#A constitutive relationship for maturing concrete

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A Constitutive Relationship for Maturing Concrete

Un rapport constitutif pour le béton au cours de son durcissement

Gesetzmässigkeit für alternenden Beton

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INTRODUCTION

In order to make a rational analysis of any structure or structural member, it is necessary to have available the constitutive relationships for the component materials. While a simple stress-strain-time relationship of the commonly used 'effective modulus' type is suitable for concrete in situations where creep does not lead to an appreciable redistribution of forces within the structure, a more general form is often required. Such instances arise, for example, when there is differential movement of external reactions, when the imposed loading is time-variable or when, as in a column, there is a gradual redistribution of stresses between steel and concrete. Where the load applied to a structure is an appreciable part of the ultimate load, the nonlinear creep:stress relationship also leads to a time-dependent redistribution of forces throughout the structure.

A number of investigators have found that apart from a few shortcomings, the well-known principle of superposition, first applied to concrete by Lorman[1] and McHenry[2], leads to a method of relating stress, strain and time in a fairly reliable way. This superposition method can be expressed precisely in terms of a time-variable linear viscoelastic equation; when the applied stress σ, varies with time t, then,

$$\varepsilon_c[t] = \int_{\tau_o}^{t} \phi[t-\tau,\ \tau] \cdot \frac{\partial\sigma}{\partial\tau} \cdot d\tau \qquad \ldots\ 1$$

where $\varepsilon_c[t]$ is the observed total creep strain at time t, τ_o is the age at stress is first applied, and $\phi[t-\tau, \tau]$ is the time-variable kernel or specific creep function which depends on the duration of loading $t-\tau$, and the age τ, at which stress is applied.

Many of the shortcomings of this theory can be traced to the three major assumptions made in deriving the above equation[3]; several important concrete characteristics are ignored. For instance, no account is taken of the nonlinear creep: stress relationship at higher levels of stress. Of greater significance perhaps, is the assumption that during a test in which the stress varies, strain increments are proportional to applied stress increments; hence no account is taken of the effects of either previous stress and environment history, or the sign of the current stress rate. In this last instance, the differing responses to positive and negative stress changes are not recognized.

Lastly, it is an additional limitation of the present form of the visco-elastic equation that it is difficult to allow directly for the marked dependence of creep on changes in the state (i.e. temperature, internal humidity and degree of hydration) of the cement-water system during the period under load.

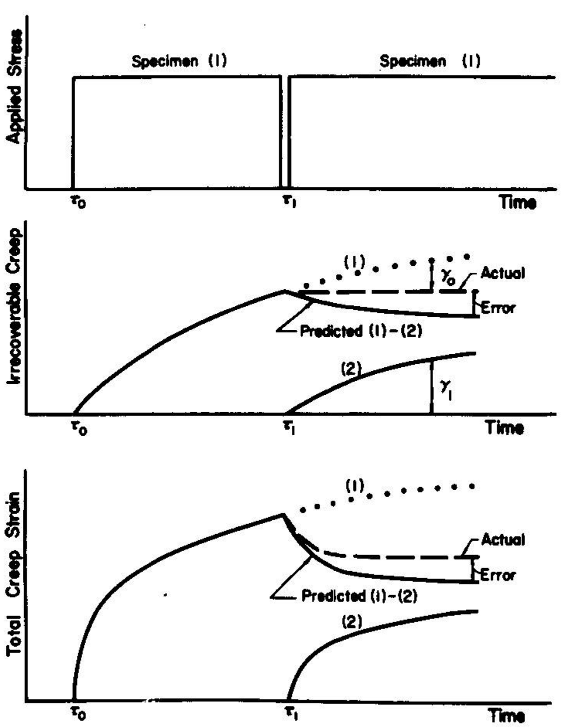
The paper is an attempt to incorporate these important effects into the governing equation.

STRESS CHANGES AND STRESS HISTORY EFFECTS

Several different tests of the superposition hypothesis have shown that under conditions of decreasing stress, the amount of creep recovery is usually over-estimated. It was shown by Gamble and Thomass[3] that this error occurs not simply as a result of the existence of an irrecoverable component of creep, but because after unloading the additional irrecoverable creep γ_o, (see Fig. 1) is not equal to the irrecoverable creep γ_1 occurring in a newly loaded companion specimen. The same analysis showed that the error involved should stabilize soon after the stress change, and thereafter the predicted and observed recovery curves would be parallel. Since this feature has indeed been observed in several tests and indicates a similarity of creep and creep recovery phenomena, a factor S, was proposed such that S is the ratio of the response of a specimen having some history of stress to that of a virgin specimen of the same age; see Fig. 2. Because of the differing responses to increasing and decreasing stress, S could be written in the form,

$$S = \begin{cases} S_c \quad ; \quad \dfrac{\partial\sigma}{\partial\tau} > 0 \\ \\ S_r \quad ; \quad \dfrac{\partial\sigma}{\partial\tau} < 0 \end{cases} \qquad \dots\dots 2$$

Incorporation of this factor overcomes, to some extent, the limitations



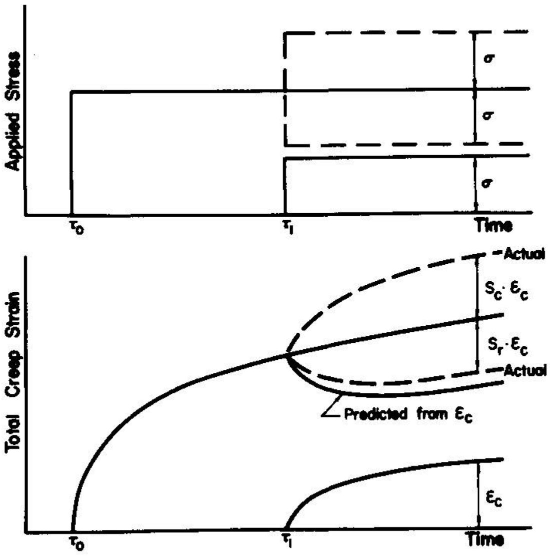


FIG. 1. (LEFT) SUPERPOSITION OF IRRECOVERABLE CREEP STRAINS

FIG. 2. (ABOVE) DEFINITION OF HISTORY FUNCTIONS FOR INCREASING AND DECREASING STRESS

imposed by the assumptions noted above.

In the investigation reported by Gamble and Thomass[3], the evaluation of S_r was carried out using experimental data, giving,

$$S_r = \begin{cases} 0.76 & ;\ t - \tau_1 < 72 \text{ hrs.} \\ 1 - 0.24 \cdot \dfrac{\varepsilon_c[72]}{\varepsilon_c[t-\tau_1]} & ;\ t - \tau_1 > 72 \text{ hrs.} \end{cases} \qquad \ldots. \quad 3$$

The error, $\varepsilon_c - S_r \cdot \varepsilon_c$, i.e. the difference between the corresponding creep and recovery responses, is a constant for values of $t-\tau_1$ greater than 72 hours.

In general, the form of S may be more complex than equation 3 would indicate and will in fact reflect all previous stress and environment history. For example, we know that events which took place either early in the life of a specimen or close to the time of observation, have a greater effect on the response to a given stress than have events which took place in the intervening period. In the present analysis it is acknowledged that a modifying factor S, dependent on stress and environment history, may exist and allowance is made for its effects.

NONLINEAR CREEP:STRESS RELATIONSHIP

For a given concrete composition, the degree to which the isochronous creep:stress relationship is nonlinear appears to depend primarily on the ratio of stress to strength. For normal concretes, some nonlinearity is apparent when the stress:strength ratio exceeds 0.30. In this same range time-dependent microcracking also commences and thus the observed nonlinearity appears to be a manifestation of the microcracking phenomenon. Photoelastic studies[4] have shown that strain concentrations of the order of 3 exist in normal concrete and we might therefore expect that at stress levels above 0.30 of the strength, cracking will begin. In mortars and pastes which may be considered to be less grossly heterogeneous than concrete, microcracking is delayed and appreciable nonlinearity may not be apparent until the stress: strength ratio exceeds 0.70. Tests by Gamble and Thomass[3] on concrete with a maximum aggregate size of ¼" indicated that the nonlinearity factor* is approximately 2 when the stress:strength ratio is 0.70. At this point the creep strain due to time-dependent cracking is equal to that due to water diffusion.

Until recently, development of the viscoelastic theory of concrete creep had been carried out assuming a directly proportionality with stress, and little attention had been given to characterising nonlinear behaviour. As early as 1941 however, Leaderman[5], in an investigation of textile fibres, proposed that if creep were a unique nonlinear function of stress, i.e.,

$$\varepsilon_c = C[t] \cdot f[\sigma] \qquad \ldots. \quad 4$$

then under varying stress conditions,

$$\varepsilon_c = \int_{\tau_0}^{t} C[t-\tau] \cdot \frac{\partial f}{\partial \tau} \cdot d\tau \qquad \ldots. \quad 5$$

Further consideration is given to this approach below.

* Defined as the ratio of the actual creep at a given stress:strength ratio to that which would have occurred had a linear relation held at all levels of stress.

STATE OF CEMENT-WATER SYSTEM

Creep is defined as being the time-dependent strain initiated or caused by stress. Although we define stress to be the primary agent causing creep (i.e. at zero stress there can be no creep) other variable factors, temperature, internal humidity and degree of hydration, will directly affect both the rate and value of creep at any moment. If the imposed stress is high enough that time-dependent micro-cracking exists, then an additional factor, related to strength will also influence the effect of a given stress. Although age is usually regarded as an important variable, it cannot, *per se*, exert a direct influence. Its effect on creep comes about indirectly through its influence on both the internal humidity and the degree of hydration of the cement gel.

If we define a set of standard values for each of these influencing factors, we will, for a given concrete subjected to unit stress, obtain a creep:time curve which by definition is a line in the conventional duration, age and specific creep surface. Until now it has been considered that a different creep:time curve will result whenever the influencing factors above are varied from their 'standard' values. In the present instance however, the author wishes to change this emphasis and indeed consider that there is a creep:time curve (or time function) which is a property of a particular concrete, unaffected by concurrent and possibly varying values of the influencing factors. This being assumed, it is necessary to account in some other way for the observed different creep responses of concrete subjected to stress in different steady environmental conditions. It is proposed that this be accomplished by the introduction of an excitation variable, not a function of stress alone as in the Leaderman equation but a composite function of stress, temperature, internal humidity, degree of hydration and, where appropriate, strength.

The traditional approach and the one here proposed do not differ at all when a creep test performed under constant conditions is being studied. In both cases the creep occurring for given constant values of stress, temperature, internal humidity and degree of hydration* will be

$$\varepsilon_c = A[\sigma] \;.\; B[T] \;.\; D[p/p_s] \;.\; G[\alpha] \;.\; C[t-\tau] \qquad \ldots. \quad 6$$

where T is the absolute temperature, p/p_s the internal humidity or relative vapour pressure, and α the degree of hydration. The quantity $t-\tau$ is the duration of loading. $C[t-\tau]$ is the creep:time curve when the other influencing factors have unit values. In the older approach, functions B, D, G and C are gathered together and represented by a temperature and humidity dependent creep:duration:age surface. Degree of hydration is not studied separately as it is usually reckoned to be a unique function of age, temperature and humidity. Furthermore for linear creep $A[\sigma] = \sigma$. Hence in terms of specific creep,

$$\varepsilon_c/\sigma = \phi[t-\tau,\ T,\ p/p_s,\ \tau] \qquad \ldots. \quad 7$$

The greatest disadvantage of this approach is that for a given concrete, a different kernel results for each set of influencing factors (usually time-varying) imposed during the test. Such information makes it rather awkward to predict behaviour under a different set of conditions.

The main point of justification for regrouping the variables is that changes of temperature, humidity or degree of hydration will affect the rate of creep in much the same way as will a change of stress. Temperature, for example,

* The most significant factor to describe the effect of continuing hydration is uncertain. A possible alternative to the degree of hydration would be the volumetric concentration of hydrated gel material.

affects the energetics of load-bearing water within the cement gel. An increase in temperature therefore increases the rate of creep. The internal humidity clearly governs the amount of water available for diffusion away from these load-bearing areas; other factors being equal, the higher the internal humidity the greater the rate of creep. When the ambient or external humidity is different from the internal humidity, the resulting moisture exchange also augments creep. For an initially saturated specimen, lower ambient humidities accelerate the migration or diffusion of water from the interior, resulting in a higher creep rate. Presumably self-dessication caused by continuing hydration of a sealed specimen has a similar effect. The direct effect of increasing hydration is to cause the level of stress per unit cross-sectional area of cement gel to diminish even though the overall force on the specimen may remain constant. This causes the creep rate of maturing concrete to decrease more rapidly than that of old concrete.

In the author's formulation, functions A, B, D, and G are grouped to give the temperature; humidity; degree of hydration-compensated stress or excitation function F, i.e.,

$$\varepsilon_c = C[t-\tau] \;.\; F[\sigma,\, T,\, p/p_s,\, \alpha] \qquad \ldots\ldots \; 8$$

Hence for steady conditions, the two approaches amount to the same thing; it is the case of time-varying conditions which is of greater interest and which is developed below.

CREEP UNDER TIME-VARYING CONDITIONS

By further consideration of equation 8, which holds for steady conditions, we may obtain a familiar viscoelastic formulation for time-varying conditions. In developing this form however, the effect of stress and environment history, not of consequence in the virgin specimen so far considered, has to be accounted for. Taking the stress and environment history to be characterised by S, the creep increment caused by an incremental change in the excitation function will be,

$$\Delta\varepsilon_c = C[t-\tau] \;.\; S \;.\; \Delta F[\sigma,\, T,\, p/p_s,\, \alpha] \qquad \ldots\ldots \; 9$$

Summation of all such increments leads to,

$$\varepsilon_c = \int_{\tau_o}^{t} C[t-\tau] \;.\; S \;.\; \frac{\partial}{\partial\tau}F\,[\sigma,\, T,\, p/p_s,\, \alpha] \;.\; d\tau \qquad \ldots\ldots \; 10$$

which is the governing or constitutive equation for the case where any of the influencing factors vary with time.

FORM OF EXCITATION FUNCTION FOR CONSTANT TEMPERATURE AND HUMIDITY

Of particular interest is the case of creep in conditions where the temperature and internal humidity remain at constant values, while the stress imposed and/or the degree of hydration vary. Having defined the constant levels of temperature and internal humidity, the excitation function may be written,

$$F = F\,[\sigma,\, \alpha] \qquad \ldots\ldots \; 11$$

Or for the more general situation where cracking produces creep strain,

$$F = F\,[\sigma,\, \alpha,\, \sigma_u] \qquad \ldots\ldots \; 12$$

As previously discussed, the stress:strength ratio logically appears to be a controlling factor at higher levels of stress, while at lower stresses where water diffusion is the probable creep mechanism, the degree of hydration or some similar variable, will be of greater fundamental importance. Hence as in Fig. 3,

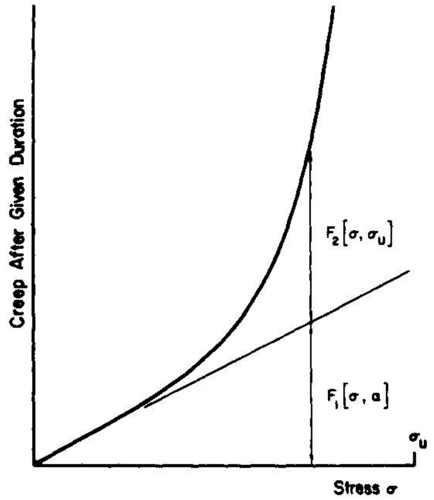


FIG. 3. DEPENDENCE OF CREEP ON STRESS LEVEL AND CONCRETE PROPERTIES.

$$F = F_1 [\sigma, \alpha] + F_2 [\sigma/\sigma_u] \quad .... \quad 13$$

In recent years numerous authorities have stated that the stress:strength ratio is the major influencing factor, although from theoretical considerations this is not at all obvious and indeed there are some refutations of this hypothesis. Certainly degree of hydration and strength will be intimately related, but this does not mean that the function of degree of hydration appearing in F_1 above must necessarily correspond with the inverse of strength; much more research is required before this point can be resolved satisfactorily.

In the present analysis the author has proceeded using the evidence in favour of the stress:strength ratio concept, i.e.,

$$F = F [\sigma/ \sigma_u] \quad .... \quad 14$$

DETERMINATION OF THE TIME FUNCTION C [t-τ]

As one test of the analysis presented in this paper, an attempt was made to obtain the time function C [t-τ], using data from twenty-two creep and creep recovery experiments which had been started at different ages and loaded to different stress levels. In these experiments, the age at loading varied between 3 and 40 days, so that although after loading the stress remained constant, continuing hydration caused the excitation function F, to diminish considerably during most tests. Temperature and ambient relative humidity were maintained at 75 ± 1.5°F and 94 ± 2 percent respectively throughout. Other experimental details are as previously given[3].

Referring to equations 10 and 14, it can be seen that if the variations of the quantities ε_c, S, and F, with time are known, then it is possible to derive the time function C[t-τ], by a simple iterative procedure. Normally of course, we will wish to find the response ε_c, using known time, history and excitation functions; then the integral is evaluated by the usual numerical methods.

The dependence of the excitation function on the stress:strength ratio was, in this case, determined from the short term creep data which was unaffected by changing hydration; see Fig. 4. In turn the variation of F with age τ, was determined.

The time-dependence of the history function S, was obtained from a comparison of creep and creep recovery data. The relationship for S_r is identical to equation 3, while S_c was taken as unity.

The calculated variation of the time function with duration is shown in Fig. 5. The results are surprisingly uniform when it is considered that twenty-two widely differing tests are involved. This means that a reliable estimate of creep under time-varying conditions of stress and hydration can be found using a single time function or unit creep curve.

At this point, attention is drawn to the rather different conclusions reached by Huggins and Timusk[6]. They deduced from tests performed that there would be significant differences in the creep of sealed specimens loaded at various ages to the same constant stress:strength ratio, i.e. to the same apparent value of the excitation function (c.f. equation 8). The clue to this

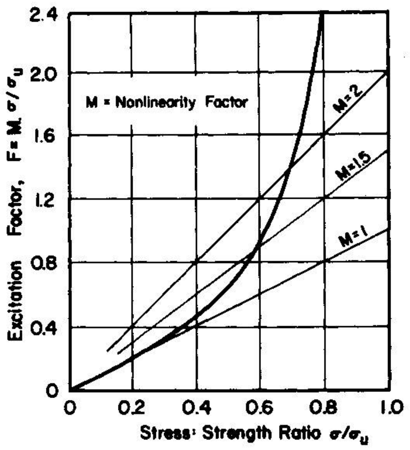


FIG. 4. RELATIONSHIP OF STRESS:STRENGTH RATIO AND EXCITATION FUNCTION

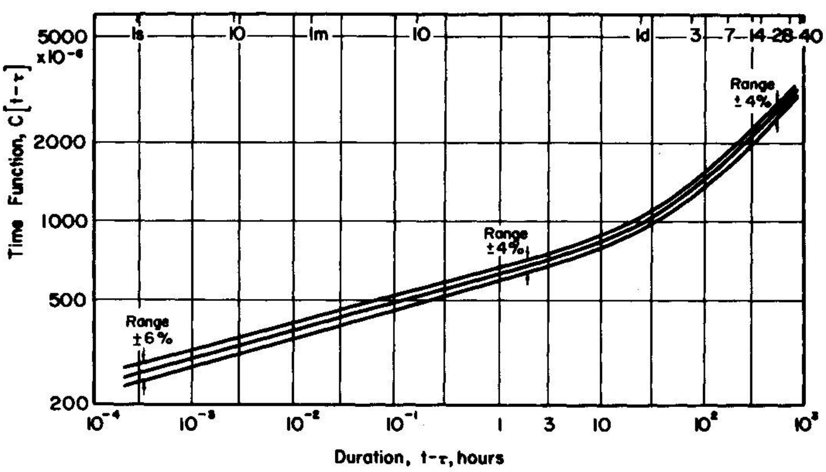


FIG. 5. TIME FUNCTION, OR UNIT CREEP CURVE CALCULATED FROM TWENTY-TWO CREEP TESTS

puzzle however, seems to lie in the fact that the specimens were sealed at an early age. This would cause progressive self-dessication and consequent lowering of the internal relative vapour pressure. In such a case, the stress: strength ratio cannot be used as the basis of comparison since the value of the excitation function depends on both σ/σ_u and p/p_s. The progressively diminishing internal relative humidity provides a logical reason for the observed differences in creep response.

CONCLUSION

The modified viscoelastic theory summarises creep behaviour under time-varying conditions in terms of three functions. The *excitation* function is a variable which, for specimens having a similar stress and environment history, is the sole creep-determining factor. Because of the simultaneous dependence of the excitation function on stress and internal humidity, the phenomenon of 'drying creep' may also be accounted for. As yet the dependence of the function on the component variables is only partially known. However, further investigation using the huge mass of existing creep data will rectify this situation. For the simple case where the creep:stress relationship is linear and the environmental conditions are unchanging, the function reduces approximately to the stress:strength ratio. The *time* function is a property of a particular concrete and is not a function of any time-variable parameters; the magnitude, and variation with duration of loading, depend only on the mix proportions and the properties of the cement and aggregates. The *history* function accounts for the differing behaviour of virgin and previously loaded specimens. It also accounts for the differing creep responses to increasing and decreasing excitations. Even a crude approximation for the history function results in much improved predictions.

The analogues of the excitation and time functions in the existing viscoelastic theory are stress and specific creep respectively. The existing formulation in terms of an age-varying creep function is an over-simplification of the actual phenomena involved. Age alone cannot be used to characterise creep, since the fundamental influencing factors, especially degree of hydration and internal humidity, may each vary independently during a test and their total effect is not therefore uniquely determined by age. Because of the form of the excitation function, the theory presented is essentially time-invariable, i.e. the time function depends only on the duration of loading; preliminary tests indicate that this dependence is probably unique for a given concrete. As a result of these properties, it will be possible in some instances to apply transformation techniques and make use of linear elastic solutions to solve the corresponding creep problems. A further advantage of the time-invariability property is that the analogue of the relaxation modulus can be defined in terms of duration alone. This is not possible for time-variable materials, even when the specific creep can be expressed as a product of age and duration functions[7]. This property is of especial importance in cases where a problem cannot be presented solely in terms of the creep formulation or the reciprocal relaxation formulation.

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SUMMARY

The existing superposition principle for concrete is modified so as to account for the nonlinear creep:stress relationship, for the dependence of creep on previous stress and environment history, and for the differing creep responses to increasing and decreasing stress. Introduction of an excitation function dependent upon stress,temperature,internal humidity,degree of hydration and strength makes it possible to account for the dependence of creep on changes in these variables during the test period. One series of experiments has provided partial verification of the analysis.

RESUME

Le principe existant de superposition pour le béton est modifié afin d'expliquer le fluage non linéaire: un rapport de force, entre d'une part la dépendance du fluage sur des efforts prévisibles et les caracteristiques du milieu et d'autre part les fluages différents, explique l'augmentation et la diminution des forces. L'introduction d'une fonction servant d'excitation dépendant de la tension,de la température, de l'humidité interne, du degré d'hydratation et de la rigidité donne une explication sur la dépendance du fluage avec le changement de ces variables. Une serie d'expérience a vérifié partiellement l'analyse.

ZUSAMMENFASSUNG

Das bestehende Ueberlagerungsprinzip für Beton ist abgeändert worden, um nichtlineares Kriechen zu erklären: Spannungsbeziehung für die Abhängigkeit des Kriechens von den vorgängigen Spannungen und der Umgebung sowie für verschiedene Kriechvorgänge infolge wachsender und abfallender Spannungen. Die Einführung einer Vergrösserungsfunktion, die von der Spannung, der Temperatur, der inneren Feuchtigkeit, des Wassergehaltes und der Festigkeit abhängt, erlaubt,die Abhängigkeit des Kriechens infolge der Aenderung dieser Variabeln während der Prüfzeit zu erklären. Eine Versuchsserie hat die Berechnungsmethode teilweise bestätigt.

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