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V

Safety Concepts for Non-Repeated and Repeated Loadings

Concepts de sécurité pour des charges non-répétées et répétées

Sicherheitskonzept für nicht wiederholte und wiederholte Belastungen

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1 - INTRODUCTION

For discussing structural safety concepts in the cases of repeated loading it is convenient to consider first the problems in which the effect of load repetitions is disregarded. Then the reasoning used can be extended so as to cover the cases in which the effects of load repetition are important. This extension is difficult both from conceptual and from practical points of view. In fact, the influence of load repetitions on limit-states is a complex problem. In most cases satisfactory definitions of the parameters to be considered do not yet exist.

On the other hand even basic safety concepts differ with the various branches of engineering. Reliability criteria used in the design of aircraft and space vehicles considerably differ from structural safety criteria in civil engineering structures. To justify and to clarify the different positions is too broad a task exceeding the scope of this report. On the other hand it is quite evident that advances in structural design depend to a large extent on such a clarification.

The classification of load variability in relation to the effect of load repetitions on resistance is a basic problem. In fact, the influence of loading history on resistance has to be considered in a simplified way. In recent years this problem has been much studied by researchers interested in fatigue. More and more sophisticated test procedures have been introduced. Nevertheless choice of the major parameters of life history of a structural system remains controversial. General principles, such as those of cumulative damage, are useful as guide lines. However the presented trend is to reproduce the main features of service loads through idealizing them by suitably combining deterministic and stochastic processes.

It is usual to distinguish low-cycle loading from high-cycle loading leading to fatigue. The actual rupture phenomena are different in the two cases, although, conceptually, the same procedure should be used in both problems and no definite demarcation can be established.

2 - GENERALIZED FORMULATION OF STRUCTURAL SAFETY

2.1 - Structural safety problems

Several criteria can be used for classifying structural safety problems as they are usually considered in Civil Engineering. The following classification, although not systematic, may give an overall view of the question.

According to the objectives to be reached the following main problems can be identified:

i) to compute the generalized cost of the structure during a given interval of time;
ii) to compute the probability of a given limit-state being reached during a given interval of time;
iii) to relate the above probabilities of reaching limit-states with the values of the ponderation factors;
iv) to derive simplified design rules.

The first problem involves initial costs, relationships between limit-states and the corresponding damage costs, and probabilities of reaching the different limit-states. For dealing with it, it is necessary to solve the second problem: i.e. to compute the probabilities of reaching the different limit-states.

Furthermore the third problem involves the solution of sets of problems of the second type, using the ponderation factors as parameters. By doing so for typical cases, basic information can be obtained from which simplified design rules can be derived. Consequently, in the following, attention is concentrated on the second problem, taken as the principal one.

Another classification criterion of safety problems may be based on the number of variables necessary for defining the different types of loads, the load-effects due to the loads, and the load-effects corresponding to the different limit-states.

In case one variable alone suffices to define both the loading and the limit-state, the probability of reaching the limit-state can be obtained simply by computating a convolution integral. This is a basic problem in structural safety, so far discussed by several authors for more than 30 years (1).

When the loads are of different types and are defined by several — uncorrelated or correlated — components, the computation of the probability of failure becomes more involved. Yet the conceptualization of this general problem is of much importance in serving as a guide for simplified solutions.

Such a problem is discussed below assuming that the behaviour relationships and the limit-states can be defined independently of the type of loading. Further on, the case in which this influence exists is also considered.

Other classification criteria of safety problems can be used. Such criteria may be based on the type of structure and on the type of loading.

Thus one must distinguish statically determinate from statically indeterminate structures, linear from non-linear behaviour, and deterministic from random idealizations of structural behaviour.

As regards loads one must distinguish problems of imposed forces, of imposed deformations and mixed problems; static and dynamic behaviour (the latter involving inertia forces); and deterministic and random definitions of loadings. Finally, as mentioned, the way loadings vary in time may also be of basic importance.

In the discussion that follows each structure is assumed to be made up of a finite number of structural elements. The safety of the structure is to be studied on the basis of the safety of the structural elements. For that purpose the loads applied on the structure are transformed into load-effects acting on the elements.

In order to give generality to the presentation, both applied loads and applied displacements are referred to as generalized loads or simply loads. Likewise, the behaviour of each structural element is defined by the relationship between generalized load-effects and generalized displacements. In this context, forces, moments of forces, and stresses are considered as generalized load-effects. On the other hand, geometrical quantities such as displacements, rotations, strains, distorsions, curvatures, crack lengths, distances between cracks and crack widths are considered as generalized displacements. The behaviour of structural elements will thus be studied in the "generalized load-effects"-"generalized displacements" space.

Under deterministic assumptions, behaviour is described by given surfaces, and limit-states can be expressed as lines or surfaces in this space. Although limit-states may be of very different nature: rupture, deformation, limitation of cracks, etc., the values of the generalized load-effects corresponding to limit-states are simply referred to as resistances.

Randomness of structural behaviour can be introduced in two ways. As a randomness of the "generalized load-effects"-"generalized displacements" relationships or as a randomness of limit-states. In a simplified way, the former is called "randomness of behaviour" and the latter "randomness of resistance".

Further randomness of loading has also to be considered. Owing to the structural behaviour this randomness is transformed into a randomness of acting load-effects or displacements. Thus the probability of reaching a given limit-state is obtained by combining the load randomness, expressed in the "generalized load-effects-generalized displacements" space, with the resistance randomness.

2.2 - Statically determinate structure under a single loading

Consider the case of a statically determinate structure, for instance a cantilever (Fig. 1), under a single force, X . In such a case both loading and resistance can be expressed by the same variable, X . The maximum value of the force and the resistance being assumed to be random, densities of probability $f(X)$ and $f(X_u)$ can be defined. The integral of these densities of probability are distribution functions $F(X)$ and $F(X_u)$, Fig. 2.

When dealing with structural safety problems, the parameters usually adopted for defining these distribution functions are: i) the mean values $\bar{X}$ and $\bar{X}_u$ (or the median values corresponding to the 0.50 fractiles) and ii) the

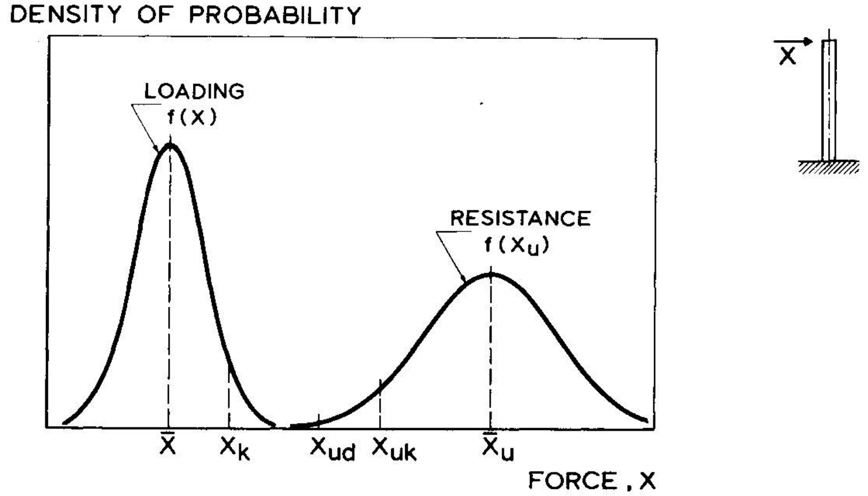

Fig. 1 - Densities of probability of loading and resistance.

characteristic values, X_k and X_{uk}, corresponding to the 0.95 and 0.05 fractiles. Furthermore a resistance value, called design value, is often

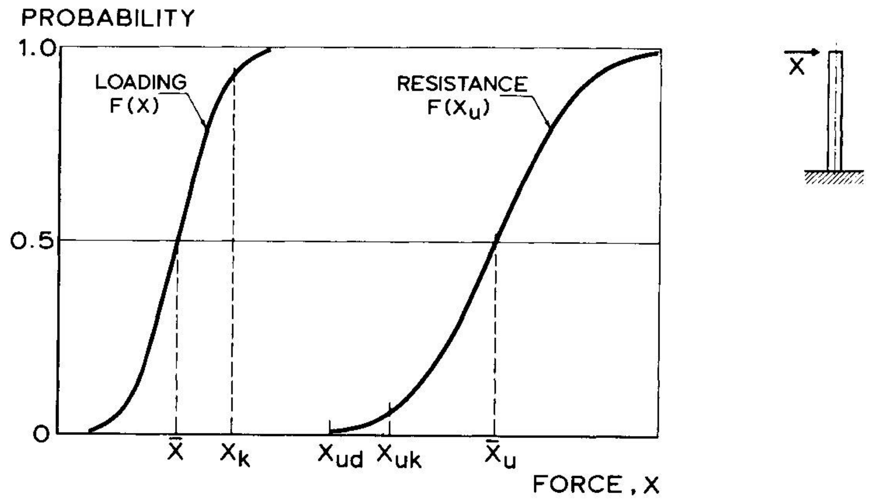

Fig. 2 - Distribution functions of loading and resistance.

defined by a fractile different from 0.05. According to the usual practice the design values of the resistances related to rupture correspond to fractiles of the order of magnitude 0.005.

The probability of not attaining a limit-state, probability of efficiency, is given by the probability of the resistance exceeding the loading. This probability, also called probability of survival when dealing with the limit-state of rupture, is given by the convolution integral:

$$P_e = \int_{-\infty}^{+\infty} F(X)\, dF(X_u) \qquad 1)$$

Given the analytical expressions of $F(X)$ and $F(X_u)$, the probability of efficiency can be related to the parameters that define the distributions and in particular to the relationships $\gamma_k = \frac{X_{uk}}{X_k}$ and $\gamma_d = \frac{X_{ud}}{X_k}$, which are called characteristic safety factor and design safety factor, respectively.

In the problem under discussion the variability of the loading in time is not considered. It is simply assumed that, given a structure, the distribution functions of the loading and of the resistance may be defined. In the present case, the structure being statically determinate the loading and the resistance can also be directly expressed in terms of load-effects.

2.3 - Statically determinate structure under several loadings

Consider further the case of a statically determinate structure, Fig. 3, now under two forces X_1 and X_2. Assume the resistance of this structure to depend on the values of the load-effects M and N at the built in section.

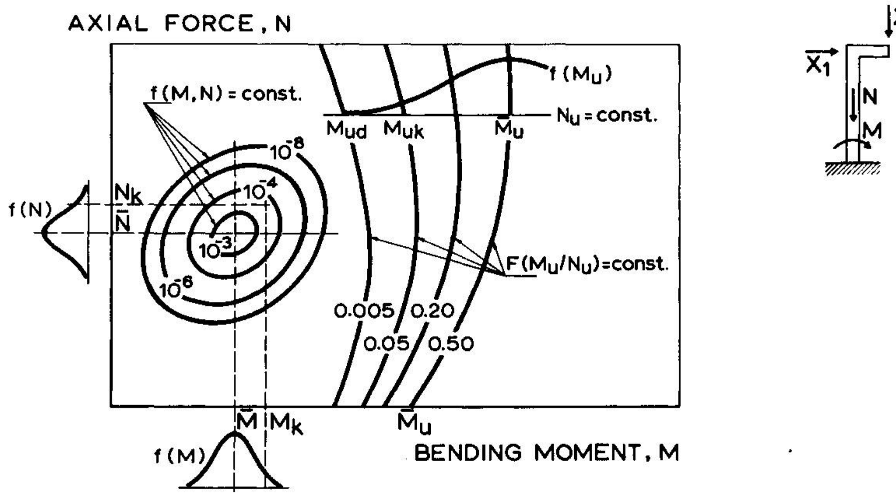

Fig. 3 - Bivariate densities of probability of loading and resistance.

Resistance being random, a density of probability $f(M_u, N_u)$ can be defined. To a given value of N_u there corresponds a density of probability $f(M_u)$ and a distribution function $F(M_u)$. As above, mean, characteristic and design values, $\overline{M}_u$, M_{uk} and M_{ud}, can also be considered, Fig. 3.

On the other hand, assuming that the densities of probability $f(X_1)$ and $f(X_2)$ are given, the bivariate density of probability $f(M, N)$ and the distribution function $F(M, N)$, expressed in terms of load-effects, are easily obtainable.

In this case the probability of efficiency (probability of the resistance exceeding the loading) is given by:

$$P_e = \int_{\Omega} F(M, N)\, dF(M_u, N_u) \qquad 2)$$

the integral being extended to the whole space of the load-effect variables, $\Omega(M, N)$.

In the present case, more than one load being applied, it is important to study the problem of load combination. Such a problem was approached in a recent paper (2) by assuming the loads to be statistically independent and by disregarding the influence of the variability of the loading on the limit-states.

2.4 - Statically indeterminate structure under a single loading

Consider the case of a simple statically indeterminate structure made up of two elements, a cantilever and a spring, Fig. 4. Assume that only region A of the cantilever is deformable, and that its behaviour is expressed

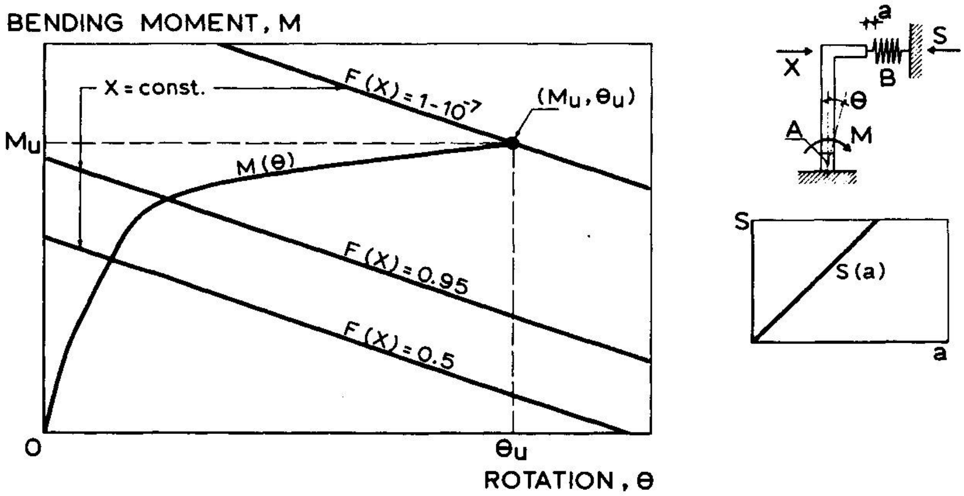

Fig. 4 - Linear statically indeterminate structure acted on by a single random loading.

by the moment-rotation relationship, $M(\theta)$, and its resistance by the position of point (M_u, θ_u). On the other hand, assume the behaviour of spring, B, to be linear and expressed by $S(a) = k\,a$. Displacement a can be directly related to rotation θ by the expression $a = L\,\theta$, L being the distance of the center of region A to the line of action of forces X and S. In this case the space of "generalized load-effects - generalized displacements" has two dimensions: the bending moment M and the rotation θ.

To a given value of the force X there corresponds in space (M, θ) a straight line with the equation:

$$M = (X - S(a))\,L = (X - k\,a)\,L = X\,L - k\,L^2\,\theta \qquad 3)$$

Assuming that the distribution function $F(X)$ exists, a probability $F(X)$ corresponds to each line $X = \text{const.}$ The value of $F(X)$ corresponding to the line $X = \text{const.}$ which contains the point (M_u, θ_u) measures the probability of efficiency.

If the behaviour of spring B is non-linear, Fig. 5, the lines $X = \text{const.}$ in the plane (M, θ) are no longer straight. Yet a probability $F(X)$ corresponds to each line, and the problem is analogous to the one above.

In the two last examples the behaviour of elements A and B were deterministic, only the loading being random. It may also be assumed that the behaviour of A, of B, or of both are random.

Assume the behaviour of A to be random and the reaching of its limit-state to be expressed by the density of probability $f(M_u, \theta_u)$, Fig. 6. The probability of efficiency is:

$$P_e = \int_{\Gamma} F(X\,(M, \theta))\; f(M_u, \theta_u)\; dM\, d\theta \qquad 4)$$

the integral being extended to the whole space Γ (M , θ).

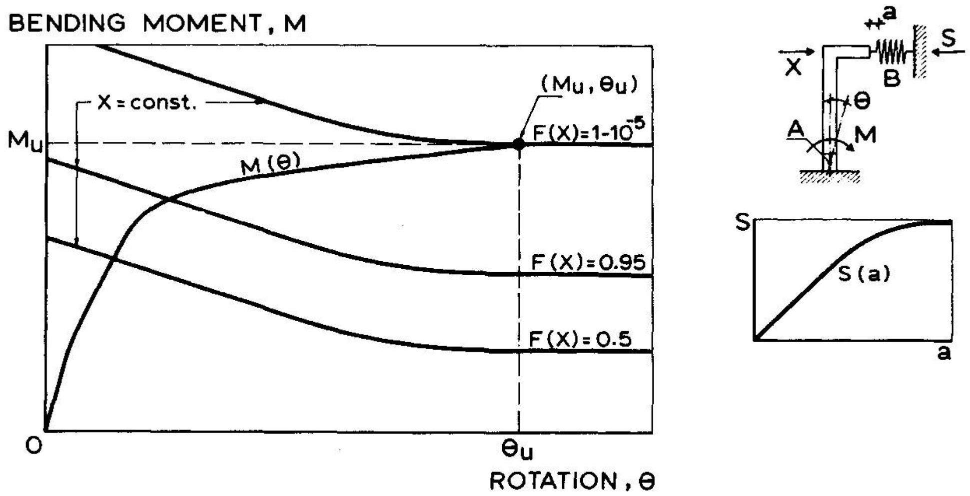

Fig. 5 - Non-linear statically indeterminate structure acted on by a single random loading.

The randomness of the behaviour of B influences the distribution function F(X) and could also be considered.

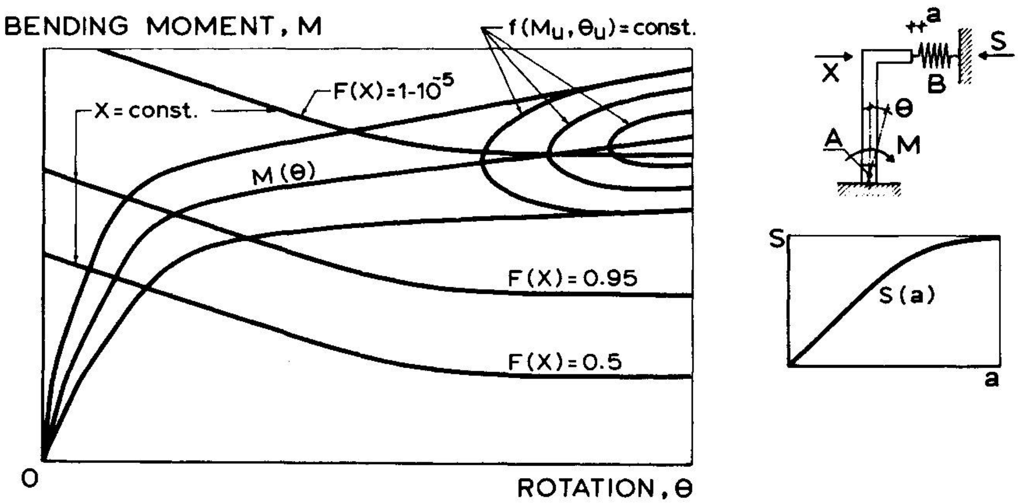

Fig. 6 - Non-linear statically indeterminate structure having random behaviour and acted on by a single random loading.

The preceding formulation covers in general terms the definition of the probabilities of efficiency (or survival) in statically indeterminate structures under one loading. In fact, assuming the structure to be made up of a finite number of elements, the probability of efficiency in each element may be obtained as above, by taking the element under consideration as element A and the whole complementary structure as element B , (3). By suitably combining the probabilities of efficiency of each element, the probability of efficiency of the whole structure can be obtained.

2.5 - Statically indeterminate structure under several loadings

By associating the reasonings presented in 2.3 and 2.4 it is possible to solve in quite general terms the problem of computing the probability of efficiency of any element of a statically indeterminate structure under several loadings.

The simple example presented in Fig. 7 indicates how to dispose of this problem. In the present case the "generalized load-effects - generalized displacements" space has three components M , N and θ , and the density of probability of the loading is expressed by f (M , N/θ), which is obtained

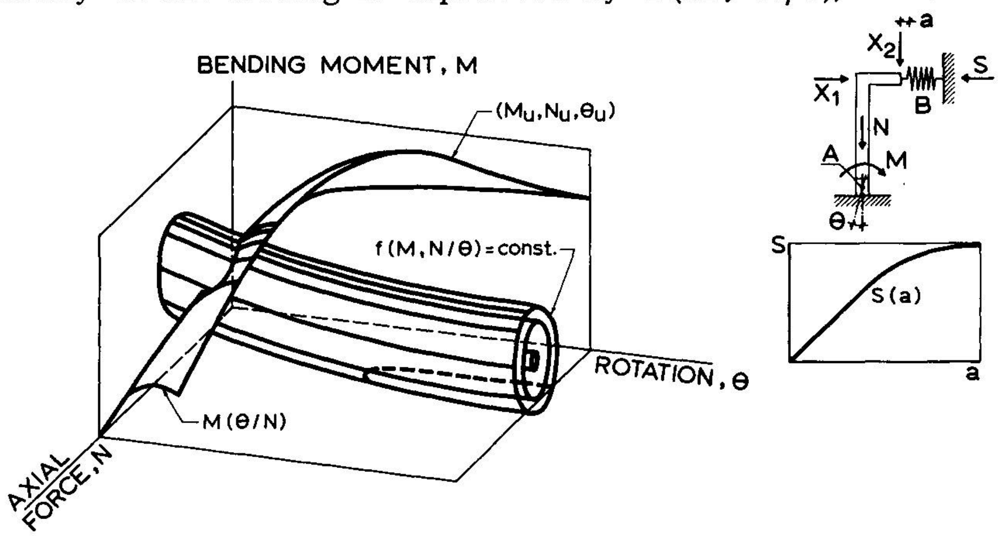

Fig. 7 - Non-linear statically indeterminate structure acted on by two random loadings.

from the densities of probability f (X_1) and f (X_2) taking into account relationships (M , N , θ) (X_1 , X_2 , S(a (θ))). The distributions f (X_1) and f (X_2) being normal, the curves f (M , N/θ) are ellipses.

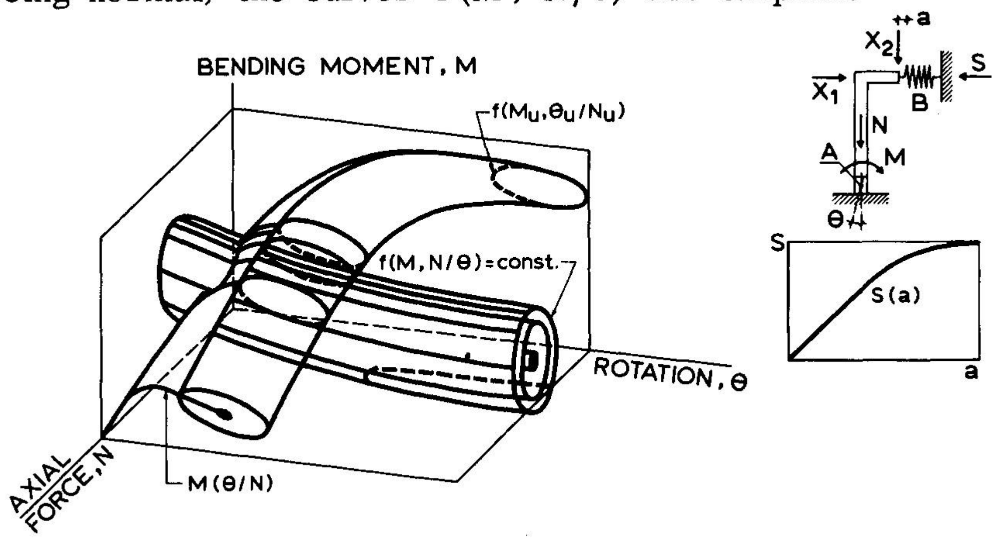

Fig. 8 - Non-linear statically indeterminate structure having random behaviour and acted on by two random loadings.

On the other hand, under deterministic assumptions, the behaviour of element A is defined by the surface Σ (M , N , θ) containing the curves M(θ, N). The limit-states of element A are defined by the curve (M_u , N_u, θ_u) also belonging to surface Σ .

The probability of efficiency is given by:

$$P_e = \int_{\Sigma} f(M, N, \theta)\, d\Sigma \qquad 5)$$

Finally, the limit-states of element A may also be statistically defined by the density of probability $f(M_u, N_u, \theta_u)$, Fig. 8. Then the probability of efficiency is obtained by the convolution of this density of probability and the density of probability $f(M, N, \theta)$ extended to the whole space.

The above problem dealing with a generalized space of three components can be easily extended to any number of components. Thus a generalized formulation of the problem of computing the probability of efficiency is obtained. In this formulation it is assumed that resistance is not affected by previous loading history.

3 - STRUCTURAL SAFETY FOR REPEATED LOADING

3.1 - Presentation of the problem

The problems of structural safety and of reliability are viewed in different terms in the different branches of engineering. As is well known reliability has been extensively studied in connexion with aircraft and space vehicles, machinery, road and railway vehicles, ship structures, etc. In all these cases load repetitions predominate (4). In civil engineering structures, load repetitions are important in relation to superimposed loads acting on bridges and industrial buildings. Moreover the repeated character of wind and earthquake loads cannot be disregarded. Many other instances in which load repetitions affect safety could be mentioned.

The main features of loading variability are strongly influenced by the type of equipment or structure.

For airflight vehicles the cycle ground-air-ground is the fundamental one, often forming the basis for fatigue testing (5). Aspects of fatigue in connexion with the aircraft industry have been discussed in several international symposia (6 to 10). It is worth mentioning that reliability problems in airflight vehicles are directly related to inspection (11). The interval between inspections is one of the main parameter in the control of safety. This concept of inspection influences the general reliability outlook in aircraft industry.

For mechanical handling equipment the main cycle corresponds to a lifting. The severity of the use of the crane is thus measured by the number of lifting operations (class of use), duly corrected by taking into account the probability of the different fractions of the maximum load in each operation (load spectrum). By combining these two classifications a final grouping is obtained, which qualitatively indicates the severity of use. This group indication is used for design purposes in both mechanical equipment and structural elements (12).

So far the influence of load repetitions in bridges has been taken into account in very simplified forms. Only recently have studies performed in Great-Britain made it possible to establish a code in which the concept of load spectrum is introduced (13). In this code, load repetitions are considered within serviceability limit-states. This is justified on the basis that load repetitions mainly affect fatigue cracking in steel structures. It is anticipated that the repairing of such cracking is ensured by suitable inspections.

The idealization of wind and earthquake loadings by stochastic processes has been adopted more than 20 years ago (14). Yet such assumptions are mainly used for studying the structural behaviour and not for defining the resistance of the structures under loads of this type.

In order to deal with structural safety problems, for repeated loading according to the general theory presented in 2), it is necessary to qualify both loadings and resistances according to their previous histories. Loads and resistances may not be compared unless their histories are analogous.

In these general terms the problem apparently has no practical solution. It is thus necessary to substitute loading history by a small number of pertinent parameters. Resistance is to be determined within the range of variation of such parameters. Additionally loadings acting on the structures have to be qualified by the same parameters.

The influence of repeated loading on resistance is first discussed in the simplest case of pure cyclic loading. The generalization for any type of variable loading is discussed thereafter.

3.2 - Constant amplitude cyclic loading

Consider a one-variable sinusoidal cyclic loading of constant amplitude. The parameters which define the loading are: i) the mean value $\overline{S}$, the peak to peak amplitude ΔS, and the circular frequency ω. At a given instant, t, the intensity of the loading is:

$$S = \overline{S} + \frac{\Delta S}{2} \sin\left(\omega t + \frac{\pi}{2}\right) \qquad 6)$$

The load varies between a maximum value $S^{max} = \overline{S} + \frac{\Delta S}{2}$ and a minimum value $S^{min} = \overline{S} - \frac{\Delta S}{2}$. During an interval of time t, the total number of cycles is:

$$N = \frac{\omega}{2\pi} t = f t \qquad 7)$$

Very often the main influences on the resistance to cyclic loading are due to the total number of cycles, N, and depend little either on the shape of the cycle function or on the frequency. In other cases this is not so (15). In the former case the effect on the resisting load-effect, R, which defines the limit-state can expressed by a function:

$$R\,(N, S^{max}, S^{min}) \qquad 8)$$

By identifying the values of R and S^{max}, expression 8) becomes:

$$R\,(N, S^{min}) \qquad 9)$$

This function is usually plotted by means of a Wöhler type diagram in function of N, which is represented in a logarithmic scale, Fig. 9. For counting the number of cycles N the first attaining of the maximum loading is usually taken as the origin.

By specifying the value of S^{min}, which can be considered as constant or as a function of R , curves R(N) are obtained. These curves have usually three branches: i) a first branch corresponding to very few repetitions, in

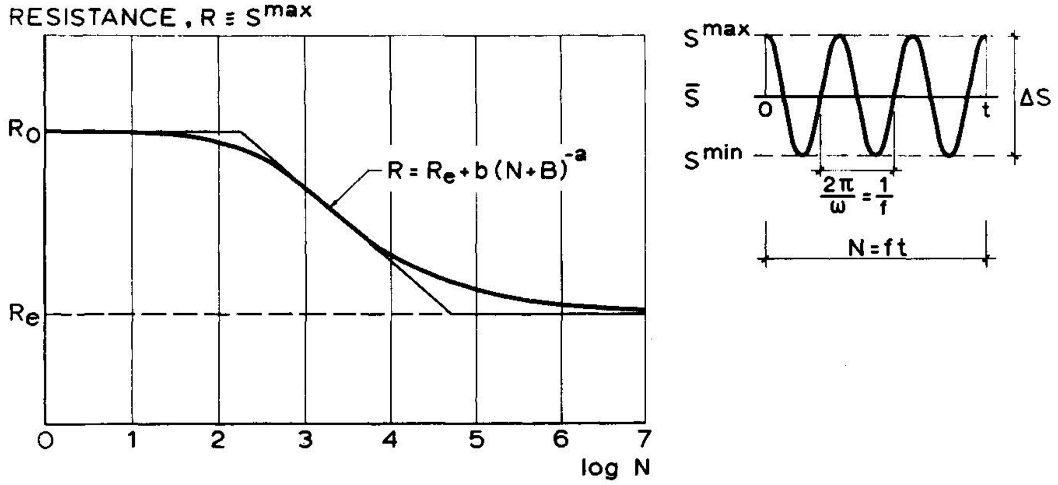

Fig. 9 - Wöhler diagram indicating resistance to constant amplitude cyclic loading.

which R is little influenced by N ; ii) a second branch in which R considerably decreases with N ; and iii) a third branch in which R no longer depends on N . The ordinate of the segment corresponding to the third branch is called fatigue or endurance limit.

The variation of R with N is usually idealized in two ways: by means of straight lines or a continuous curve. Among the different continuous curves suggested, the one proposed by Weibull (16, 17) has been extensively used:

$$R - R_e = b(N + B)^{-a} \qquad 10)$$

In this expression a , b and B are experimentally determined constants.

Another way of representing the relationship of R , $\bar{S}$ and N consists in using a Goodman diagram, Fig. 10. In this diagram the extreme

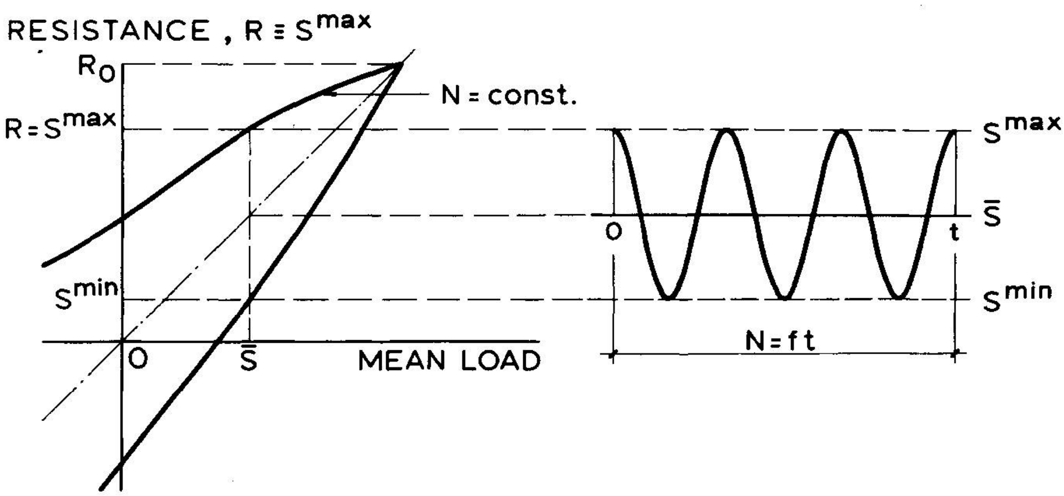

Fig. 10 - Modified Goodman diagram.

value of the applied load-effect leading to fatigue rupture is marked in ordinates. The mean value $\bar{S} = \frac{S^{max} + S^{min}}{2}$ is marked in abscissas and a curve corresponds to each number of cycles necessary to reach the considered ultimate state.

Assuming relationship 8) to be random, the probability of reaching a limit-state, can also be studied by using diagrams of the types indicated in Figs. 9 and 10.

Fig. 11 indicates how Wöhler diagrams can be expressed in statistical terms. The deterministic curve $R(N, S^{min})$ is substituted by lines of equal probability. For representing the distribution functions $F(R | N, S^{min})$,

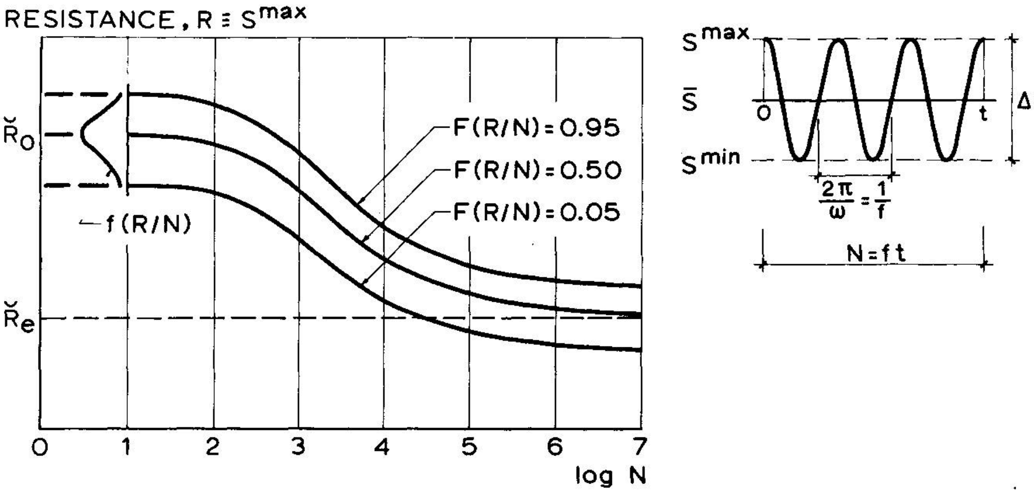

Fig. 11 - Wöhler type diagram expressed statistically.

extreme distributions of for instance Weibull's type, may be used. The curve $F(R | N) = 0.5$ corresponds to the median values and still may be expressed by equation 10).

The description above does not include the effect of load repetitions on the generalized displacements. This effect, which may be disregarded in some materials, may be important in other cases, e.g. the limit-states of cracking or deformation in reinforced concrete.

As indicated, the general solution of structural safety problems implies the location of the limit-state condition in the "generalized load-effect - - generalized displacements" space. For the case under consideration, this location may be expressed in function of the maximum and the minimum values of the cyclic load-effect and the number of cycles.

3.3 - Variable-amplitude cyclic loading. Cumulative damage

The simplest way of dealing with the problem of variable-amplitude cyclic loading consists in accepting the linear cumulative damage hypothesis. This hypothesis is expressed by the Palmgren-Miner rule. This rule was proposed by Palmgren in 1924 for the life evaluation of ball bearings (18) and, independently, by Miner in 1945 for the design of aircraft components (19). According to this rule damage due to different loading cycles is additive and measured, for each type of cycles, by the ratio of the number of applied cycles n_i to the number of cycles necessary for reaching the limit-state, N_i. Consequently by applying k different types of load cycles the limit-state is reached when:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} = 1 \tag{11}$$

The ratio $\frac{n_i}{N_i}$ is called damage. The linear cumulative hypothesis implies reciprocity and transitivity of the damage. The damage due to a given number of cycles is independent of its order in the test sequence, Fig. 12.

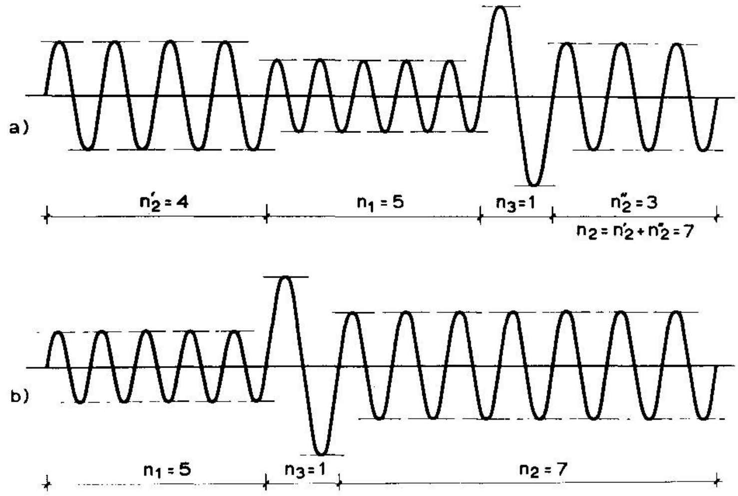

Fig. 12 - Different test sequences corresponding to the same damage.

Several attempts have been made for generalizing the theory of cumulative damage in order to taking the effect of the loading history into account. For instance, Bolotin (20) indicates two ways of performing this generalization: i) by defining a damage function whose derivative depends not only on the damage, as defined above, but also on the amplitude of the cycle; and ii) by distinguishing two different damage measures: a measure of desintegration and a measure of fatigue-crack development.

The need to generalize the Palmgren-Miner rule derives from the fact that very often experimental data do not fit it. In fact it may occur that small amplitude cycles initially applied contribute to increasing longevity. On the contrary the initial application of high amplitude cycles may result in a very considerable weakening in relation to the effect of future low cycles. Consequently the validity of the hypothesis of linear cumulative damage is only to be accepted under broad limits.

By studying several hundred cases related to aircraft fatigue, Jacobi (21) indicates that the rule:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} = 0.3 \tag{12}$$

gives life values on the safe side in 95% of all possible cases.

The concept of damage presented as deterministic can be extended so as to be given a statistical character also (22).

3.4 - General variable loading. Loading as a stochastic process

Useful results concerning loading idealization have been obtained by considering the loading variability as a combination of deterministic and stochastic processes. In some cases, loading can even be idealized simply as a Gaussian stationary or quasi-stationary process. Yet the quantities used to define a stationary stochastic process - correlation or power spectral density diagrams - have seldom been used in direct studies of the effect of load variability on resistance. In fact, the damage effect of load repetitions is traditionally associated with the loading cycles. It is possible from theoretical results to relate the power spectral density and the mean number of excedances for each load level in a given interval of time (23). However this is not a direct information concerning the amplitude of the cycles. For obtaining such an information special instruments have been suggested for the analysing the records (24) making it possible to construct a matrix which indicates the number of cycles classified according to classes of maximum and minimum values. However such technique involves a subjective identification of the cycles.

A procedure for resolving complex variation patterns in order to apply a generalized Miner's rule is presented by Crede (25), Fig. 13. According

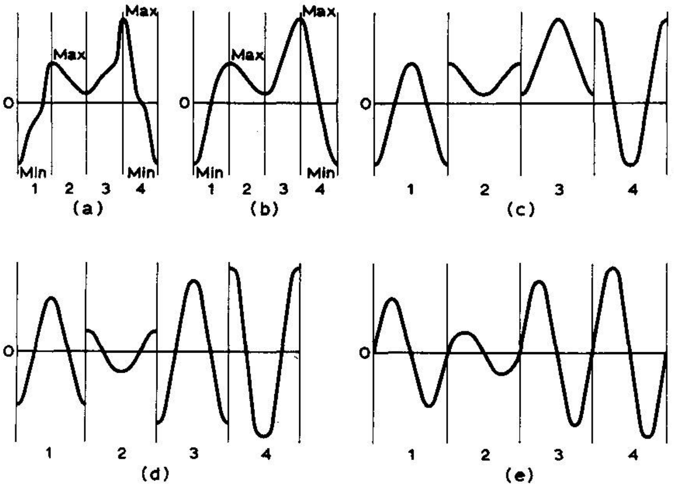

Fig. 13 - Resolution of a complex time-history into a variable-amplitude cyclic loading, after Crede (25).

to this procedure the complex load variation, a) , is split into quasi sinusoidal half cycles, b) , and the damage of each half cycle is assumed to correspond to half the damage of a full cycle, c). The cycles are then displaced d) and their phase changed in order to get a variable-amplitude cyclic loading, e).

The influence of the loading history, expressed as deviations from Palmgren-Miner rule, and the difficulty to identify the parameters which control resistance in the case of repeated loading imply in practice that these resistances will be determined according to loading testing schemes that closely follow the main features of real loading. By doing so, resistance to repeated loading has to be determined in view of a specific use of the structure or a type of loading, and cannot be specified in general terms.

Due to the reasons above, for instance in aircraft studies, numerous testing schemes have been suggested, which may be classified in the following main categories:

i) program load testing directly following flight records;
ii) randomized program tests including random sequences of cycle blocks, cycles, half-cycles or peaks, statistically independent or correlated;
iii) random process tests, reproducing stationary or quasi-stationary stochastic processes with specified power spectra.

Lifetimes determined according to these different techniques may be very different. It is very controversial which of these schemes is the most convenient.

3.5 - General formulation of the safety problem for repeated loading

One way of dealing with the general problem of structural safety under repeated loading resorts to a discretization in time by considering successions of fixed elementary intervals or successions of peaks. The probability of efficiency for a given interval of time would be obtained by considering the probabilities of efficiency at all elementary intervals of time before the considered time. As the probability at each step may depend of what occurred at all the preceding steps, the general problem formulated in these terms is too involved.

Yet, according to the type of limit-state considered, it may be assumed that reaching the limit-state puts or does not put the structure out of action. In the former case, and considering for instance the limit-state of rupture, the structure can only survive at the nth step if it has survived all preceding steps.

According to the analysis presented by Turkstra (26), the density of probability of the loading S_n and of the resistance R_n at step n are given by the conditional distributions of probability $f(S_n | S_1, \ldots, S_{n-1}, R_1, \ldots, R_{n-1})$ and $f(R_n | R_1, \ldots, R_{n-1}, S_1, \ldots, S_{n-1})$, respectively.

The probability of efficiency till step n is the probability that $S_1 < R_1$ at step 1, $S_2 < R_2$ at step 2, and so on until $S_n < R_n$ at step n. This probability is given by:

$$P_e = \int_0^{\infty} \int_0^{R_1} \int_0^{\infty} \int_0^{R_2} \cdots \int_0^{\infty} \int_0^{R_n} f(R_1)\, f(S_1)\, f(R_2 | R_1, S_1)\, f(S_2 | S_1, R_1\, R_2) \ldots$$

$$\ldots f(R_n | R_1, \ldots, R_{n-1}, S_1, \ldots, S_n)\, f(S_n | S_1, \ldots, S_{n-1}, R_1, \ldots, R_{n-1}) \ldots$$

$$\ldots dS_n\, dR_n \ldots dS_2\, dR_2\, dS_1\, dR_1 \qquad 13)$$

Conceptually this formulation is simply a generalization of computations of the type presented in Chapter 2. Yet in practice the information required for taking into account the full loading history will very seldom be available and drastic simplifications will be necessary.

Assuming that the successive loads are independent and that load and time do not affect resistance, the probability of efficiency at all cycles is:

$$P_e = \int_0^{\infty} f(R) \left(\int_0^{R} f(S)\, dS \right)^n dR \qquad 14)$$

Assuming that R is a function of n only, given by $R_n(R_1, n)$, the probability of efficiency is given by:

$$P_e = \int_{R_1}^{\infty} f(R_1) \int_0^{R_1} \cdots \int_0^{R_n} f(S_1) \ldots f(S_n)\, dS_n \ldots dS_1\, dR_1 \qquad 15)$$

The assumption that R_n is a function of n only may be convenient for studying aging effects. However, as mentioned, it is too crude a hypothesis for studying the effect of load repetitions.

For high cyclic loading, the most convenient way of considering the influence of load repetition on limit-states, and particularly on failure, is by means of the concept of cumulative damage.

Assume that failure occurs when cumulative damage reaches the value 1. For a given lifetime, let n(S) be the number of cycles in which the maximum loading exceeds S . If ΔS is sufficiently small the product $\frac{dn}{dS} \Delta S$ indicates the number of cycles in interval ΔS. Thus by generalizing expression 11), cumulative damage can be calculated from the integral:

$$D = \int \frac{dn}{dS} \frac{1}{N(S)}\, dS \qquad 16)$$

extended to the whole range of variation of S . The survival condition corresponds to $D < 1$. Function N(S) indicates the number of cycles of intensity S which lead to rupture when applied alone. As mentioned in Chapter 3.2 $N(S) \equiv N(R)$ may be taken as a deterministic or a random function. If considered as random, a distribution function $F(S|N)$ can be defined which measures the probability of load leading to failure being less than S for a given number of cycles N , Fig. 14. Physical considerations make it likely that distributions $F(S|N)$ are closely correlated for the different values of N. Supposing a complete correlation, to each structural element there corresponds a line F = const. Thus for computing the probability of efficiency (or of survival) the following procedure can be used:

i) S(N) is expressed by means of an auxiliary random variable α in such a way that, for a given N , S increases with α;
ii) the cumulative damage equation:

$$\int_{-\infty}^{\infty} \frac{dn}{dS} \frac{1}{N(S,\alpha)}\, dS = 1 \qquad 16)$$

is solved yielding a root α_o.
iii) the probability of efficiency is given by $F_{\alpha}(\alpha_o)$.

It was assumed above that randomness derives from the failure condition N(S) alone. Randomness could also be due to the function n(S) or/and the cumulative damage condition. In fact, a distribution function $F_D(D)$ can be defined giving the probability of failure for values smaller than D. In the

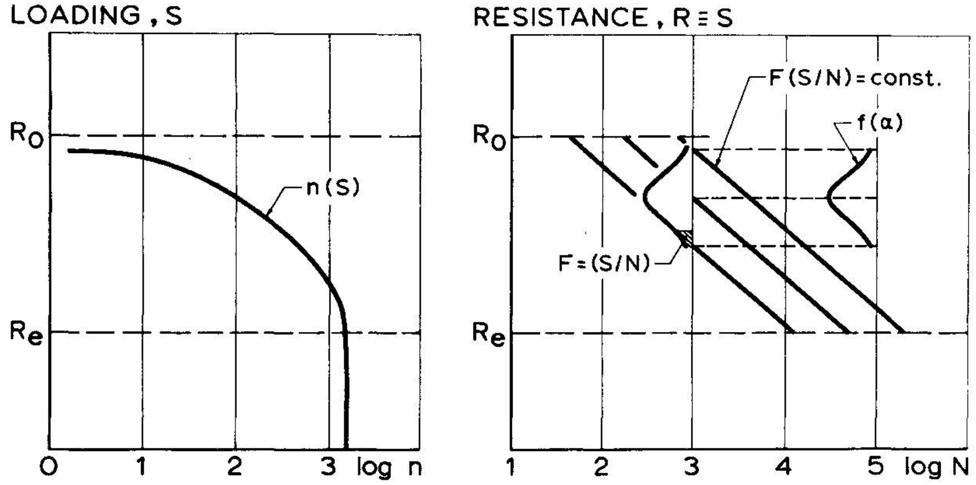

Fig. 14 - Diagrams indicating the number of loading cycles and fatigue life.

latter case the probability of efficiency should be computed by a convolution of the distribution functions $F_\alpha(\alpha(D))$ and $F_D(D)$.

Experimental results have shown that the variability of cumulative damage for an overall group of tests is centred around a mean value close to the unit. Consequently the statistical considerations presented emphasize the interest of the linear cumulative damage hypothesis for determining probabilities of efficiency.

In problems in which loading is not purely cyclic but defined by arbitrary load spectra, the function N(S) can be corrected by introducing overall interaction factors as suggested by Freudenthal (27) and Shinozuka (28).

4 - RULES FOR STRUCTURAL DESIGN

4.1 - Limit-states for repeated loading

In civil engineering structures the limit-states usually considered are: rupture, deformation, and cracking. It is very difficult to discuss in general terms the effect of load repetitions on these different limit-states for the different types of materials, elements and structures.

For steel and reinforced concrete structures acted on by a single type of loading, the effect on resistance under cyclic load repetitions may be expressed by Wöhler diagrams, Figs. 9 and 11. A large amount of research has been performed and is in progress (29) for obtaining information of this type. Yet results are still scarce in many aspects. Difficulties in the field of fatigue are even increased for low cyclic loading, owing mainly to the Bauschinger effect. As indicated in 2.4 a complete definition of rupture limit-state should be based on statistical terms and should cover both loading and deformation. Most results of repeated loading tests at present available yield no information on ultimate generalized displacements: strains, curvatures, displacements, rotations. Attention is called to the urgent need to obtain data in this area.

Also, as mentioned, cyclic repeated loading tests do not yield sufficient information for dealing with all repeated loading problems. The cumulative damage principle is a very useful hypothesis if not misused. In fact, testing techniques have to be suitably chosen according to the purpose in view. This principle leads to the types of testing techniques used in aircraft industry, although conveniently adapted in order to reproduce the loading conditions in civil engineering structures. These considerations apply to both high and low-cycle loading. For instance testing procedures for studying resistance (and ultimate deformability) of structures acted on by earthquake loads should be much more carefully discussed than they have been till now.

Theorization at present available for dealing with low-cycle loading is based on the accumulation of plastic strain and uses the Manson-Coffin's hypothesis. According to Coffin (see e.g. Massonnet (30)) n loading cycles under constant plastic strain amplitude lead to rupture according to the law.

$$N = \left(\frac{C}{\Delta \epsilon_p}\right)^2 \qquad 17)$$

where C is a constant and $\Delta \epsilon_p$ the width of the hysteretical cycles.

Coffin's law can be extended to a linear cumulative damage criterion by means of the expression:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} \left(\frac{\Delta \epsilon_p}{C}\right)^{\alpha} = 1 \qquad 18)$$

which is analogous to Miner's law.

A criterion of this type was used by Tang and Yao (31) for studying low cycle damage of structures under seismic loads.

A review of the different rutpure criteria, under low and high-cyclic repeated loading, to be applied in steel structures, was recently prepared by Yamada (32). A report on the effect of load repetitions, both in steel and in reinforced and prestressed concrete structures, was presented in 1966 by Saillard (33). A report on fatigue in partially prestressed structures is due to Baus (34).

The effect of load repetitions on deformations has a twofold interest. In fact deformations have to be limited by comparison with allowable values, and additionally, in statically indeterminate structures, deformations affect the distribution of load-effects, thus influencing rupture. This influence may also be felt in statically determinate structures owing to second-order effects.

Park's report (35) presented at this Symposium summarizes different idealizations by means of which it is possible to study the load-deformation behaviour in reinforced, prestressed, and steel structures. The complexity of the real behaviour explains why satisfactory idealizations must also be complex.

Existing information on limit-states of deformation is very scarce. This subject should be given particular attention in future research. In fact, structural safety, for instance in Earthquake Engineering, is in most cases controlled by ultimate deformations, although very little is known about the effect of low-cycle load repetitions on ultimate deformations.

For high-cycle load repetitions leading to fatigue, a linearization of behaviour is to be expected. Thus linear structural theories are usually adopted when dealing with fatigue.

The problem of cracking limit-states is entirely different in steel and in reinforced concrete structures.

In steel or other metallic structures attention is paid to the laws governing the growth of cracks (36). This problem may be particularly important in welded structures.

For reinforced and partially prestressed structures the crack width is limited mainly to avoid corrosion. Present theories of cracking (37 and 38) allow the crack width to be estimated in both extreme cases of monotonic and highly repeated loading. However information on the evolution of crack width in function of time and of the number and the amplitude of loading cycles is still missing.

4.2 - Design rules for repeated loading

In general, codes on design of steel, reinforced and prestressed concrete structures contain very little information on repeated loading. This fact is due to the fields of application of these codes, which are limited to types of structures little affected by load repetitions. This is not entirely correct as in fact these design rules cover for instance wind and earthquake loadings, cases in which the effect of load repetitions may be very important.

Having mainly in mind load repetitions in industrial buildings, the Soviet Code for steel structures (39) indicates reduction coefficients of allowable stresses given by:

$$\gamma = \frac{1}{(a\beta \pm b) - (a\beta \mp b)\rho} \tag{19}$$

where: β is a concentration effect coefficient defined in function of the connection types (rivets, welds, bolts) and the detailing of the connection;

a and b depend on the type of steel and $\rho = \dfrac{\sigma_{min}}{\sigma_{max}}$ measures the relative amplitude of the stress cycles. This expression is established having in mind 2 million load repetitions. A revision of this code, now in progress, will include a factor to take into account variable number of repetitions.

An analogous approach is used for the design of lifting equipment, such as cranes (12). Diagrams of allowable stresses are given in function of: i) the relative amplitude of stress cycles, ii) the types of elements and of connections, iii) the types of steel and iv) the groups classifying the severity of use. Such diagrams refer to tension, compression and shear stresses. Fig. 15 exemplifies one of these diagrams.

For many years, allowable stresses in the design of steel railway and road bridges have been reduced in function of the amplitude of the stress cycles. However only recent studies (40 and 41) have attempted to improve the quantification of the load repetitions and duly to take them into consideration.

CEB-FIP Recommendations on the design of reinforced and prestressed concrete structures (38) do not include load repetition effects. A proposal concerning design for fatigue was presented by Gvozdev et al. (42). According

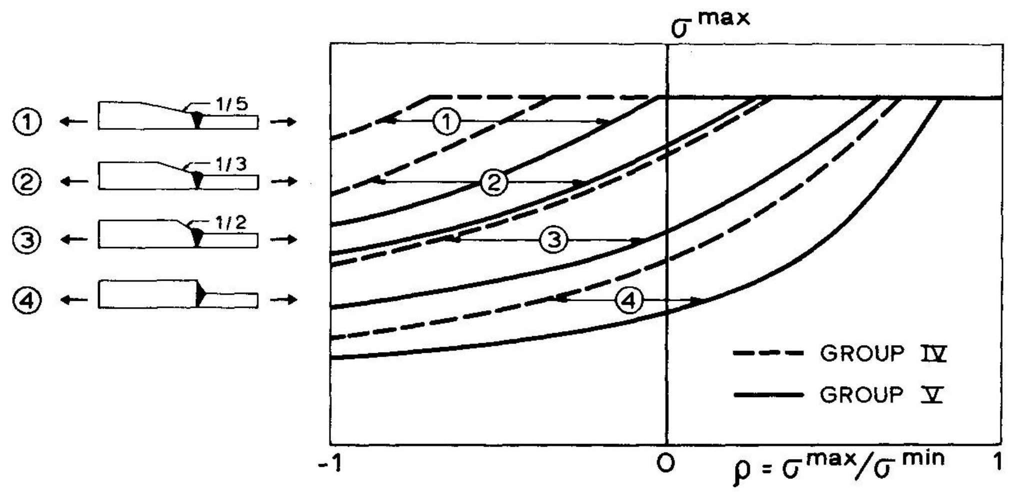

Fig. 15 - Design rules for fatigue, after (12).

to this proposal linear structural behaviour and linear stress distribution in transverse sections are assumed. Coefficients for the reduction of design stresses are given for different types of steel, of concrete and of welding of bars, in function of the relative amplitude coefficient of stress variation only. Although no explicit indication of the number of repetitions is given, repetitions of the order of two million cycles were taken as a basis.

In this proposal two different independent verifications are recommended, one disregarding load repetitions and the other concerning fatigue. The justification for this independency derives from the fact that a large number of load repetitions, thus far as 50% to 90% of those corresponding to life-time little affects resistance to monotonic loading.

Attention is called to the fact that endurance limits for steel bars cannot be established considering steel quality alone. Rib arrangements and other factors considerably affect the endurance limits, forcing the experimental determination of fatigue behaviour. Construction details in particular the diameter of curvature of the bars, may be very important (43 to 45). Even small holes on the surface of reinforced concrete elements may be the main factor affecting their fatigue behaviour (46).

5 - CONCLUSIONS

Progress in the design of structures depends very much on the clearness and on the coherence of structural safety concepts. Once these concepts are established, there is a long way to go in order to obtain simple and appropriate design rules. Along this way, large amounts of technological information must be introduced.

Consequently, research along the following lines is recommended:

5.1 - Studies on the basic structural safety concepts for repeated loading, compatible with those in which the effect of load repetitions are disregarded;

5.2 - Idealization of repeated loadings and of their structural effects for different type of situations, ranging from small to large number of repetitions;

5.3 - Definition of structural behaviour for different types of structural elements. Such behaviour should be studied adopting load repetitions schemes congruent with the problems under consideration;

5.4 - Improvement in the statement of the different limit-states, and, particularly, of those related to deformations.

5.5 - Theoretical and experimental studies on the overall behaviour of structures, particularly of the statically indeterminate ones, acted on by repeated loading.

5.6 - Derivation of improved design rules, in which a convenient equilibrium between simplicity and accuracy be reached.

It is expected that the papers to be presented at the Symposium and the discussing to be held at the meetings will contribute to improve the knowledge on these subjects.

6 - ACKNOWLEDGMENT

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SUMMARY

The basic structural safety problem when the limit-states are not affected by load repetitions is first formulated in the general case.

Then the influence of load repetitions on limit-states is taken into account for constant and variable amplitude cyclic loading and for general variable loading. The possibility of extending to repeated loading the formulation first presented is discussed. Considerations on limit-states and design rules for repeated loading follow.

Finally general lines of research on this subject are recommended.

RESUME

On présente en premier lieu une formulation générale du problème de base de la sécurité des constructions, lorsque les états-limites sont indépendants de la répétition des charges.

Ensuite on tient compte de l'influence des charges répétées sur les états-limites, pour des cycles de charge ayant des amplitudes soit constantes soit variables et pour le cas d'une variation aléatoire. On discute la possibilité de généraliser au cas de charges répétées, la formulation présentée en premier lieu. On fait alors des considérations au sujet des états-limites et des règles de projet pour les charges répétées.

Enfin on présente des recommandations sur l'orientation des recherches futures.

ZUSAMMENFASSUNG

Das Grundproblem der Bauwerksicherheit bei nicht beeinflussten Grenzzuständen durch Wiederholung der Belastungen wird erstmalig allgemein präsentiert.

Ferner wird der Einfluss wiederholter Belastungen auf die Grenzzustände bei konstanten und veränderlichen zyklischen Amplituden, sowie bei dem allgemeinen Fall veränderlicher Belastungen in Betracht gezogen. Die Möglichkeit, die erstgenannte Formulierung auf die wiederholten Belastungen auszudehnen, wird diskutiert. Es folgen Betrachtungen über Grenzzustände und Entwurfsregeln im Falle wiederholter Belastungen. Schliesslich werden allgemeine Forschungsrichtlinien empfohlen.