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Autor(en): **Rockey, K.C.**

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An Ultimate Load Method for the Design of Plate Girders

Méthode de dimensionnement à la ruine des poutres à âme pleine

Ein Traglastverfahren zur Bemessung von Blechträgern

K.C. ROCKEY
M.Sc., C.Eng., Ph.D., F.I.C.E.
Professor of Civil and Structural Engineering
University College, Cardiff, England

1. INTRODUCTION

The economic design of plate girders frequently involves the use of thin web plates reinforced by longitudinal and vertical stiffeners. Since more often than not these stiffened webs will buckle before the girders collapse, the use of ultimate load methods of design are essential if full advantage is to be taken of the post buckling load carrying capacity of the web plates. The present paper presents an ultimate load design method for plate girders which has been developed from an extensive study into the ultimate load behaviour of plate girders (1 - 5).

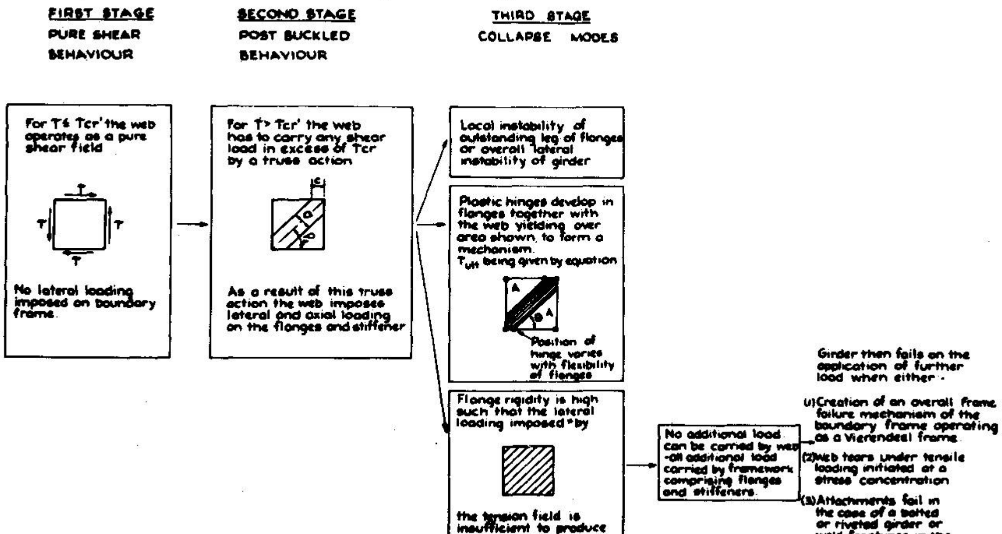


FIG 1 POSSIBLE MODES OF FAILURE.

An extensive study of the behaviour of shear webs carried out by Skaloud and the writer (1 - 3) has shown that a shear web may fail in a number of different ways, see figure 1. From figure 1 it will be noted that if the webplate has no initial imperfections then prior to buckling it does not impose any lateral loading upon the boundary members. However, once the web has buckled it is no longer capable of carrying any further compressive loading across the diagonal ab and as a result the web has to carry all additional shear loads by a diagonal tensile membrane action, this action being referred to as a 'truss type action'. This membrane action imposes a lateral load upon the flanges and it is possible for this membrane action to cause the flanges, and therefore the girder, to fail due to the development of plastic hinges in the flanges. Whether or not this type of failure will occur depends upon the stiffness of the flanges and upon the magnitude of the membrane loading which varies with the elastic buckling stress of the web; with an increase in the buckling stress there is a decrease in the membrane action. If the flanges are sufficiently stiff then the boundary members comprising of the flanges and the stiffeners permits the web to develop a full membrane action until it yields. Once this stage has been developed, the web cannot carry any further shear load and any further loads imposed upon the girder have to be carried by the flanges and stiffeners acting as a Vierendeel frame.

The first significant step in the development of plastic methods of design for plate girder webs, loaded in shear, occurred when Basler and his associates at Lehigh University (6 - 9) presented their design method some ten years or so ago. They assumed that the webplate would fail due to the development of an inclined plastic band anchored against the vertical stiffeners, see figure 3. In developing this mechanism, Basler et al assumed that the flanges of most girders were too flexible to withstand the membrane

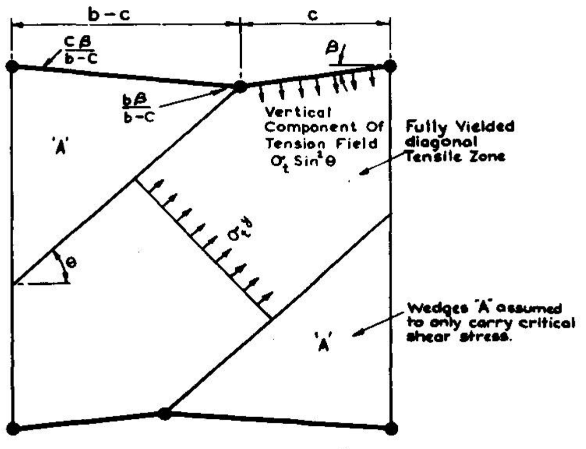


FIG. 2 COLLAPSE MECHANISM PROPOSED BY ROCKEY & SKALOUD (3)

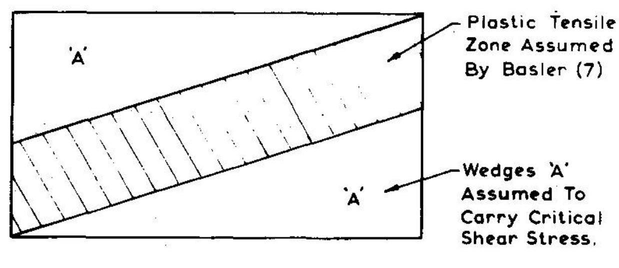


FIG. 3. COLLAPSE MODE ASSUMED IN BASLER THEORY.

action imposed by the buckled web. However, it has been shown in references (1 - 3) that their "collapse model" can both significantly underestimate and overestimate the strength of plate girders.

More recent research at Lehigh by Ostapenko et al (10 - 12) has developed the Basler model and shown how it can be applied to the design of plate girder webs reinforced by both transverse and longitudinal stiffeners. One very important contribution being the development of formulae for flange and stiffener design. Unfortunately, the model assumed still does not allow for the influence of transverse flange rigidity upon the behaviour of buckled webs.

The present paper indicates how the ultimate load method of design for transversely stiffened shear webs as proposed by Skaloud and the writer (3, 4) can be adapted to deal with the loading cases of combined shear and bending and also to deal with the design of plate girder webs reinforced by both transverse and longitudinal stiffeners.

2.1. ULTIMATE LOAD DESIGN OF WEBS LOADED IN SHEAR

In reference (1 - 3), Skaloud and the writer have shown that when a shear web plate buckles prior to yielding, it fails with the development of a diagonal tension band which is fully yielded together with the development of plastic hinges in the flange members to form a mechanism, see figures 1 and 2. It was established experimentally in reference (1) that the width of the diagonal band and therefore the position of the plastic hinges varies with the I/b^3t ratio where I is the flexural rigidity of the compression flange about an axis through its centroid and perpendicular to the web plate, b is the spacing of the transverse stiffeners and t is the thickness of the web.

Subsequently in reference 4 a theoretical solution based on the collapse model shown in figure 2 was developed and it was established that this method of analysis was capable of predicting with very good accuracy the failure load of transversely stiffened plate girders loaded in shear. In section 2.2, this solution will be briefly presented, whilst in section 2.3 and 2.4 it will be extended to deal with the design of plate girder webs reinforced by both longitudinal and transverse stiffeners and also with the design of hybrid girders.

2.2. DESIGN OF SHEAR WEBS

The behaviour of a plate girder web loaded in shear can be divided into three stages.

Stages I and II

In Stage I, which only applies to a perfectly flat plate, the applied shear stress is less than the critical shear buckling stress and therefore the web panel carries the applied load by a pure shear action.

The second mode of action results from the fact that in a buckled web the compressive stresses cannot increase and any additional load has to be carried by a tensile truss action.

With normal welded plate girders which have webs with significant permanent deformations, no buckling phenomena will be observed and the loadings which are associated with Stage II occur as soon as load is applied to the girder.

Failure occurs when the diagonal tension band, see figure 1 and 2 yields and the boundary members develop sufficient plastic hinges to result in a failure characterised by one of the three possible forms of failure listed under Stage III.

Stage III

(a) If the lateral membrane loading on the flange is sufficient to develop plastic hinges in the flanges, then failure will be due to the development of a mechanism consisting of a yielded diagonal strip together with plastic hinges in the tension and compression flanges; see figures 1 and 2.

(b) If, however, the membrane loading corresponding to a yielded web is not sufficient to develop plastic hinges in the flanges then failure will occur when either

(1) the web material fractures, such as occurs in an aluminium web

(2) the framework comprising the flanges and the stiffeners, acting as a Vierendeel frame develops a 'frame' mechanism

(3) the compression flange buckles laterally or torsionally

2.2a. THEORETICAL BASIS

Stage I

For an initially plane web, for loading below the buckling stress τ_{cr}, the stress state is assumed to be one of pure shear. Obviously the value of τ_{cr} will vary with the flexural and torsional rigidity of both the flanges and the stiffeners. However, since most conventional welded steel girders have flanges of low torsional rigidity it is reasonable to assume that the shear web is simply supported on all edges. If, however, a tubular flange is employed it would be necessary to use the corresponding buckling stress (13).

As stated earlier, following buckling, the web is unable to withstand any further compression loading and any additional loading has to be carried by a tension field action. The present solution does not attempt to deal with the very complicated stress field which occurs in the elastic post buckled range, it is solely concerned with the final collapse mode. This is essential, if a comparatively simple design procedure is to be developed since observation of the collapse behaviour of girders indicates that the stress and deflection distributions vary quite rapidly at loads close to the ultimate.

The experimental evidence resulting from the earlier study by Skaloud and the writer (1 - 3) is that at collapse the web develops a tension band as shown in figure 2 in which the angle of the tension band is equal to the inclination of the geometrical diagonal and that the tension band is symmetric with respect to the geometric diagonal. The width of this diagonal tension load is assumed to be such that the intercept of its boundary with the flange coincides with position of the plastic hinges in the flanges. The above assumptions will clearly result in slightly lower bound solutions for values of $\alpha > 1$, in the case of girders with very stiff flanges, since in such cases the inclination of the tension field will be closer to 45^{o} than the inclination of the diagonal. This point will be discussed again later in the paper.

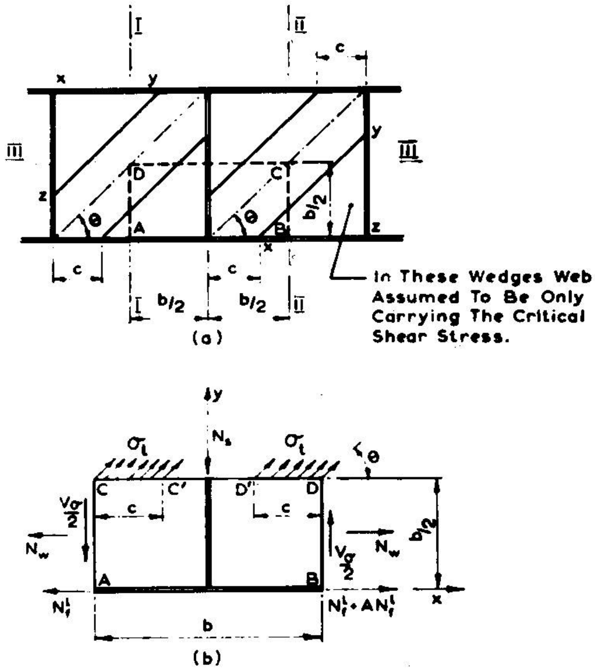
Thus we see that there are two stress regions, see figure 4.

(1) two triangular wedges in which the critical shear stress is assumed to act

(2) a yielded diagonal strip.

The tension stress σ_t is assumed to act uniformly over the diagonal band, yielding occurring when σ_t reaches a value σ_t^{y}.

The stress condition, see figure 5, in the diagonal web strip is given by



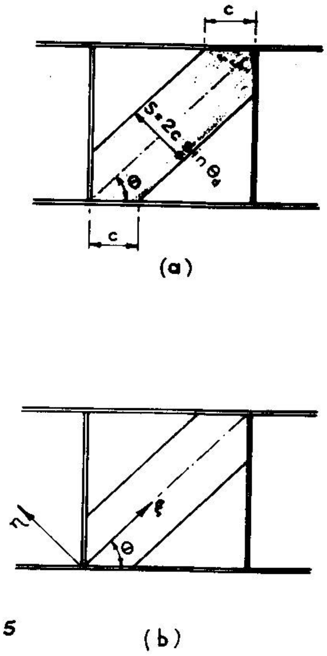


FIG 5

$$\sigma_\zeta = \tau_{cr} \text{ Sin } 2\Theta + \sigma_t^{\,y}$$

$$\sigma_\eta = \tau_{cr} \text{ Sin } 2\Theta \tag{1}$$

$$\tau = \tau_{cr} \text{ Cos } 2\Theta$$

Using the Huber Von Mises plasticity condition, the material yields when $\sigma_{mc} = \sigma_{yw}$ where

$$\sigma_{mc} = \sqrt{\sigma_\zeta^{\,2} + \sigma_\eta^{\,2} - \sigma_\zeta\sigma_\eta + 3\tau^2} \tag{2}$$

Substituting equations (1) into (2) and rearranging yields

$$\sigma_t^{\,y} = -\frac{3}{2}\tau_{cr} \text{ Sin } 2\Theta + \sqrt{\sigma^2_{yw} + \tau_{cr}^{\,2}\left[\left(\frac{3}{2} \text{ Sin } 2\Theta\right)^2 - 3\right]} \tag{3}$$

The vertical component V_σ of the diagonal stress $\sigma_t^{\,y}$ is given by equation (4)

$$V_\sigma = 2ct \text{ Sin}^2\Theta \left(-\frac{3}{2}\tau_{cr} \text{ Sin } 2\Theta + \sqrt{\sigma^2_{yw} + \tau^2_{cr}\left(\frac{3}{2}\text{Sin } 2\Theta\right)^2 - 3}\right) \tag{4}$$

The total shear force V_{ult} is equal to the sum of V and the shear force V_{cr} necessary to cause the plate to buckle.

$$V_{ult} = V_{cr} + V_\sigma = \tau_{cr}\, dt + 2ct \text{ Sin}^2\Theta \left(-\frac{3}{2}\tau_{cr} \text{ Sin } 2\Theta + \right.$$

$$\left.\sqrt{\sigma^2_{yw} + \tau^2_{cr}\left(\frac{3}{2} \text{ Sin } 2\Theta\right)^2 - 3}\right) \tag{5}$$

Since $\tau_{yw} = \sigma_{yw}/\sqrt{3}$, equation (5) may be rewritten as equation (6).

$$\frac{\tau_{ult}}{\tau_{yw}} = \frac{\tau_{cr}}{\tau_{yw}} + 2\sqrt{3}\,\frac{c\alpha}{b}\,\mathrm{Sin}^2\Theta\left(-\frac{\sqrt{3}}{2}\,\mathrm{Sin}\,2\Theta\,\left(\frac{\tau_{cr}}{\tau_{yw}}\right) + \sqrt{1 + \left(\frac{\tau_{cr}}{\tau_{yw}}\right)^2\left(\frac{3}{4}\,\mathrm{Sin}^2\,(2\Theta) - 1\right)}\right) \tag{6}$$

The position of the plastic hinge in the flanges may be theoretically determined using the collapse mechanism shown in figure 2. This mechanism assumes that the hinge coincides with the edge of the diagonal strip and that the loading consists of the vertical component of the diagonal tensile membrane stress $\sigma_t^{\,y}$. The solution of this simple mechanism reduces to the solution of the cubic equation given in equation (7).

$$\left(\frac{c}{b}\right)^3 - \left(\frac{c}{b}\right)^2 + \frac{4\,z_f\sigma_{yf}}{b^2 t\,\mathrm{Sin}^2\Theta\,(\sigma_t^{\,y})} = 0 \tag{7}$$

where z_f denotes the plastic modulus for flange assembly. It is proposed that when the web buckling stress is less than half the shear yield stress, a depth of web plate $z = 30\left(1 - \frac{2\tau_{cr}}{\tau_{yw}}\right)$ be assumed to act with the flange assembly.

It is of interest to consider how equation (6) satisfies a number of the limiting conditions.

<u>(1) Very Thin Webs and Rigid Flanges</u>

For very thin webs $\tau_{cr} \to 0$, in which case

$$\frac{\tau_{ult}}{\tau_{yw}} = 2\sqrt{3}\alpha\frac{c}{b}\,\mathrm{Sin}^2\Theta$$

Since for rigid flanges, $\frac{c}{b} = 0.5$, then for square web panels in which $\Theta = \frac{\pi}{4}$ one obtains the value for τ_{ult} of

$$\frac{\sqrt{3}}{2}\,\tau_{yw} \quad \text{or} \quad \frac{\sigma_{yw}}{2}$$

which agrees with the value obtained from Wagner's Theory (14) for a complete tension field.

<u>(2) Very Thick Webs</u>

When webs are very thick, then $\tau_{cr} \to \tau_{yw}$ and the terms inside the main brackets reduces to zero so that equation (6) reduces to $\tau_{ult} = \tau_{yw}$; which again is as to be expected.

<u>(3) Very Flexible Flanges</u>

Finally, if $\frac{c}{b} \to 0$, as would be the case if the flanges had zero stiffness, and could not withstand any lateral loading, then

$$\tau_{ult} = \tau_{cr}$$

Thus we have seen that equation (6) satisfies the extreme boundary conditions exactly.

When $\sqrt{3}\tau_{cr}$ exceeds the limit of propor tional stress of the material then the effective modulus E_r is less than the modulus of Elasticity E. This reduces the critical shear stress; and to allow for this Basler and his colleagues have recommended that τ_{cr} be replaced by τ_{cre} when $\tau_{cr} > \frac{0.8\sigma_y}{\sqrt{3}}$, τ_{cre} being obtained from equation 8.

$$\frac{\tau_{cre}}{\tau_{yw}} = 1 - \frac{0.16\tau_{yw}}{\tau_{cr}} \tag{8}$$

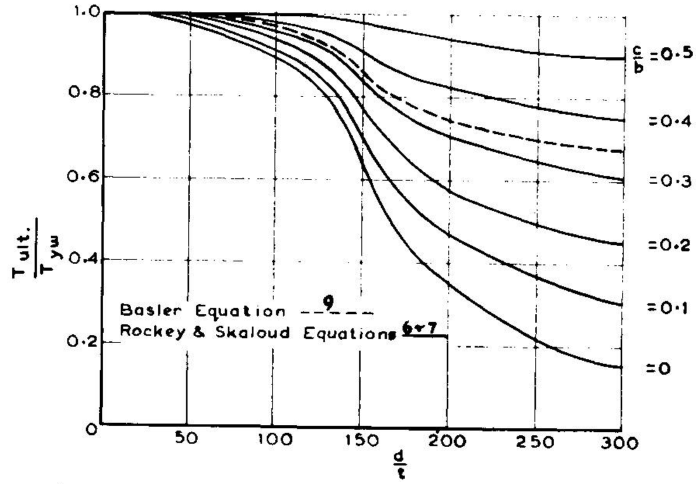


FIG. 6

Using equation (8) in conjunction with equation (6) the relationship between the ratio τ_{ult}/τ_{yw} and the depth to thickness ratio for different values of c/b have been plotted in figure 6 for the case of α equal to 1. The values of τ_{ult}/τ_{yw} as derived from Basler's ultimate load expression, see equation 9, have also been plotted using the same relationship between τ_{cre}, τ_{cr} and τ_{yw}, and it is clearly seen that for very flexible flanges Basler's equation overestimates the strength of the girder and for relatively stiff flanges it underestimates the strength, this being particularly true for larger values of α.

$$W_{ult} = dt\left[\tau_{cr} + \frac{\sqrt{3}\tau_{yw}}{2\sqrt{1+d^2}}\left[1 - \frac{\tau_{cr}}{\tau_{yw}}\right]\right] \tag{9}$$

The present design procedure, see equations (7) and (8), has been checked (4, 5) against existing experimental data and as will be noted from Figure 7 very good correlation has been obtained.

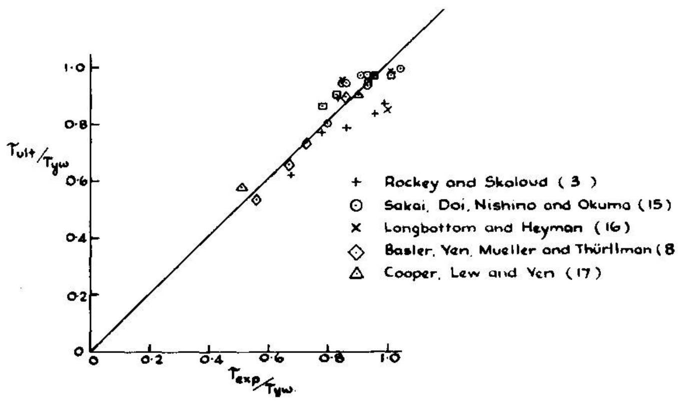


FIG: 7 COMPARISON OF PREDICTED ULTIMATE SHEAR STRESS (τ_{ult}) AND EXPERIMENTAL ULTIMATE SHEAR STRESS (τ_{exp}) FOR WEBS LOADED IN SHEAR.

2.2. Transversely stiffened web plates loaded in shear and bending

Web plates are normally subjected to a combination of shear and bending and in the present section an ultimate load method of design is proposed for this case of loading.

Three important additional factors have to be considered when determining the failure load of a web plate loaded in shear and bending, these are

(1) the reduction in the buckling stress of the web due to the presence of a bending stress or a direct stress

(2) the influence of the inplane bending stresses upon the value of the diagonal tensile membrane stress σ_t^y which is developed in the diagonal strip

(3) the reduction in the magnitude of the plastic modulus z_f of the flanges due to the presence of the axial compressive and tensile stresses.

For the case of a webplate subjected to combined shear and bending the reduction in the buckling stress τ_{cr} due to the presence of a bending stress σ can be calculated with reasonable accuracy from equation (10).

$$\left(\frac{\sigma_{mb}}{\sigma_{crb}}\right)^2 + \left(\frac{\tau_m}{\tau_{cr}}\right)^2 = 1 \qquad (10)$$

where σ_{crb} = critical bending stress when the plate is subjected to pure bending

τ_{cr} = critical bending stress when the plate is subjected to pure shear

σ_m, τ_m = critical bending and shear stresses when acting together

For the case of all edges being simply supported σ_{cr} and τ_{cr} can be determined from equations (11) and (12)

$$\sigma_{crb} = 23.9 \left(\frac{\pi^2 E}{12(1-\mu^2)}\right)\left(\frac{t}{d}\right)^2 \qquad (11)$$

$$\tau_{cr} = \left(5.35 + \frac{4d^2}{b^2}\right)\left(\frac{\pi^2 E}{12(1-\mu^2)}\right)\left(\frac{t}{d}\right)^2 \quad \text{when } b \geqslant d \qquad (12a)$$

$$\tau_{cr} = \left(5.35\,\frac{d^2}{b^2} + 4\right) \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{d}\right)^2 \quad \text{when } b \leqslant d \qquad (12b)$$

The plastic modulus z_f will be reduced by the presence of the axial force and for flanges having a simple rectangular cross section the following relationship may be employed to determine the reduced modulus z_{fr}

$$z_{fr} = z_f \left[1 - \left(\frac{\sigma}{\sigma_{yf}}\right)^2\right] \qquad (13)$$

where σ is the axial stress in the flange and σ_{yf} is the yield stress for the flange material.

As stated earlier when a plate girder web buckles in shear it loses its capacity to carry any additional compressive load, likewise when a panel loaded in direct compression buckles the central area of the panel is unable to carry any further direct stress, and any additional direct load has to be carried by the web material adjacent to the flanges and stiffeners.

Since the stress distribution in a yielded panel subjected to shear and bending is very complex it is assumed in the proposed design procedure that after the plate buckles, the flanges alone carry the additional bending moments. Furthermore, it is assumed that the web carries the additional shear loads by the development of a diagonal membrane stress σ_t^Y.

When a webplate is loaded by direct bending stresses as well as by shear stresses, the value of the diagonal stress σ_t^Y at which yielding occurs is changed from that given in equation (3) to the value obtained from equation (14).

$$\sigma_t^Y = \frac{1}{2}\Bigg[-(3\tau_m \operatorname{Sin} 2\theta + \sigma_m \operatorname{Sin}^2\theta - 2\sigma_m \operatorname{Cos}^2\theta) + \sqrt{(+3\tau_m \operatorname{Sin}2\theta + \sigma_m \operatorname{Sin}^2\theta - 2\sigma_m \operatorname{Cos}^2\theta)^2 - 4\left[\sigma_m^2 + 3\tau_m^2 - \sigma_{yw}^2\right]}\Bigg] \quad (14)$$

Figures 8 and 9 show how the value of σ_t^Y varies with the presence of a direct stress. It will be noted that the presence of

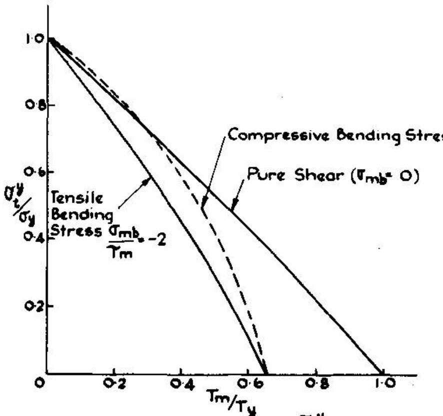


FIG. 8 VARIATION OF σ_t^y/σ_y RATIO WITH τ_m/τ_y RATIO

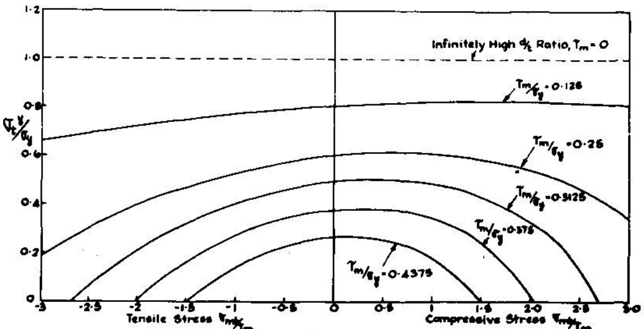


FIG. 9 VARIATION OF THE RATIO σ_t^y/σ_y WITH THE RATIO σ_{mb}/τ_m FOR DIFFERENT VALUES OF THE BUCKLING STRESS τ_m.

a tensile bending stress reduces the capacity of σ_t^Y more significantly than does the compressive bending stress. Because of these factors one would expect to observe a spreading of the diagonal band either side of the neutral axis and that the diagonal band in the tension area, because of its reduced σ_t^Y value being wider than the band in the compression zone.

It should be appreciated that the bending stresses, σ in the tension zone continue to grow after the plate buckles and therefore the value of σ_t^Y in the tension zone will continue to be affected, and that this area will yield first.

The experimental tests conducted by Rockey and Skaloud have shown that the distance 'c' giving the position of the 'central' plastic hinges in the tension flanges is larger than the distance 'c' which occurs in the compression flange. This is to be expected because of the reduced σ_t^Y value and the presence of the tension forces in the tension flange which will have the tendency to keep the flanges straight.

In the present design procedure the values of τ_m and σ_m which are to be used in equation (14) are the stresses at which the plate buckles under the combined loading, τ_m being the average shear stress across the web and σ_m is the compressive bending stress at the web/flange junction.

A general expression for the ultimate load can be obtained by combining equations (7), (13) and (14). In section 2.2 the position of the central hinge was obtained by solving equation (7)

$$\left(\frac{c}{b}\right)^3 - \left(\frac{c}{b}\right)^2 + \frac{4z_f\sigma_{yf}}{b^2 t \sin^2\theta \;(\sigma_t{}^Y)} = 0 \qquad (7)$$

When a bending stress acts with the shear stress, the value of z_f which has to be used in equation (7) is the reduced value z_{fr} as given by equation (13) and $\sigma_t{}^Y$ is the modified value of $\sigma_t{}^b$ given by equation (14). Equation (7) thus becomes

$$\left(\frac{c}{b}\right)^3 - \left(\frac{c}{b}\right)^2 + \frac{4\sigma_{yf}}{b^2 t \sin^2\theta} \; \frac{z_f\left[1 - \left(\frac{\sigma}{\sigma_{yf}}\right)^2\right]}{\sigma_t{}^Y} = 0 \qquad (15)$$

It will be noted from figures 8 and 9 that $\sigma_t{}^Y$ varies with the value AM which is the ratio of the applied bending stress to the applied shear stress before buckling occurs. The value of the flange bending stress σ in equation (15) can, for values of σ up to σ_{yf}, be obtained from equation (16).

$$\sigma = \frac{M_Y}{I_z} = \frac{(WF)(d + 2t_f)}{2I_z} = W_{ult}(q) \qquad (16)$$

Where I_z = Moment of inertia of a section comprising the flanges only, F is a factor depending upon the type of loading, which for the case of a centrally loaded simply supported girder = $\frac{l}{2}$ where l is the distance of the section to the nearest support. Now for a centrally loaded, simply supported girder,

$$W_{ult} = 2dt\left[\tau_{cr} + \frac{2c}{d}\sin^2\theta\sigma_t{}^Y\right] \qquad (17)$$

Substituting (15) and (16) into (14) yields

$$\left(\frac{c}{b}\right)^3 - \left(\frac{c}{b}\right)^2 + \frac{4}{b^2 t \sin^2\theta} \; \frac{z_f\left[\sigma_{yf}^2 - q^2 d^2 t^2\left[\tau_{cr} + 2\frac{c}{d}\sin^2\theta\sigma_t{}^Y\right]2\right]}{\sigma_t{}^Y \;\sigma_{yf}} \qquad (18)$$

which reduces to an equation of the form

$$\left(\frac{c}{b}\right)^3 - (A)\left(\frac{c}{b}\right)^2 + B\left(\frac{c}{b}\right) + D = 0 \qquad (19)$$

The solution of this equation leads to the solution of $\frac{c}{b}$ and hence the position of the hinge c which when substituted into equation (17) will give the ultimate load W_{ult}.

The above solution does not require the use of any assumed interaction relationship between the shear load ratio (v/v_u) and the moment ratio (M/M_u), since this relationship is incorporated in the solution.

Figure 10 gives typical interaction curves between the ratio v/v_u and M/M_u, which have been obtained using equation for two

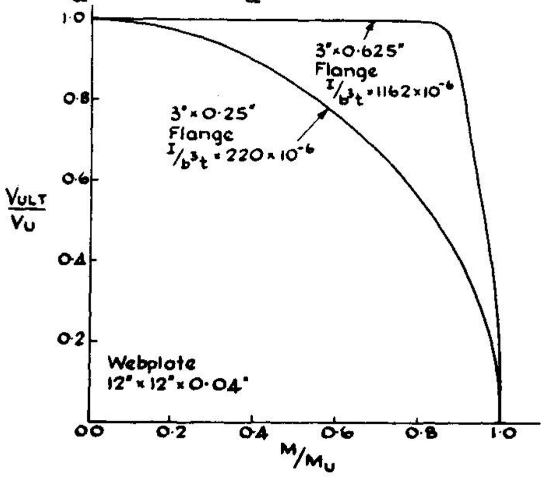


FIG.10 INTERACTION DIAGRAMS - DEMONSTRATING INFLUENCE OF FLANGE RIGIDITY UPON SHAPE OF DIAGRAM

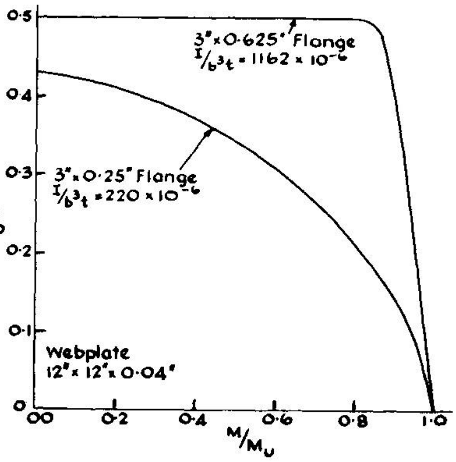


FIG 11 VARIATION OF THE c/b RATIO WITH THE MOMENT RATIO M/M_u

girders from which it will be noted that the shape of the interaction curves is greatly influenced by the flange rigidity parameter and the slenderness of the web $(d/_t)$. For girders with very stiff flanges the loss in shear strength with applied bending stresses is not significant, but with the case of girders with relatively flexible flanges, the influence of the bending stresses upon the z_f value becomes critical and there is a steady loss in shear load carrying capacity. The manner in which the position of the central hinges varies the M/M_u ratio is clearly shown in figure 11. It will be noted that with 'flexible' flanges, as the applied bending stress increases the effective shear stress as buckling decreases and the position of the central hinge moves towards the stiffeners. The reduction of the width of the diagonal tension band together with the reduction in the critical shear stress, which is the stress acting in the triangular areas either side of the diagonal band, means that the shear load capacity decrease steadily with the (M/M_u) ratio.

2.3. Web plates reinforced by both transverse and longitudinal stiffeners and subjected to shear

2.3.1. Pure shear

Figure 12 shows the typical collapse pattern which can be assumed to occur in a longitudinally reinforced web plate subjected to shear.

Consider Panel 1, this panel will impose lateral loading on the flange and it can be assumed that a yield zone will develop as indicated with hinges forming in the flanges as shown, the position of the internal hinge (C_1) in the flange varying with the rigidity of the flange and the buckling stress in the panel. However, since

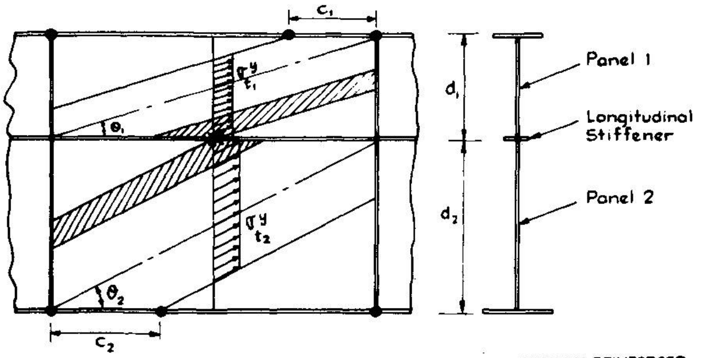


FIG.12 PROPOSED COLLAPSE MECHANISM FOR A LONGITUDINALLY REINFORCED WEB SUBJECTED TO SHEAR

the adjacent panel will act as a very stiff flange at the position of the longitudinal stiffener, the position of the hinge can be assumed at 0.5b. Thus for a panel such as 1, the shear load V_1 will be given by equation (18). In equation (18), the subscripts 1 signify Panel 1.

$$V_1 = \left[\tau_{cr_1} d_1 t + t \sin^2\theta_1 \sigma_{t_1}{}^{Y} (C + 0.5b)\right] \tag{20}$$

For Panel 2, a similar procedure can be followed, the load V_2 for this panel can be calculated from equation (19).

$$V_2 = \left[\tau_{cr_2} d_2 t + (0.5b + C_2) \sin^2\theta_2 \sigma_{t_2}{}^{Y}\right] \tag{21}$$

where C_2 is the position of the hinge in the tension flange,

$$V = \left[\tau_{cr_1} d_1 t + \tau_{cr_2} d_2 t + \left[C_1 + 0.5b\right] \sigma_{t_1}{}^{Y} \sin^2\theta_1 + (C_2 + 0.5b)\, \sigma_{t_2}{}^{Y} \sin^2\theta_2\right] \tag{22}$$

When two or more longitudinal stiffeners are employed the shear load carried by the internal panels can be calculated from equations (6) and (7) assuming c/b = 0.5 in these cases.

In such cases if the inclination of the diagonal Θ is used then a lower bound solution would be expected, since for such internal panels it would be reasonable to expect that the inclination of the

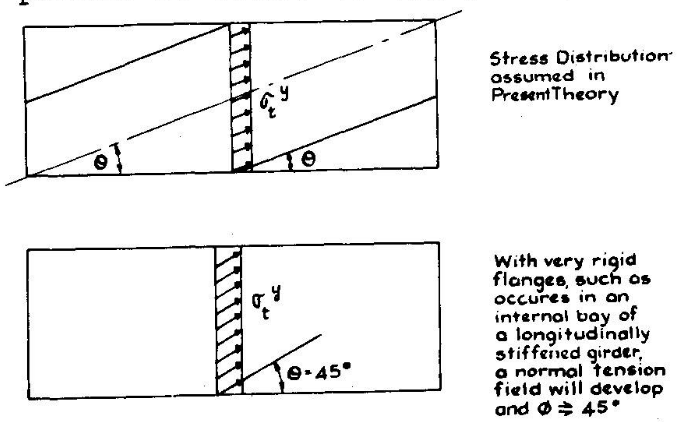


FIG.13 PROPOSED COLLAPSE MECHANISM FOR A LONGITUDINALLY REINFORCED WEB SUBJECTED TO SHEAR.

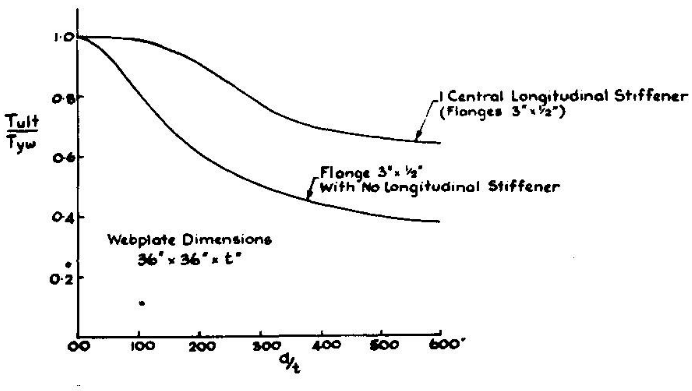


FIG.14 INFLUENCE OF A LONGITUDINAL STIFFENER ON THE SHEAR ULTIMATE STRENGTH OF A WEB

tensile membrane field would approach 45°, as indicated in figure 13. However, further research studies are required before this further

Figure 14 shows the significant gain in ultimate shear load which can be achieved by employing a longitudinal stiffener at mid depth.

2.4. Webplates reinforced by both transverse and longitudinal stiffeners and subjected to shear and bending

Figure 15 shows a typical panel subjected to a combined linearly varying axial direct stress and a shear stress τ. The linearly varying axial stress distribution can be replaced by an axial stress together with a pure bending stress as shown. For example, in Panel 1, there will be a direct compressive stress of σ_{mc1} and a pure bending stress at the flange/web junction of σ_{mb1}.

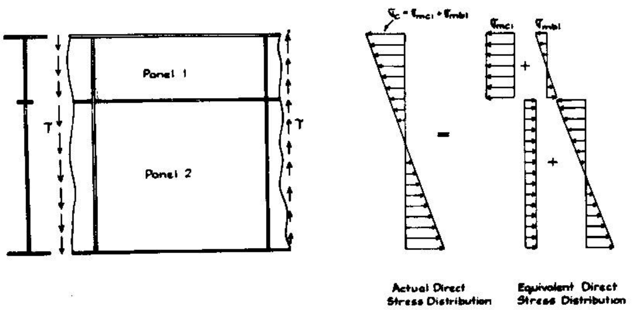


FIG.15 STRESS DISTRIBUTION IN PANELS OF A PLATE GIRDER SUBJECTED TO SHEAR AND BENDING

The critical stresses σ_{mc}, σ_{mb} and τ_m at which will cause buckling under their combined action occurs can be predicted with reasonable accuracy by equation (23).

$$\left(\frac{\sigma_{mc}}{\sigma_{crc}}\right) + \left(\frac{\sigma_{mb}}{\sigma_{crb}}\right)^2 + \left(\frac{\tau_m}{\tau_{cr}}\right)^2 = 1 \qquad (23)$$

where :

σ_{crc} = the critical uniform direct axial stress to cause buckling see equations (24) and (25)

σ_{crb} = the compressive edge stress causing buckling in the panel when loaded in pure bending, see equations (11) and (26)

τ_{cr} = the uniform shear stress to cause buckling, see equations (12a), (12b) and (27)

$$\sigma_{crc} = 4\left[\frac{\pi^2 E}{12(1-\mu^2)}\right]\left[\frac{t}{d}\right]^2 \quad \text{when all edges are simply supported} \qquad (24)$$

$$\sigma_{crc} = 5.41\left[\frac{\pi^2 E}{12(1-\mu^2)}\right]\left[\frac{t}{d}\right]^2 \quad \text{when one longitudinal edge is clamped, the others simply supported} \qquad (25)$$

$$\sigma_{crb} = 41.7\left[\frac{\pi^2 E}{12(1-\mu^2)}\right]\left[\frac{t}{d}\right]^2 \quad \text{when the compressive longitudinal edge is clamped, the other simply supported} \qquad (26)$$

$$\tau_{cr} = \left[7.07 + \frac{3.91}{(b/d)^{\frac{3}{2}}}\right]\left[\frac{\pi^2 E}{12(1-\mu^2)}\right]\left[\frac{t}{d}\right]^2 \quad \text{when one longitudinal edge is clamped, the others being simply supported} \qquad (27)$$

Once the critical stresses, σ_{mcl}, σ_{mb} and τ_m for the individual panels have been determined, the stress distribution at buckling will be known and the collapse load for each of the panels determined using the basic equations (7) and (14). In a longitudinally stiffened web plate this will involve an iterative procedure since the axial stresses in the flanges will vary with the shear load. Thus the use of either a desk calculating machine or a small computer is highly desirable.

CONCLUSION

The paper establishes an ultimate load method of design for plate girders having transversely and longitudinally reinforced webplates. In particular, it is shown that the flexural stiffness of the flange members has a significant influence upon the post buckled behaviour of webplates and the design method allows for the interaction which occurs between the buckled webplate and the flanges.

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SYMBOLS

t	thickness of web plate
t_f	thickness of flange plate
d	clear depth of webplate between flanges
b	clear width of webplate between stiffeners
$\alpha = \frac{b}{d}$	aspect ratio of panel
I	flexural rigidity of flange members about an axis passing through their centroid and perpendicular to the web plate
c	position of plastic hinge, see Figure 2
V_B	ultimate shear load provided by Basler collapse mechanism
V_{exp}	experimental ultimate shear load
V_{ult}	theoretical ultimate shear load
W	applied load
W_{ult}	theoretical ultimate shear load
W_B	collapse load according to Basler mechanism
W_{exp}	experimental ultimate load
M	applied bending moment
M_u	applied bending moment to cause collapse when acting alone
τ	applied shear stress
τ_{cr}	critical shear stress $= K \dfrac{\pi^2 E}{12(1-\mu^2)} \left(\dfrac{t}{d}\right)^2$ where K is a non dimensional parameter
τ_{cre}	reduced critical shear stress - see equation (8)
τ_{yw}	shear yield stress of web material
τ_{ult}	ultimate shear stress

σ_{yw}	tensile yield stress of web material
σ_{mc}	maximum comparison stress in Huber Von Mises plasticity condition
σ_{yf}	tensile yield stress of flange material
z_f	plastic modulus of flange
E	Young's Modulus of Elasticity
μ	Poisson's ratio
Θ	inclination of diagonal of panel with respect to flanges
σ_{crb}	compressive critical bending stress for a panel loaded in pure bending
σ_{crc}	critical direct stress for a panel loaded in pure compression
σ_{mb}, σ_{mc}, τ_m	compressive bending stress, direct compressive stress and shear stress which when acting in a combined system cause buckling

All other terms are defined as they appear in the text

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