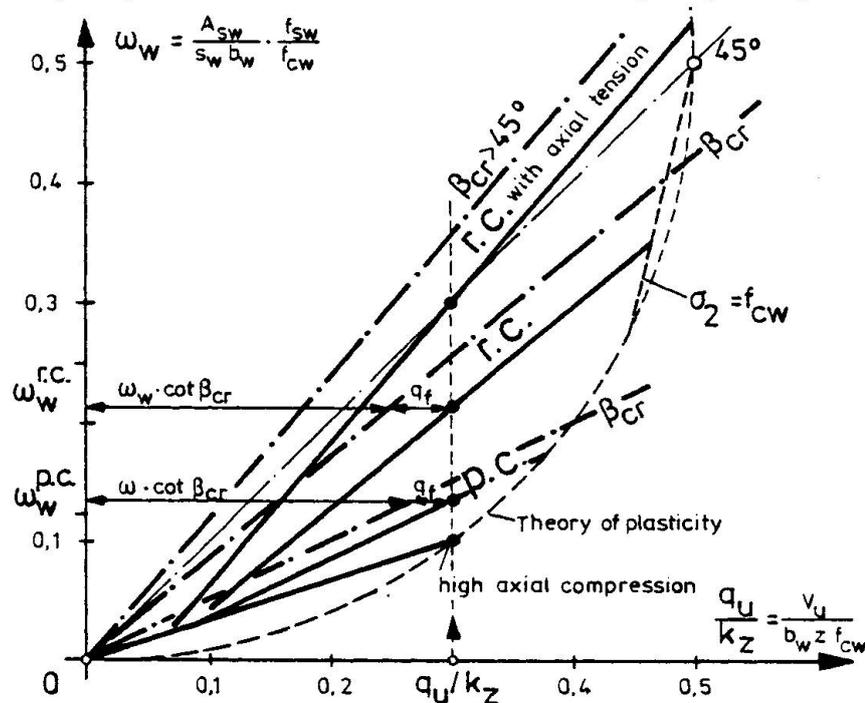


Fig.6: Principal sketch of a dimensioning diagram for the transverse reinforcement in B-regions and influence of prestress and axial tensile and compressive forces

Since p.c.-beams exhibit a flatter crack inclination than r.c. beams, less stirrups are required although the value for  $q_f$  is smaller. For medium values of  $q_u$  the friction capacity defines the ultimate load, and only for high values  $q_u$  the concrete between the cracks really fails in compression. This occurs at slightly lower shear forces than according to the theory of plasticity because the web is here in biaxial compression (due to high stresses  $\sigma_f$  on the crack face). Of course, in test beams the difference between both "failure types" is not always clearly recognizable, especially not for p.c. beams and webs or panels with small crack spacing.

Axial forces are also constantly considered in the equilibrium equations (see Fig.1), leading e.g. to higher axial strains  $\epsilon_x$  as well as steeper crack inclinations for tension flanges of box-girders. It is especially worth mentioning that for high axial compression very small crack inclinations and therefore also small inclinations  $\Theta$  for the compression field are possible; therefore a lower limit for  $\Theta$  is not necessary according to this proposal, since it is taken into account by the limitation of the friction transfer. (Why not  $10^\circ$  for box-columns of bridges with high axial compression? Here the model is that of a very flat strut as explained by Schlaich /1/). However, since structural concrete is capable of some redistribution, also higher strut inclinations than that of the cracks can occur as pointed out by Kupfer/Guckenberger /17/, who tested and clearly explained the structural behaviour of highly compressed structural concrete members.

Finally it may be concluded that the presented model explains the " $V_c$ -term" and that the simple demanded ways of accounting for it (MacGregor /1/) are possible, as demonstrated by a joint proposal from Kupfer and Reineck for the CEB MC 90.

## 6. EFFECTIVE CONCRETE STRENGTH

When applying the theory of plasticity to structural concrete, an effective concrete strength for the compression field is taken, which is lower than the uniaxial compression strength (Marti and MacGregor /1/). For a compression field crossing cracks it is now obvious from before, that this value is not constant but depends on the strain condition as well as on the direction and spacing of the cracks, if the friction capacity is decisive. If the compression struts between cracks govern the failure, the reduction of the strength of struts

between cracks is only up to  $0,85 f_c$  and  $0,8 f_c$  according to many tests by different researchers as explained in /14/; therefore lower values are unnecessarily restrictive.

If now the ultimate loads according to the diagram (Fig.6), are interpreted by the well-known truss with a uniaxial compression field as shown in Fig.7, then the so-called effective concrete strength ( $v \cdot f_{cw}$ ) varies with the ultimate load or the inclination  $\theta$ . The value  $v$  now cannot be constant, since the friction characteristics cannot only be formulated in terms of strength values, like normally in "shear-friction theories", but a complete description of the strength as well as relative crack displacements is required. Therefore a crack direction has to be assumed and the crack spacing has to be calculated, in order to evaluate the crack width for a given strain condition.

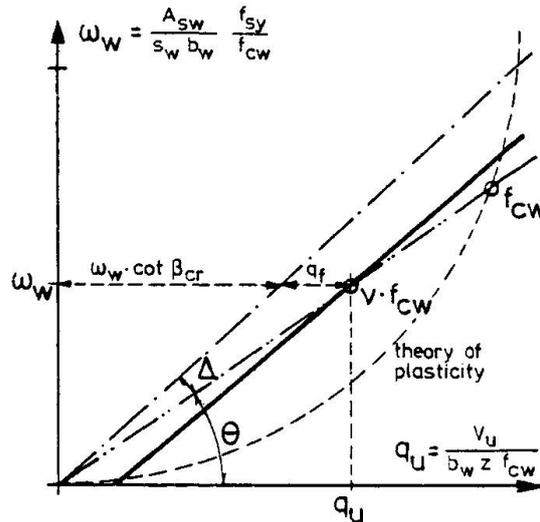


Fig.7: Interpretation of the dimensioning diagram in Fig.6 in terms of an effective concrete strength

All these influencing parameters may simply be described by their effect on the friction component  $q_f$ , acc. to Fig.7, and this is shown in Fig.8. The influence of the crack spacing as well as of the crack inclination follows from Fig.8a; thereby the crack spacing is of principal importance because by this parameter a "size-effect" is induced: for larger member depths the crack widths are larger for same rotations or curvatures (as explained in /15/) and also larger bar diameters are used resulting in larger crack widths and smaller values for the friction component  $q_f$ . This also follows from the works of Gambarova /5/ and Kupfer et al /4/. The vertical or stirrup strains (Fig.8b) also limit the friction capacity, if large values are reached (unless a second crack field appears and the original cracks close). The longitudinal strains (Fig.8c) reflect mainly the influence of axial forces, which furthermore have an additional effect on the crack inclination. In case of axial compression the crack inclination and also  $q_f$  acc. to Fig.8a or 8c decreases, but this is more than made up by the capacity increase of the truss (see Eq.(1) and Fig.2a).

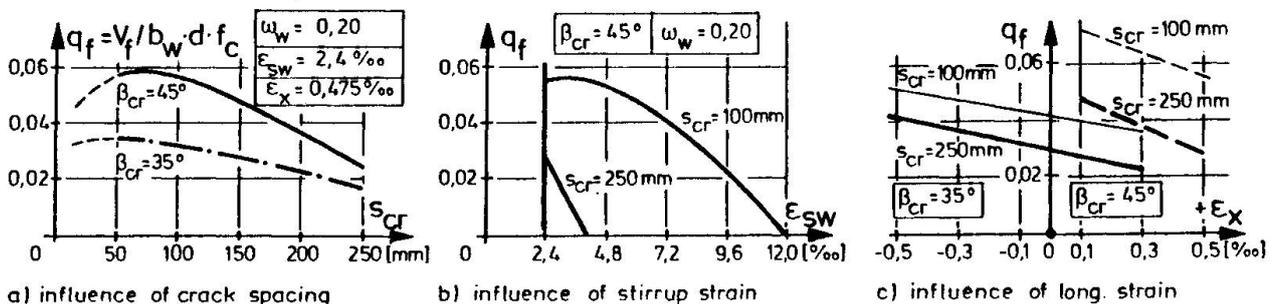


Fig.8: Influence of crack spacing, crack inclination and strains on the shear force component  $q_f$  carried by friction

All this shows, that the limited friction transfer also explains the reduction of the effective concrete strength with increasing transverse strain, which was pointed out by Collins et al /9, 10, 11/; a direct comparison was given in /14/. Furthermore however, the crack spacing and thereby the crack width have an influence, since friction is involved and it is not only a problem of "compression with transverse tension" (as also only in most tests) or of "compression-softening". Consequently it means reversing cause and effect if the "shear



transfer strength" is explained by the "softening" of the concrete as done by Hsu /18/. The force transfer over cracks is a discrete problem and must be dealt with as such, similar to the force transfer over joints as discussed by Ruth /19/. Both problems require a model for the structural behaviour of the whole member, but they also have an influence on it, e.g. in terms of a low "effective strength". Finally it must be mentioned that there might be further causes for a strength reduction as shown by Thürlimann /20/.

## REFERENCES

1. IABSE Colloquium "Structural Concrete". Introductory Reports. Stuttgart, April 1991
2. Reineck, K.-H.: A mechanical model for the behaviour of reinforced concrete members in shear (in German), Dr.-thesis., Univ. Stuttgart, 1990, 1-273
3. Kupfer, H.; Mang, R.; Karavesyrogrou, M.: Failure of the shear-zone of r.c.- and p.c.-girders - an analysis with consideration of interlocking of cracks (in German). Bauingenieur 58 (1983), 143-149
4. Kirmair, H.; Mang, R.: The structural behaviour of the shear-zone of slender r.c.- and p.c.-girders with bending and axial force (in German). Bauingenieur 62 (1987), 165-170
5. dei Poli, S.; Gambarova, P.G.; Karakoc, C.: Aggregate interlock role in r.c. thin-webbed beams in shear. ASCE-Jour. Struc.Div. 113 (1987), 1-19
6. Fenwick, R.C.; Paulay, T.: Mechanisms of shear resistance of concrete beams. ASCE-Journal Struct.Div. V.94, 1968, ST 10, Oc., 2325-2350
7. Lipski, A.: Poutres a àme mince en béton armé ou précontraint. Ann. d. Travaux Publ. d. Belgique No.1-2, 1971/72
8. Schlaich, J.; Schäfer, K.; Jennewein, M.: Toward a consistent design for structural concrete. PCI-Jour. 32 (1987), No.3, p.75-150
9. Vecchio, F.J.; Collins, M.P.: The modified compression field theory for reinforced concrete elements subjected to shear. ACI Struct.Jour. 83 (1986), March-April, 219-231
10. Bhide, S.B.; Collins, M.P.: Influence of axial tension on the shear capacity of reinforced concrete members. ACI Struct.Jour. 86 (1989), Sept.-Oct., 570-581
11. Collins, M.P.; Mitchell, D.: Shear and tension design of prestressed and non-prestressed concrete beams. PCI Jour. 25 (1980), Sept.-Oct., 32-100
12. Hardjasaputra, H.: Consideration of the state of strain in the shear design of r.c.- and p.c.-girders (in German). Dr.-thesis, University of Stuttgart, 1987
13. Reineck, K.-H.; Hardjasaputra, H.: State of strain in the shear design of r.c.- and p.c.-girders (in German). Bauingenieur 65 (1990), H.2, 73-82
14. Reineck, K.-H.: Theoretical considerations and experimental evidence on web compression failures of high strength concrete beams. CEB Bull. 193, 61-73, Lausanne 1989
15. Reineck, K.-H.: A mechanical model for structural concrete members without transverse reinforcement. IABSE Colloquium "Structural Concrete", Stuttgart, April 1991
16. Leonhardt, F.; Walther, R.: Shear tests on simple r.c.-beams with and without shear reinforcement (in German). DAfStb H.151, Berlin 1962
17. Kupfer, H.; Guckenberger, K.: Tests on the structural shear behaviour of profiled r.c.- or p.c.-girders with compressed flanges (in German). DAfStb H.377, Berlin, Ernst & Sohn, 1986
18. Hsu, T.T.C.: Softened truss model theory for shear and torsion. ACI Struct. ACI Struct.Jour. 85 (1988), Nov.-Dec. 624-625
19. Ruth, J.: Influence of contract surface problems on design practice. IABSE Colloquium "Structural Concrete", Stuttgart, April 1991
20. Thürlimann, B.: Theory of plasticity for reinforced concrete (in German). Course summerterm 1985, Univ. Stuttgart, Institut für Massivbau.