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Theoretical Evaluation of Remaining Fatigue Life of Steel Bridges

Evaluation théorique de la durée de vie restante de ponts en acier

Theoretische Abschätzung der Restlebensdauer von Stahlbrücken

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SUMMARY

An operative methodology for the assessment of remaining fatigue life of existing railway bridges is presented. Either the Miner rule or the Paris crack propagation law can be used. Examples are provided in order to show the method's sensitivity to variation of input data. A critical examination of the results leads to justification for field measurements of structural response.

RÉSUMÉ

L'article présente une méthode d'évaluation de la durée de vie restante de ponts de chemin de fer existants en acier. Cette méthode permet aussi bien l'application de la loi de Miner que celle de la loi de Paris pour la propagation de fissures. Quelques applications numériques sont présentées ensuite afin de montrer la sensibilité de la méthode par rapport à la variation des données. Un examen critique des résultats justifie les mesures sur la structure.

ZUSAMMENFASSUNG

Vorgestellt wird eine operationelle Methode zur Abschätzung der Restlebensdauer bestehender Eisenbahnbrücken aus Stahl. Hierfür eignen sich sowohl das Verfahren von Miner als auch die Gleichung von Paris für das Risswachstum. Anhand von Beispielen wird die Empfindlichkeit der Methode für Änderungen der Eingabe-Parameter gezeigt. Eine kritische Prüfung der Resultate rechtfertigt die Durchführung von Messungen im Feld.

1. INTRODUCTION

In the recent years the attention toward durability has considerably increased, as many structures are approaching the end of their service lives and damages due to materials' deterioration are reported [1]. In this frame the problem of the evaluation of the fatigue damage and of the corresponding ramaining life begins to set up concretely, especially in the case of the bridges.
The analytical procedure for the evaluation of remaining fatigue life has been well known for a long time [2,3] but its practical application demands for reliable data and probabilistic methods.
In the present paper the attention is paid to the case of steel railway bridges because in these structures it is often possible to detect the fatigue cracking and to known with better approximation the magnitude and the cycles' number of past loads.

2. STATEMENT OF THE PROBLEM

In the present paper the problem of the evaluation of residual life is dealt with at Level 2, for an assigned value of the safety index β, with reference to a steel railway bridge. The structure is supposed to have been designed for fatigue with the criteria of Eurocode 3 [4].
It is in general possible to get additional information on the time history of the stress cycles in service, by means of strain measurements. The measures must be carried out during time intervals long enough to make possible the delicate process of extrapolation of measured data to the whole future life of the bridge. In this phase it is also necessary to set the correspondence between number of cycles and duration of time. It is often possible to examine in situ the hot spots of steel structure, in order to detect the presence and the length of eventual fatigue cracks by means of suited instruments .
The evaluation of the residual life is performed at an assigned time T, which corresponds to a known fraction of the duration of the design life. The stress collective, on the basis of field measurements, is assumed to be different from the one used at the time of the design. The corresponding damage is computed applying the Miner rule.
Then the influence of the load sequence is investigated applying the Paris law. The comparison of the numerical results encourages in carrying out in situ strain measurements, in order to improve the knowledge of the stress collective.

3. THE EVALUATION OF RESIDUAL LIFE

In the frame of the Level 2 methods for the assessment of safety, the residual fatigue life may be expressed as:

$$T_{res} = T_{res}(\mathbf{X}, \beta_0) \qquad (1)$$

where T_{res} is the duration of the residual life, $\mathbf{X}$ is the m-component vector of the design variables and β_0 is the prescribed value of the safety index.
If the problem parameters are:

$$\mathbf{X} = \{T_{past}, \Delta\sigma_{past}(t), \Delta\sigma_{fut}(t), R(N)\}^T \qquad (2)$$

where T_{past} is the duration of the past life, $\Delta\sigma_{past}(t)$ is the time history stress cycles applyed in the past, $\Delta\sigma_{fut}(t)$ is the time history of future stress cycles, $R(N)$ is the law of fatigue behaviour of material, then the (1) reduces to:

$$T_{res} = T_{res}(T_{past}, \Delta\sigma_{past}(t), \Delta\sigma_{fut}(t), R(N), \beta_0) \qquad (3)$$

Everyone of these parameters, except β_0, is in general a random variable. The safety index β depends on the same parameters and on T_{res}:

$$\beta = \beta(T_{res}, T_{past}, \Delta\sigma_{past}(t), \Delta\sigma_{fut}(t), R(N)) \quad (4)$$

The solution of the eq. (1) may be found by iteration solving the (4) in the form $\beta = \beta_0$. The (4) expresses a conventional Level 2 safety problem. Its solution requires to write down the limit state condition in the case of fatigue failure $Z(\underline{X})=0$. If the fatigue damage is computed using the Miner rule, then it is particularly simple:

$$Z = \Delta - \sum_{i=1}^{k} \frac{n_i}{N_i} = 0 \quad (5)$$

where n_i is the number of the stress cycles having amplitude $\Delta\sigma_i$, N_i is the number of the stress cycles of the same amplitude which would lead to failure and Δ is the ultimate value of the damage.
If the Paris law is used to evaluate the propagation of a critical fatigue crack, the eq. (5) modifies into the:

$$Z = a_f - a \quad (6)$$

where a is the current length of the crack and a_f is the corresponding ultimate value.
Then in both cases the basic variables $\underline{X}$ are transformed in a corresponding set of uncorrelated normal standard variables $\underline{x}$. Also the limit state condition is projected in the space of the variables $\underline{x}$ and becomes $z(\underline{x})=0$.
The limit state condition is approximated by means of an hyper-plane tangent to it in the point nearest to the origin:

$$z(\underline{x}) \approx \sum_{i=1}^{m} \alpha_i x_i + \beta = 0 \quad (7)$$

where $\underline{\alpha}$ is the vector of direction cosines of the normal to that hyper-plane. Then it follows:

$$\beta = -\sum_{i=1}^{m} \alpha_i x_i \quad (8)$$

and β is computed by iteration.
The value of Tres corresponding to $\beta = \beta_0$ is the solution of the eq. (2).

4. SOME REMARKS ON INPUT PARAMETERS

4.1 The stress collective

The time history of the loads applied to a structural component of a steel bridge is a random process, as it is the response to a random load, the traffic. In general it is a wide-banded process, whose power spectral density exhibits several maxima, corresponding to the structural natural modes and to the maxima of input process. In the study of the fatigue phenomena it is useful to know the statistic properties of the stress cycles of the response process.

It is well known that in the case of narrow-banded response the amplitudes of the stress cycles are Rayleigh distributed:

$$p(\Delta\sigma)=(\Delta\sigma)/var(\Delta\sigma))\exp(-\Delta\sigma 2/2var(\Delta\sigma)) \qquad (9)$$

In the opposite case when the response process is infinitely wide-banded the distribution of the amplitudes is approximately normal.
By means of integration it is possible to find the corresponding stress collective.
In the case of narrow band the integration of eq. (9) leads to:

$$N = N_{max}\exp(-\Delta\sigma 2/2var(\Delta\sigma)) \qquad (10)$$

with N_{max} = vT, where v is the frequency of the process and T is its duration.
It is easy to see that eq. (10) is a parabola in semilogarithmic scale (Fig.1a).
In the case of wide band the stress collective is computed by numerical integration and shows the trend of Fig.1b.
Also when the distribution of the cycles amplitudes is neither Normal, nor Rayleigh it is necessary to resort to numerical integration.

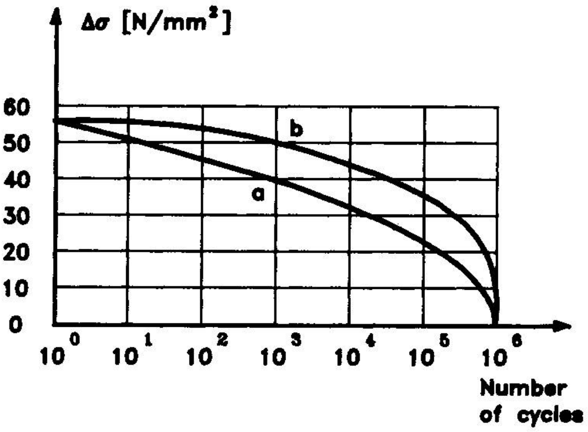

Fig.1 Stress collective: a) narrow band and b) wide band structural response

4.2 The damage criterion

The simplest and most widely used damage criterion for the evaluation of the fatigue life under variable amplitude loading is due to Palmgren e Miner. It expresses the fatigue damage D as:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} \qquad (11)$$

On the basis of laboratory tests Miner [5] reports that the range of values for D was from 0.61 to 1.49, with an average value close to unity.
In 1954 Marco e Starkey [6] report that in the case of sequences of ascending stress amplitudes the average value for D was 1.49 and in the case of descending loading the average value for D was found to be 0.78. To simulate this behaviour they propose the following non-linear damage criterion:

$$D = \sum_{i=1}^{k} \left(\frac{n_i}{N_i}\right)^{\delta} \qquad (12)$$

with δ depending on the order of application of loading blocks.
In the '60 the methods of assessment of damage based on fracture mechanics assert themselves, especially in the field of mechanical engineering. Among the others we remember the Paris - Erdogan law [7]:

$$da/dN = C(\Delta K_I)^n \qquad (13)$$

where da/dN is the rate of crack propagation and ΔK_I is the opening mode stress intensity factor. The crack does not grow if ΔK_I is lower than a threshold value K_{IC}. The fatigue failure occurs when the crack length a reaches the critical value a_f. Further improvements to this criterion are due to Klesnil e Lukas [8], who consider the threshold value as a function of the stress history and to Forman, Kearney , Engle [9], who take into account the ratio between the minimum stress and the maximum one in every cycle $R=\sigma_{min}/\sigma_{max}$.
In 1978 Hashin e Rotem [10] propose a non-linear damage criterion which remembers the sequence of stress cycles. It is expressed by the recurrent relation:

$$D = \mu_i \qquad (14)$$

$$\mu_i = \mu_{(i-1)}^A + n_i/N_i$$

with $A = (\Delta\sigma_i - \Delta\sigma_e)/(\Delta\sigma_{i-1} - \Delta\sigma_e)$
where $\Delta\sigma_e$ is the material endurance limit.

5. NUMERICAL APPLICATIONS

The proposed method of analysis was applied to an actual case, to test its validity and the sensitivity to input parameters.
A steel railway bridge was considered. The evaluation of the remaining fatigue life was performed at a welded joint of a secondary beam of the deck.
As far as the material is concerned, the SN law is assumed to be a straight line in logarithmic scale, without endurance limit, following the equation:

$$N\Delta\sigma^l = B \qquad (15)$$

with $l = 5$ and $B = 10^{16}$.
the ordinate at 10^6 cycles is therefore 100 N/mm^2. It corresponds to the mean value minus two standard deviations.
The crack propagation law is the Paris-Erdogan one with $n = 3.3$, $C = 2.43 \times 10^{-12}$ and $\Delta K_{IC} = 5.8\ MN/m^{3/2}$. These figures correspond to the mean value of variables.
The stresses applied to the structural member during its past life were extimated to fit into a collective of parabolic shape having a total duration of 100×10^6 cycles and maximum ordinate $\Delta\sigma_{max}$ of 50 N/mm^2. The collective of future stresses was supposed to have the same shape with the same values of duration and maximum ordinate.
The evaluation of fatigue damage was performed using the Miner rule, adopting as its critical value 0.8, as suggested by Augusti [11]. The residual life was computed at Level 2 applying the procedure presented in the chapter 3., for the value $\beta_o=3$ of the safety index. Also the values $\beta_o=2$ and $\beta_o=4$ were considered as a reference. Input data are presented in Table 1.
The analysis led to the following extimates of the residual life T_{res}, expressed as number cycles:
for $\beta_o=2$ $T_{res} = 3.30 \times 10^{10}$,
for $\beta_o=3$ $T_{res} = 6.53 \times 10^9$,
per $\beta_o=4$ $T_{res} = 1.17 \times 10^9$.
Then a sensitivity analysis was stated, varying some of the most important parameters.
First of all the influence of the mean value of the

random variable	distribution	expected value	c.o.v.
$\Delta\sigma_{max}$	log-normal	50 N/mm^2	0.20
T_{past}	log-normal	10^8 cycles	0.10
$\Delta\sigma_{SN}$	log-normal	167 N/mm^2	0.20
Δ	log-normal	0.8	0.20

Table 1 Input data of the basic case (Miner rule)

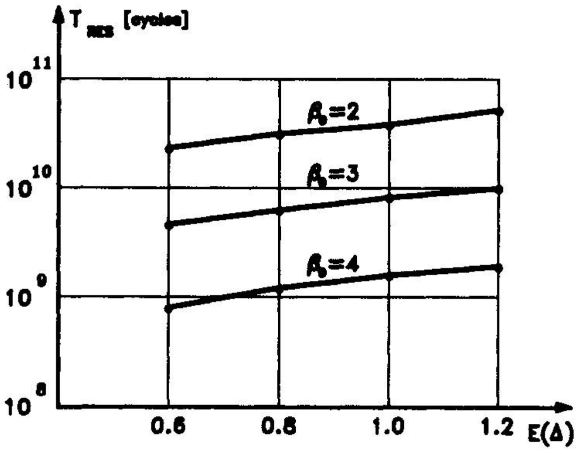

Fig.2 *Remaining life as a function of the mean value of the limiting figure of the Miner sum*

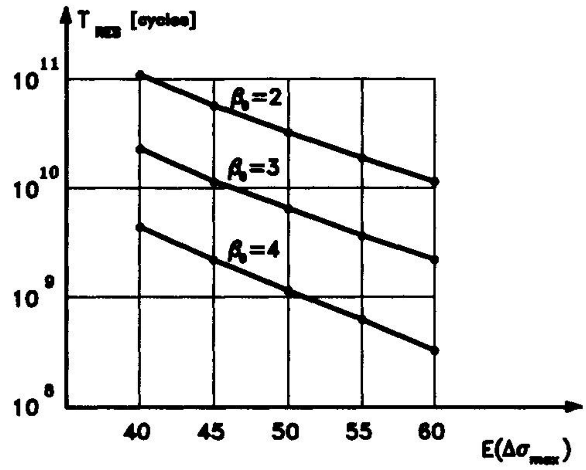

Fig.3 *Remaining life as a function of the mean value of $\Delta\sigma_{max}$ expressed in N/mm²*

limiting value of the Miner sum was examined, varying it in the range between 0.6 and 1.2.
The corresponding results are reported in Fig.2. In all the three cases considered for $ß_o$ a nearly twofold variation of the residual life was found between the most favorable case and the least one.
Then the influence of the parameters relating to the stress collective of the past was investigated. In the Figs. 3 and 4 the results of the sensitivity tests toward the variation of the expected value of $\Delta\sigma_{max}$ and of its coefficient of variation $V(\Delta\sigma_{max})$ are reported. For the mean value a range between 40 e 60 N/mm² was assumed. It lead to variations of about ten times on the evaluation of the residual life for all the three cases of $ß_o$. As far as the coefficient of variation is concerned, value ranging between 0.10 and 0.25 were considered. The corresponding variation on the residual life was 3.4 times for $ß_o=2$, 5.2 times for $ß_o=3$ and 11 times for $ß_o=4$.
Then the sensitivity to the duration of the past life was investigated, separately considering the effect of its mean value variation (Fig.5) and of the c.o.v variation (Fig.6). The mean value varyed between 10^6 and 10^9 cycles. For $ß_o=2$ and $ß_o=3$ a reduced sensitivity was found and for $ß_o=4$ a sensitivity was found only for very high number of cycles, near 10^9.
As far as the coefficient of variation is concerned (investigated range: 0.1 to 0.25), very little sensitivity was found for all the figures of $ß_o$ considered.

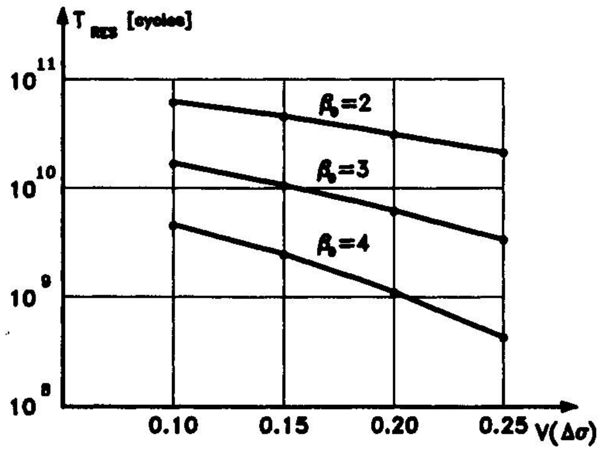

Fig.4 *Remaining life as a function of the c.o.v. of $\Delta\sigma_{max}$*

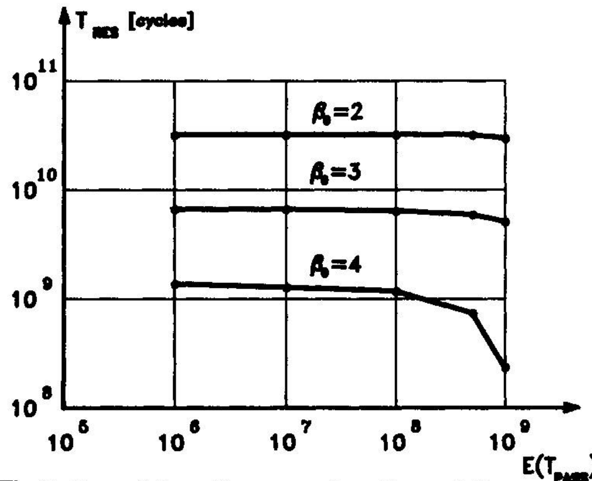

Fig.5 *Remaining life as a function of the mean value of the past life*

At the end of this part of the investigation the case of simultaneous variation of all input data was considered: first in the most favorable direction, then in the least favorable. The results are reported in Table 2.
Then an analysis of residual life was carried out applying the Paris-Erdogan law Input data are presented in Table 3 and the results are reported in Fig.7.
The numerical computations were performed under two different hypotheses of load application: ascending loading and descending loading. Four values of the initial crack length were also considered: 2,3,4,5 mm.

Case	$\beta=2$	$\beta=3$	$\beta=4$
the most favorable	3.44×10^{11}	9.77×10^{10}	2.76×10^{10}
the basic case	3.30×10^{10}	6.53×10^{9}	1.17×10^{9}
the least favorable	4.36×10^{9}	0.0	0.0

Table 2 Duration of the remaining life expressed as number of cycles

random variable	distribution	expected value	c.o.v.
$\Delta\sigma_{max}$	normal	50 N/mm^2	0.20
C	normal	1.43×10^{-12}	0.10
a_f	normal	40 mm	0.10

Table 3 Input data of the basic case (Paris law)

6. COMMENTS ON THE NUMERICAL RESULTS

Examining the results of numerical computations we can observe that the selected figure for β_0 has a great influence. In fact changing from the value 3.0, which is suggested by the Joint Committee on Structural Safety [12] and is the basic value of our analyses, to 2 the residual life increases an average 5 times and changing to 4 the life reduces an average 5 times.
During the numerical analysis the influence of the parameters describing the stress collective and of the limiting value of the Miner sum was investigated. The major sources of uncertainty are the mean value of the maximum stress range, its coefficient of variation and the damage criterion.
The idea rises from this to reduce the sources of uncertainties with experimental in situ observations oriented toward two directions. The first one is to detect

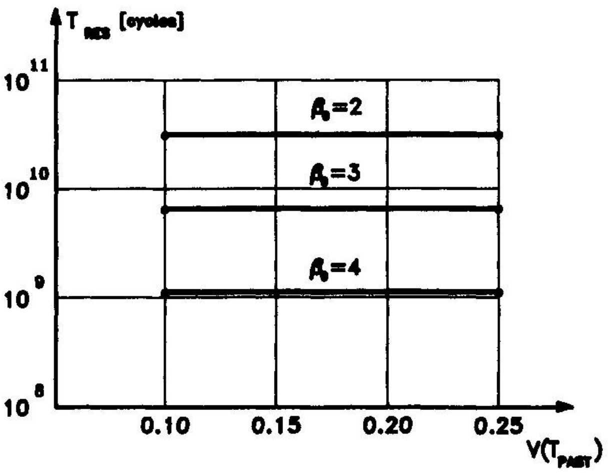

Fig.6 Remaining life as a function of the c.o.v. of the past life

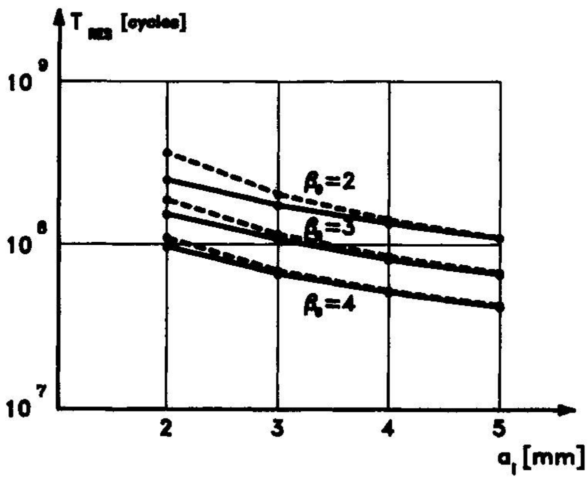

Fig.7 Remaining life computed with the Paris law: continuous line decreasing load, dashed line increasing load. ai is the initial value of the fatigue crack

the fatigue damage of the material by means of the measurement of the length of fatigue cracks. The second one is to improve the extimate of future stress history by means of sampling observations of bridge vibrations, extended to time intervals sufficient to warrant measure stationarity.
The application of the Miner rule with the improved data leads to better results. An alternative way is to use the Paris law, which allows to directly use load data without statistical reordering and the results of damage measurements.
The presented methodology is well suited to be a consistent procedure for the calibration of partial safety factors employed by the Level 1 methods.

7. CONCLUSIONS

The previous examination showed the differences between the procedures for the safety analysis of brand new structures and the existing ones. The first are methods connected to conventional probabilistic models and are intended to be support for the design process, the second ones can take advantage of experimental in situ tests that give improved information on the structural behaviour. Also in the case of fatigue problems it is often possible to gain better information whether using the Miner rule or applying the Paris law.
The theoretical fundamentals of the estimate of the residual fatigue life at Level 2 were reviewed.
The numerical experiments, which were carried out with reference to the case of a steel railway bridge, showed that the application of the method based on the Miner rule leads to estimates highly dependent on the uncertainties enbedded into the parameters that describe the past stress collective. This result encourages to undertake field tests to ascertain the damage accumulated in the past.
The use of the method based on fracture mechanics allow to improve the estimate quality and to take advantage of the information from field control of structural behaviour.
Than it seems reasonable the suggestion to subject the steel railway bridges to periodic control in order to detect incipient fatigue damage and to progressively update the evaluation of residual life. So it is possible to plan eventual strengthening works.

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