I

Cyclic Load-deflection Curves of Multi-storey Strain-hardening Frames Subjected to Dead and Repeated Alternating Lateral Loadings

Courbes cycliques charge-déformation de cadres à plusieurs étages soumis à des charges latérales dynamiques alternées

Zyklische Lastausbiegungs-Kurven von mehrstöckigen aussteifenden Rahmen unter Eigengewicht und wiederholter, wechselnder horizontaler Last

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1. INTRODUCTION

A rational theoretical investigation of the nonlinear behavior of a framed structure subjected to complicated alternating repeated loads must be based upon (1) an accurate stress-strain relation of the constituent structural material and the corresponding accurate cross-sectional force-generalized strain relations and (2) an accurate numerical method of analysis which is able to incorporate therein the complicated constitutive equations of the material and of the members. So far as the overall nonlinear behavior of a frame is to be investigated theoretically, any approximate formulation for the problem (1) must always be made, with the aim of and in a form convenient for, generating an accurate member-or element-stiffness or flexibility matrix which can be used on a computer currently available. While some complicated equations may be indispensable for describing complex nonlinear behaviors of a member with a considerable accuracy, it will be necessary to introduce some approximation in accordance with its aim.

The present contribution to the prepared discussion describes first an efficient computational method of analyzing nonlinear static and dynamic behaviors of multi-story plane steel frames. The method takes into account the gravity effect due to large deflection, incorporates the bilinear or nonlinear hysteretic stress-strain relation for a structural steel and is able to trace gradual spreading or diminishing of strain-hardening regions along member axes. Since the general idea of the authors' (Nakamura and Ishida) method has been presented in [1], the details of the procedure of generating an elastic-plastic member-stiffness matrix applicable to incremental large deflection analysis are described here. It is at this stage that the appropriateness of an approximate formulation of the stress-strain relation and of the corresponding axial force-moment-curvature relation is assessed with respect to its applicability to an overall frame. Although the numerical results in this prepared discussion are based upon the bilinear hysteretic stress-strain relation, the proposed method is of such a formulation that is able to incorporate a nonlinear hysteretic stress-strain relation. The senior authors' (Yokoo and Nakamura) contribution to the prepared discussion on Theme III presents a nonstationary hysteretic stress-strain relation and an approximate formulation of the moment-curvature relation under the presence of an axial force with the intention of incorporating them in the present method of generating the member-stiffness matrix. Some numerical results of the static load-deflection analysis of multi-story frames subjected to dead and alternating repeated lateral loads and of the dynamic analysis of the frames subjected to an amplified earthquake disturbance of a recorded wave form are presented in order to illustrate the efficiency of the method and to clarify the gravity effect and the effect of strain-hardening.

2. ELASTIC-PLASTIC MEMBER-STIFFNESS MATRIX FOR INCREMENTAL LARGE DEFLECTION ANALYSIS

Each member having an idealized sandwich section is treated as one element for its elastic response and then subdivided automatically in the program into as many elements as are necessary as the strain-hardening regions spread thereover. For the purpose of generating an accurate member-stiffness matrix, the transfer matrix technique in a form extended so as to incorporate the effect of accumulated large deflection, is applied to the member as a subsystem consisting of one-dimensionally connected elements. The essential steps of generating a member-stiffness matrix are as follows:(1) Derivation of an element-stiffness matrix [1] for a cantilever element as shown in Fig.1 in a local coordinate system, in a form excluding the rigid-body displacements; (2) Transformation of the cantilever element-stiffness matrix into an expanded form with respect to a global coordinate system of the member so as to incorporate the rigid-body displacements; (3) Sequential interconnection of the expanded element-stiffness matrices by the transfer matrix technique; (4) Transformation of the contracted transfer matrix into the member-stiffness matrix with respect to the member end forces and displacements. For the convenience of and in view of the accuracy consistent with the numerical integration with respect to time, the rate of a field variable is directly approximated by a finite increment and the problem for any incremental step is linearized without iteration for the nonlinear effect within the step.

2.1 TRANSFORMATION OF INCREMENTAL ELEMENT-NODAL DISPLACEMENT VECTOR

The nodal displacement vector $\{d_a\}=\{u_a, v_a, \theta_a\}^T$ for a cantilever element shown in Figs.1 and 2 in the local coordinates is transformed into the vector $\{D\}=\{U_a, V_a, \Theta_a, U_b, V_b, \Theta_b\}^T$ in the global coordinates by

$$\{d_a\}=[[T_R]^T \,\vdots\, -[T_R]^T]\{D\}+[T_R]^T\{1,\ 0,\ 0\}^T-\{1,\ 0,\ 0\}^T, \tag{1}$$

where all the displacement components have been nondimensionalized with respect to l, the length of the undeformed element and where

$$[T_R]=\begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad c = \cos\Theta_b \text{ and } s = \sin\Theta_b \tag{2}$$

The linear incremental transformation equation may be written as

$$\{\Delta d_a\}=[[T_R]^T \,\vdots\, -[T_R]^T]\{\Delta D\}+[[\Delta T_R]^T \,\vdots\, -[\Delta T_R]^T]\{D\}+[\Delta T_R]^T\{1\ 0\ 0\}^T, \tag{3}$$

where

$$[\Delta T_R]=\begin{bmatrix} -s & -c & 0 \\ c & -s & 0 \\ 0 & 0 & 0 \end{bmatrix}\Delta\Theta_b=\begin{bmatrix} -s & -c & 0 \\ c & -s & 0 \\ 0 & 0 & 0 \end{bmatrix}[0\ 0\ 0\ 0\ 0\ 1]\{D\}. \tag{4}$$

In view of Eq.(4), Eq.(3) may be reduced to the form

$$\{\Delta d_a\}=[T]\{\Delta D\}, \tag{5}$$

where

$$[T]=\begin{bmatrix}[T_R]\\ \hline -[T_R][H]\end{bmatrix}^T \quad \text{and} \quad [H]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ -v_a & 1+u_a & 1\end{bmatrix}. \tag{6a, 6b}$$

2.2 TRANSFORMATION OF INCREMENTAL ELEMENT-NODAL FORCE VECTOR

The transformation law between the nondimensionalized nodal force vector $\{p_a\}=\{p_a\ q_a\ r_a\}^T$ for the cantilever element in the local coordinates and the vector $\{P\}=\{P_a\ Q_a\ R_a\ P_b\ Q_b\ R_b\}^T$ in the global coordinates may be written, directly from the contragradient law, as

$$\{P\}=[T]^T\{p_a\}. \tag{7}$$

Eq.(7) can of course be derived directly by writing equilibrium equations. The linear incremental transformation equation may be written as

$$\{\Delta P\}=[T]^T\{\Delta p_a\}+[\Delta T]^T\{p_a\} \tag{8}$$

The second term of the right hand side of Eq.(8) may be transformed into the expression in terms of $\{\Delta D\}$, i.e.

$$\{\Delta T\}^T\{p_a\}=[P_a]\{\Delta D\} \tag{9}$$

where

$$[P_a]=\left[\begin{array}{c|c}[0] & \{F_a\}\\ \hline \{F_a\}^T & M_a\end{array}\right], \quad \{F_a\}^T=\{-Q_a\ P_a\ 0\ Q_a\ -P_a\},\quad M_a=-q_a v_a-P_a(1+u_a) \tag{10a~c}$$

2.3 EXPANDED ELEMENT-STIFFNESS MATRIX

Let $[\kappa]$ denote the 3×3 element-stiffness matrix in the local coordinates, as has been derived in [1]. The stiffness equation

$$\{\Delta p_a\}=[\kappa]\{\Delta d_a\}. \tag{11}$$

incorporates not only the effect of large deflection but also the strain-hardening effect. Substitution of Eqs.(9), (11) and (5) into Eq.(8) provides

$$\{\Delta P\}=[\kappa_G]\{\Delta D\}, \tag{12}$$

where

$$[\kappa_G]=\left[[T]^T[\kappa][T]+[P_a]\right] \tag{13}$$

is the desired expanded element-stiffness matrix in the global coordinates.

2.4 INCREMENTAL MEMBER-STIFFNESS EQUATION

Eq.(12) may further be rewritten in terms of the state vectors to define the field transfer matrix for the element. By applying the standard procedure of the transfer matrix method to a member j consisting of several elements, the field transfer matrix in terms of the state vectors at the left and right ends of the member can be derived. The resulting state equation may readily be reconverted into the desired incremental member-stiffness equation.

3. COMPUTATIONAL METHOD

The displacement increment method developed by the present authors (Nakamura and Ishida [2,3]) in 1969 for the second-order analysis of elastic-perfectly plastic frames, has been applied to obtain load-displacement curves of the strain-hardening frames subjected to piecewise proportional loading. The method is simply to convert the ordinary system stiffness equation

$$[K]\{\Delta u\}=\Delta\lambda\{f\} \tag{14}$$

in terms of the nodal displacement vector $\{\Delta u\}$ and the nodal force vector $\Delta\lambda\{f\}$ prescribed by the load factor $\Delta\lambda$, into the form

$$[K^*]\{\Delta u^*\}=-\Delta u_i\{k_i\} \tag{15}$$

where

$$[K^*]=[\{k_1\}\{k_2\}\cdots\cdots\{k_{i-1}\}\{f\}\{k_{i+1}\}\cdots\cdots\{k_n\}], \tag{16a}$$

$$\{\Delta u^*\}=\{\Delta u_1\ \Delta u_2\cdots\cdots\Delta u_{i-1}\ -\Delta\lambda\ \Delta u_{i+1}\cdots\cdots\Delta u_n\}^T, \tag{16b}$$

and to solve Eq.(15) for a prescribed increment Δu_i of a representative displacement u_i. This method enables one to trace load-displacement curves beyond their elastic-plastic limit points at which [k] becomes "singular" or computationally "nearly singular". For dynamic analysis, Wilson-Clough's method of numerical integration with respect to time has been utilized.

4. NUMERICAL RESULTS

4.1 VERIFICATION OF THE ACCURACY OF THE MEMBER-STIFFNESS MATRIX

Fig.4 shows the load-deflection curve of a cantilever column (shown in Fig.3) subjected to a constant axial force and a repeated alternating lateral load. The numerical result agrees almost precisely with the analytical result due to Nakamura [4]. Fig.6 shows the load-deflection curve for a roller-supported beam (Fig.5) subjected to a constant axial force and a repeated alternating lateral load. The result appears to exhibit a fairly good numerical simulation of the actual behavior shown in Fig.7 which was obtained experimentally by the senior authors [5]. While the value 0.01E of the linear strain-hardening coefficient has not been derived by any approximation theory for equivalence and while the accuracy of simulation of the behavior near the shoulder portions can not be said to be good, yet the result seems to promise the effect of refinement by incorporating the nonlinear hysteretic stress-strain relations for the flanges of equivalent sandwich sections.

4.2 NUMERICAL INVESTIGATION OF THE LARGE-DEFLECTION ELASTIC-PLASTIC BEHAVIORS OF LINEAR STRAIN-HARDENING MULTI-STORY FRAMES OF ONE-BAY

Fig.8 shows the dimensions of the model frame. Three frames have been designed for the base shear coefficient values $S_B=0.05$, 0.10 and 0.15. The method of minimum weight design [6] was applied in a modified form to the frames with the assumed inverted triangular lateral force distribution and with an *a priori* estimate of the $P\Delta$-effect based upon a linear sidesway mode. The numbers in the round bracket in Fig.8 denote the cross-sectional areas of the members of the frame designed for $S_B=0.10$. Fig.9 shows the load-top displacement curves of the three frames under uniformly distributed one-way lateral loads. Fig.10 shows the *overall* force-displacement curves [3] of the same result. Fig.11 and 12 show respectively the load-top displacement curves and the corresponding *overall* force-displacement curves of the three frames under uniformly distributed alternating lateral loads repeated with a constant top-displacement amplitude. It can readily be observed that the first maximum loads are almost proportional to S_B and that the load-displacement curves for a frame designed for a smaller S_B exhibit more deterioration. The hysteresis loop for $S_B=0.15$ has converged more rapidly to a steady-state loop, whereas the loop for $S_B=0.05$ exhibits a gradual cyclic deterioration.

The numerical experiments for the effect of member stiffness distribution on the hysteresis loops have also been carried out but no significant effect has been found. Incidentally, the computation time for one incremental step was about 1~2 sec. on a FACOM 230-60 computer.

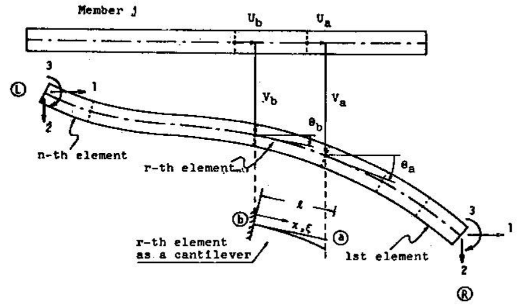
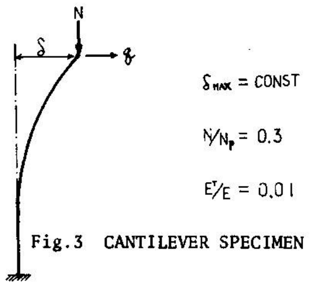


Fig.1 COORDINATES FOR A MEMBER AND FOR AN ELEMENT



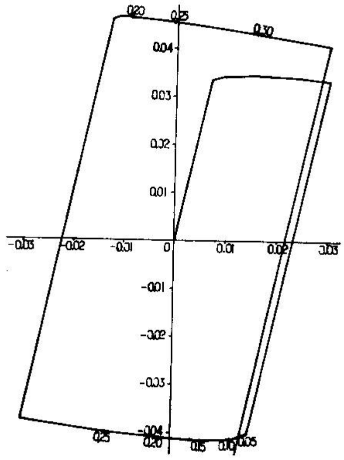


Fig.4
LOAD-DEFLECTION CURVE
FOR CANTILEVER

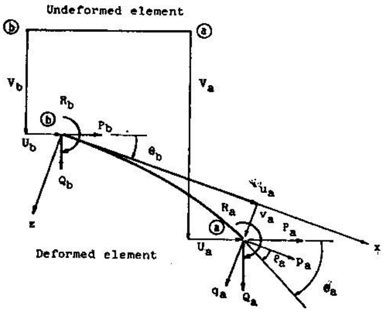
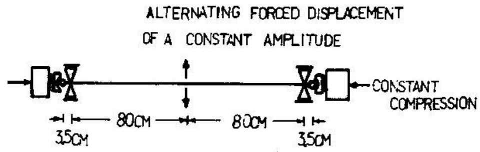


Fig.2 LOCAL AND GLOBAL COORDINATES FOR AN ELEMENT



SPECIMEN DIMENSION	
AREA	23.6 cm^2
SECTIONAL MOMENT OF INERTIA	376 cm^4
HALF SPAN LENGTH	80 cm
COMPRESSIVE YIELD STRESS	3.02 t/cm^2
TEST CONDITION	
AXIAL FORCE RATIO	0.567
FORCED DEFLECTION AMPLITUDE	14.8 cm

Fig.5 SIMPLE BEAM SPECIMEN

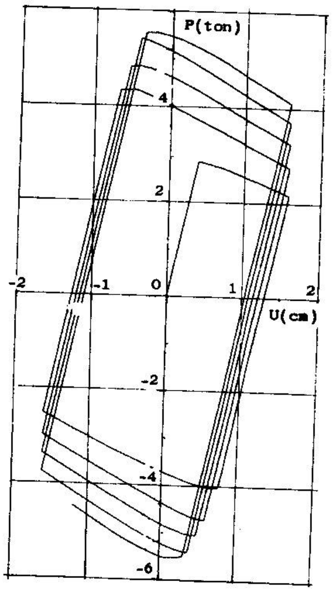


Fig.6 COMPUTED LOAD-DEFLECTION CURVE FOR SIMPLE BEAM SPECIMEN

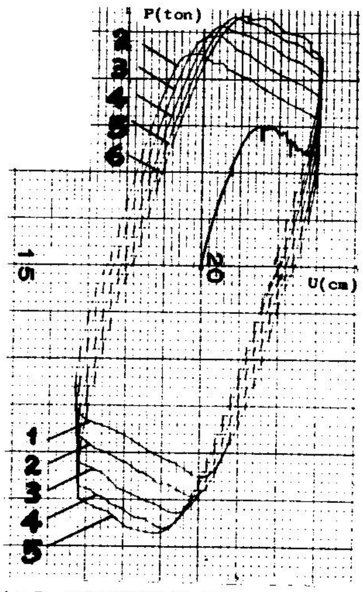


Fig.7 EXPERIMENTAL LOAD-DEFLECTION CURVE FOR SIMPLE BEAM SPECIMEN

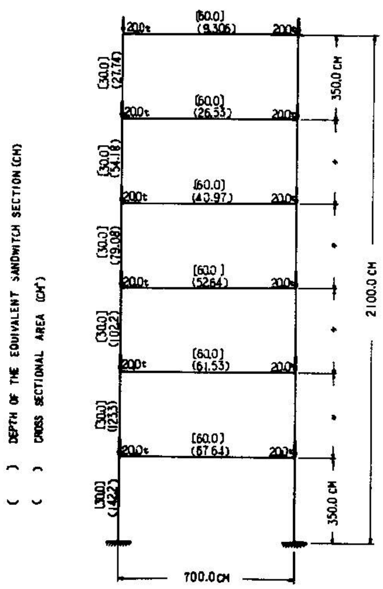


Fig.8 FRAME DIMENSION

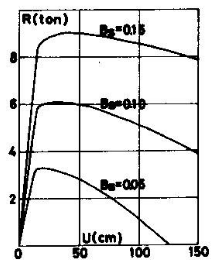


Fig.9 ONE WAY LOAD-DEFLECTION CURVES

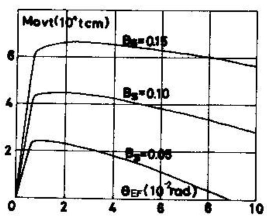


Fig.10 ONE WAY OVERALL FORCE-DEFLECTION CURVES

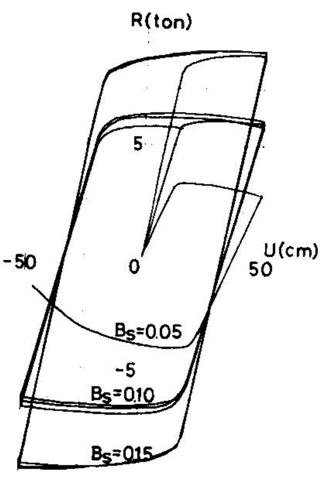


Fig.11 CYCLIC LOAD-DEFLECTION CURVES

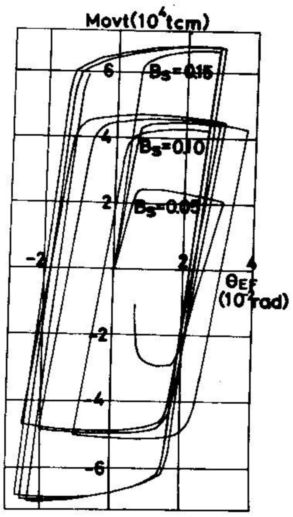


Fig.12 CYCLIC OVERALL FORCE-DEFLECTION CURVES

4.3 RESPONSE ANALYSIS OF FRAMES SUBJECTED TO A STRONG-MOTION EARTHQUAKE DISTURBANCE

The three frames with the dimensions shown in Fig.8 and described in 4.2 have been subjected to the amplified earthquake excitation of 1.0g with the wave patterns of VERNON S82°E. The stiffnesses of the springs shown in Fig.13 representing the foundation stiffnesses were determined by Barkan's method. The internal damping coefficients for the members and the foundation springs were assumed to be 0.01 and 0.20, respectively. As an example of the numerical results, the story shear-relative story displacement diagram for S_B=0.05 has been shown in Fig.14.

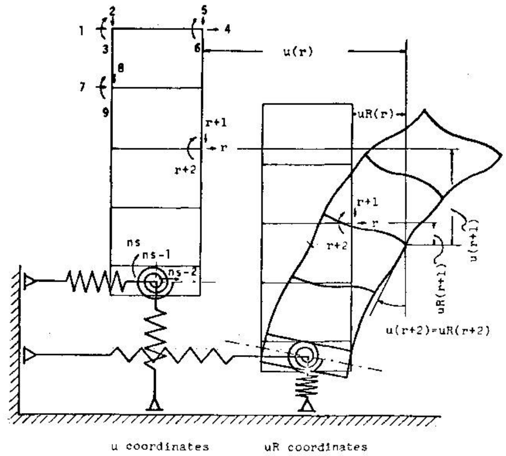


Fig.13 IDEALIZED FRAME AND COORDINATE SYSTEM

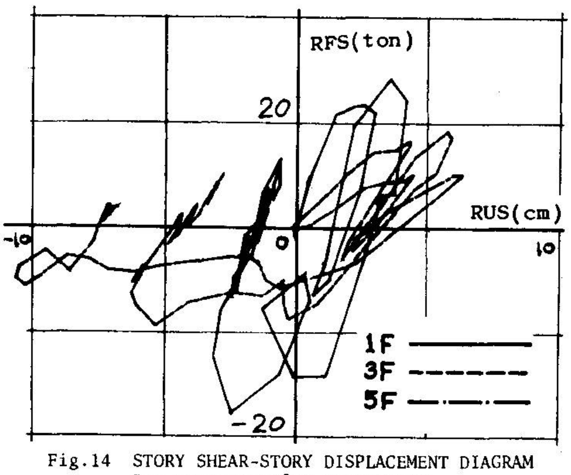


Fig.14 STORY SHEAR-STORY DISPLACEMENT DIAGRAM FOR VERNON(S82°E) EXCITATION

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SUMMARY

An efficient computational method of static and dynamic large-deflection analysis of strain hardening plane frames has been outlined. The load-displacement curves of a cantilever, a beam-column and three six-story frames of one-bay subjected to alternating repeated loads have been presented and the effect of cyclic alternating plastic deformation on the load-carrying behaviour of frames has been investigated. The effect of gravity and of strain-hardening upon the response of frames to an amplified earthquake disturbance has been clarified to some extent through the numerical examples.

RESUME

On esquisse dans cette étude une méthode d'analyse numérique pour le calcul des cadres plans dont certaines sections travaillent dans le domaine d'écrouissage, en considérant les grandes déformations statiques et dynamiques. On présente les courbes charge-déplacement pour une poutre en porte à faux, une colonne et trois portiques multiples à six étages et une travée, soumis à des charges alternantes répétées. On étudie l'effet des déformations plastiques alternées sur le comportement des portiques chargés. L'influence de la pesanteur et de l'écrouissage sur le comportement des portiques soumis à une perturbation sismique amplifiée a été analysée jusqu'à un certain degré dans les exemples numériques.

ZUSAMMENFASSUNG

Es wird eine leistungsfähige Computer-Methode zur Berechnung statisch und dynamisch grosser Auslenkungen von versteifenden ebenen Rahmen vorgeführt. Die Last-Ausbiegungskurven eines Kragarms, einer Stütze und dreier zweistieliger sechsstöckiger Rahmen unter wechselseitiger zyklischer Belastung werden gezeigt und der Effekt von zyklischer wechselseitiger plastischer Deformation auf das Tragverhalten der Rahmen untersucht. Die Wirkung der Gravitation und der Verfestigung auf die Reaktion der Rahmen auf eine verstärkte Erdbebenstörung wird zum Teil an numerischen Beispielen erklärt.