Structural safety and optimum proof load

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II

STRUCTURAL SAFETY AND OPTIMUM PROOF LOAD

Sécurité des constructions et charge d'essais optimale

Bauwerksicherheit und optimale Prüflast

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1. Introduction

In a recent paper[1] dealing primarily with aerospace structures, the author pointed out the importance of proof-load test in conjunction with the optimum structural design based on reliability concept. In fact, Ref. 1 developed an approach to an optimum design (either minimum weight design or minimum expected cost design) introducing the proof load as an additional design parameter and demonstrated the advantage of the use of proof load in terms of weight saving (under constraint of expected cost). From the view point of probabilistic safety analysis, it was also pointed out, the advantage of performing the proof-load test was two fold; it could improve not only the reliability value itself but also the statistical confidence in such a reliability estimate since the proof-load test eliminates structures with strength less than the proof-load. In other words, the structure which passes the proof-load test belongs to a subset, having the strength higher than the proof load, of the original population. The fact that the proof-load test truncates the distribution function of strength at the proof load alleviates the analytical difficulty of verifying the validity of a fitted distribution function at the lower tail portion where data are usually non-existent. Evidently the difficulty still remains in the selection of a distribution function for the load. However, the statistical confidence in the reliability estimation now depends mainly on the accuracy of the load prediction. The question of how to deal with the statistical confidence of the load distribution was also discussed in Ref. 1.

Consider now civil engineering structures such as bridges, transmission towers and buildings. Because of their characteristic construction processes, these structures usually undergo tacit processes of proof-load test during the construction. If a structure does not fail during and upon completion of construction, it implies that all of its structural components and therefore the structure itself have sufficient strength to withstand at least the dead load. This is the information that must be taken into consideration as the lower bound of the strength distribution for the reliability estimation of an existing structure, although the lower bound thus established may in some cases be too small to be of any practical significance. Furthermore, if a structure under construction survives a live load due to severe wind or earthquake acceleration, which are referred to as secondary live load in many design codes but of primary importance for safety consideration of existing structures, the combined action of such a live load and of the dead load (existing at the time of occurrence of the live load) can be interpreted as a proof-load test. The fact that the partially completed structure has survived such a proof-load test should be taken into consideration in the reliability analysis since this fact usually makes it possible to establish a better lower bound of the strength of each of structural components (existing in the partially completed structure).

Although the subject of such implicit processes of proof loading appears to be an interesting item for future study, the present paper places an emphasis on the explicit proof-load test for civil engineering structures to be performed before the structures are placed into service, and examines the conditions under which the explicit proof-load test is economically advantageous.

An important implication of the above argument is that separate considerations are given to the safety of a structure during and after completion of its construction. This seems quite reasonable since the cost of detection possibly by means of proof-load test and the cost of the replacement of that part of the structure which failed because of a member or members with insufficient strength may be absorbed as the construction cost or otherwise, whereas any failue after the structure is placed into service by the client would produce much more serious contractual and socioeconomic problems, possibly involving human lives.

2. Expected Cost and Optimum Proof Load

The present discussion deals with a structure designed under a conventional design code with a specified design load S_d. The structure is supposed to withstand a system of proportional loads with a reference value S which is statistical. This system of loads is hereafter referred to as the load S, and the design load is meant by the same system of loads with a particular reference value S_d. Furthermore, it is assumed that the proof load to be applied is also the same system of loads with a reference value mS_d, in which a positive number m indicates the magnitude of the proof load in terms of the design load. For example, when a bridge is designed for a design uniform load w, the proof load is the uniform load with intensity mw. This assumption is made essentially for simplicity of discussion and does not imply the limitation of the proof load approach presented here. An obvious example in which the proof and the design loads cannot be of the same type is a tower structure designed for wind pressure. In such a case, how to specify a system of (proportional) loads as well as its magnitude that should most effectively (in some sense) be used as a proof load, is not a trivial problem. Evidently, it is possible to proof-test structural components individually before they are assembled (an approach discussed in Ref. 1). This approach, however, appears to be too expensive to be applied to civil engineering structures.

Under these circumstances, it seems reasonable, for the purpose of presenting the essential idea of optimum proof load, to assume the following form of expected cost EC of a structure.

$$EC = q_o C_o + p_f C_f \quad \text{or} \quad EC^* = q_o \gamma + p_f \qquad (1)$$

where $EC^* = EC/C_f$ = the relative expected cost, $\gamma = C_o/C_f$, q_o= the expected number of the (candidate) structures that fail under the proof load before the one that can sustain it is obtained, C_o = the cost of a proof load test including the cost of loss of a (candidate) structure (during the proof load test), p_f = the probability of structural failure (that might occur after the structure is placed into service) and C_f = the cost of structural failure (that might occur after the structure is placed into service) such as cost of the structure, loss of prestige, etc. It is noted that Eq. 1 takes only the costs of failure and of proof-load test into account, although more elaborate forms are obviously possible and may even be desirable depending on the specific problem at hand.

Since the proof load is applied to the (entire) structure, not to its components individually as in Ref. 1, there is a probability p_o that it will produce a failure of the entire structure unless a method is devised to replace the component that exhibits an initiation of failure at a magnitude of proof load less than the prescribed value before the structural failure developes. If the proof load can produce only component failures because of such a device or otherwise, it seems reasonable to consider that the ratio γ is as small as 10^{-4} or even smaller. If, however, the proof load can lead only to structural failures, the ratio does not seem to be so small. In the present discussion, it is assumed that the proof load may produce only structural failures and that the ratio γ ranges from 10^{-4} to 10^{-1}.

The expected number q_o of candidate structures that will fail under the proof load can be shown to be

$$q_o = p_o/(1-p_o) \tag{2}$$

in which the probability p_o (defined previously) is given by

$$p_o = \int_0^{mS_d} f_{R_0}(x)dx = F_{R_0}(mS_d) \tag{3}$$

with $f_{R_0}(\cdot)$ and $F_{R_0}(\cdot)$ being respectively the density and the distribution functions of the resistance R_0 of the structure on which the proof-load test has not been performed yet.

The probability of failure, p_f, of the structure which has passed the proof-load test can be written in the following well-known form:

$$p_f = \int_0^{\infty} F_R(x)\ f_S(x)dx \tag{4}$$

where $F_R(\cdot)$ is the distribution function of the resistance R of the structure which has passed the proof-load test and $f_S(\cdot)$ is the density function of the load S.

Under further simplifying assumptions, as used in most of previous papers including Ref. 2, that the pertinent resisting strengths (such as yield strength) of the individual structural members and therefore the resistances (load carrying capacities) of the same members are statistically independent of each other as well as of the load S, the distribution functions $F_{R_0}(\cdot)$ and $F_R(\cdot)$ can be written as

$$F_{R_0}(x) = 1 - \prod_{i=1}^{n} \left[1 - F_{oi}\left(c_i x/a_i\right)\right] \tag{5}$$

$$F_R(x) = 1 - \prod_{i=1}^{n} \left[1 - F_i\left(c_i x/a_i\right)\right] \tag{6}$$

where n is the number of members constituting the structure. Eqs. 5 and 6 are to be used respectively in Eqs. 3 and 4. In Eq. 5, $F_{oi}(\cdot)$ is the distribution function of the ("parent") resisting strength τ_{oi} of the i-th member of the structure which has not been subjected to the proof-load yet. Also, $F_i(\cdot)$ in Eq. 6 indicates the distribution function of the resisting strength τ_i of the i-th member of the structure which has passed the proof-load test. Quantities c_i and a_i are such that the load S_i acting in the i-th member can be obtained from the load S as

$$S_i = c_i S \tag{7}$$

and the resistance of the same member can be computed as

$$R_i = a_i \tau_i \tag{8}$$

For example, τ_i and a_i are respectively the yield strength and the cross-sectional area of the i-th member if a truss structure is considered.

As was discussed in detail in Ref. 2, the following points are to be noted in deriving Eqs. 4, 5 and 6; (1) the definition of structural failure is in accordance with the weakest link hypothesis, that is, the failure will take place if at least one of the components fails, (2) the assumption that the member strengths are statistically independent to each other is a conservative one, (3) p_f in Eq. 4 indicates the probability of structural failure due to a single application of the load S. Also, in deriving Eq. 7, the effect of the dead load is neglected for simplicity. Any method of structural analysis can be employed to obtain Eq. 7 including the finite element method.

By applying the proof load mS_d, each member is subjected to a force $c_i mS_d$. Therefore, if the structure (and therefore all the members) survives the proof load, a lower bound $c_i mS_d$ is established for the resistance of the i-th member. Because the force and the stress are related by Eq. 8, this in turn establishes a lower bound

$$\tau_{mi} = c_i mS_d / a_i \tag{9}$$

for the parent resisting strength τ_{oi} of the i-th member. Then, the distribution function $F_i(\cdot)$ of the ("truncated") resisting strength τ_i of the same member of the structure having passed the proof-load test can be shown to be

$$F_i(x) = \frac{F_{oi}(x) - F_{oi}(\tau_{mi})}{1 - F_{oi}(\tau_{mi})} H(x - \tau_{mi}) \tag{10}$$

where $H(\cdot)$ is the Heaviside unit step function.

Eq. 10 indicates that the distribution function of the (truncated) resisting strength of the structure which passed the proof load test is obtained from that of the parent strength by "truncating" it at the lower bound established by the proof load (and normalizing it).

The standard design requires that the nominal resistance $a_i\tau_{ai}$ be equal to the nominal applied load c_iS_d:

$$a_i\tau_{ai} = c_iS_d \quad \text{or} \quad a_i\tau_{pi}/\nu_i = c_iS_d \tag{11}$$

where τ_{ai} = the allowable stress, τ_{pi} = the specified minimum resisting strength and ν_i = the safety factor of the i-th member (these quantities are functions not only of the material but also of the mode of failure, e.g. in bending, in tension, in stability, etc.).

From Eq. 11, it follows that

$$c_i/a_i = \tau_{pi}/(\nu_iS_d) \tag{12}$$

The right hand side of Eq. 12 consists of quantities specified in the design code. Therefore, Eq. 12 makes it possible to replace c_i/a_i in Eqs. 5, 6 and 9 by known quantities.

Eqs. 2 and 4 (together with Eqs. 3, 5, 6, 9, 10 and 12) can now be used in Eq. 1 to compute the relative expected cost if $F_{oi}(\cdot)$ and $f_S(\cdot)$ are known. The optimum intensity of the proof load is then obtained as that value of m which minimizes the relative expected cost EC^*.

3. Example

In the following, the assumptions are made that (1) the allowable stresses (or both the specified minimum strengths and the safety factors) and (2) the distribution functions $F_{oi}(x)$ of the parent strengths are identical for all the members; $\tau_{oi} \equiv \tau_o$ and

$F_{oi}(x) \equiv F_o(x)$. These assumptions are made purely for simplicity. The analysis presented in the preceding section can easily accommodate the situations in which this is not the case; e.g. consider different allowable stresses specified for tension and compression members and also consider the fact that in reality, different distribution functions of the parent strengths are needed for tension and compression members.

The immediate consequences of these assumptions are that (1) c_i/a_i in Eq. 12 and hence τ_{mi} in Eq. 9 become independent of the subscript i; $c_i/a_i = \tau_p/(\nu S_d)$ and $\tau_m = m\tau_p/\nu$, and (2) the truncated strength distribution $F_i(x)$ also becomes independent of i; $F_i(x) \equiv F_\tau(x)$.

In the present paper, the parent strength distribution is assumed to be distributed according to the Weibull distribution:

$$F_o(x) = 1 - \exp\left[-(x/\tau_c)^b\right] \tag{13}$$

where τ_c is the characteristic strength and b is a positive constant.

From Eqs. 10 and 13, it follows that

$$F_\tau(x) = \left\{1 - B\exp\left[-\left(\frac{x}{\tau_c}\right)^b\right]\right\} H\left(x - \frac{m\tau_p}{\nu}\right) \tag{14}$$

with

$$B = \exp\left[\left(\frac{m\tau_p}{\nu\tau_c}\right)^b\right] \tag{15}$$

Therefore, Eqs. 5 and 6 can be respectively written as

$$F_{R_o}(x) = 1 - \exp\left[-\left(\frac{x}{R_c}\right)^b\right] \quad x > 0 \tag{16}$$

$$F_R(x) = 1 - B^n \exp\left[-\left(\frac{x}{R_c}\right)^b\right] \quad x \geq mS_d \tag{17}$$

and from Eq. 2,

$$q_o = B^n - 1 \tag{18}$$

where $R_c = hS_d/n^{1/b}$ with $h = \nu\tau_c/\tau_p$ is the characteristic resistance of the structure which has not proof-load-tested yet. The parameter b is a measure of dispersion of the distributions of τ_0 and R_0; the coefficients of variation in terms of their characteristic values are 0.46, 0.33, 0.25 and 0.21 respectively for $b \doteq 2, 3, 4$ and 5.

For the distribution function $F_S(x)$ of the load S, the first asymptotic distribution function of largest values is assumed. However, since only the upper tail portion of the distribution is significant, the following exponential form is used as an approximation for larger values of the load;

$$1 - F_S(x) = r \exp\left[-a\left(x-kS_d\right)\right] \qquad x > kS_d \tag{19}$$

where "a" is a positive constant and $kS_d (0 < k < 1)$ is the lower bound above which such an approximation is valid and r is such that the probability that S will be larger than kS_d is r.

The final expression for the probability of failure is

$$p_f = ra \int_{mS_d}^{\infty} \left\{1 - B^n \exp\left[-\left(\frac{x}{R_c}\right)^b\right]\right\} \exp\left[-a\left(x - kS_d\right)\right]dx \tag{20}$$

Although this integral cannot be evaluated in closed form unless $b = 1$ or 2, an asymptotic approximation can be obtained by expanding the first term of the integrand and integrating term by term as long as $\lambda \gg 1$ where $\lambda = sh/n^{1/b}$ with $s = (1-k)^{-1}\ell n(r/q)$. The result is

$$p_f \doteq Ar \exp[-s(m-k)] \tag{21}$$

with

$$A = (2ms + 2)/\lambda^2 \qquad (b=2) \tag{22a}$$

$$A = \left\{3(ms)^2 + 6(ms) + 6\right\}/\lambda^3 \qquad (b=3) \tag{22b}$$

$$A = \left\{4(ms)^3 + 12(ms)^2 + 24(ms) + 24\right\}/\lambda^4 \qquad (b=4) \tag{22c}$$

$$A = \left\{5(ms)^4 + 20(ms)^3 + 60(ms)^2 + 120(ms) + 120\right\}/\lambda^5 \qquad (b=5) \tag{22d}$$

where ms should be smaller than λ and q is the probability that the load S will be larger than S_d. The result does not contain the parameter "a" (Eq. 19) explicitly. It however, appears in the preceding equations implicitly since $a = s/S_d$.

The validity of such asymptotic approximations is checked by comparing the result using Eq. 21 with that of the exact integration for $b = 2$. The agreement is more than satisfactory.

A number of sets of parameters are considered for numerical examples. Among these, the result for the case where the structure consists of 7 members $(n=7)$, $b = 4$, $q = 0.02$, $r = 0.1$, $k = 0.6$ (thus $s = 4.03$) and $\nu = 1.67$, is presented. The specified minimum strength τ_p is defined so that the probability of the parent strength τ_o being less than τ_p is p. Therefore, from Eq. 13, $\tau_c/\tau_p = [-\ell n (1-p)^{-1/4}]$. For the present example, $p = 0.1$ is used (hence $h = 5.15$). The assumption that $q = 0.02$ implies that the design load with a return period of 50 years is considered if the distribution $F_S(x)$ is that of the annual largest load.

The result is illustrated in Fig. 1 where the relative expected cost EC^* is shown as a function of $\mu = m/h$. The value μ indicates a magnitude of the proof load relative to hS_d at which the loads (the stresses) acting within the individual members are equivalent to their characteristic values $a_i\tau_c(\tau_c)$. Since the optimum proof-load is the one at which EC^* becomes minimum, Fig. 1 indicates that the proof-load becomes optimum when $\mu = 0.2$, 0.38, 0.55 and 0.67 (or $m = 1.03$, 1.95, 2.83 and 3.45) respectively for $\gamma = 10^{-1}$, 10^{-2}, 10^{-3} and 10^{-4}. The locus of those points at which EC^* assumes minimum values (Curve 1) is also plotted as a function of μ in Fig. 1. Since $b = 4$, the coefficient of variation with respect to the characteristic value of the parent strength τ_o is 0.25. Therefore, these optimum proof loads truncate the strength distribution at 3.2σ, 2.5σ, 1.8σ and 1.3σ below its characteristic value respectively for $\gamma = 10^{-1}$, 10^{-2}, 10^{-3} and 10^{-4}. Also plotted in Fig. 1 is the probability of failure as a function of μ. The probability decreases monotonically as μ increases; the reliability increases as a larger proof load is applied.

The above result indicates that, for this particular example, performing the proof-load test may not be justified if γ is of the order of 10^{-1} because (1) the optimum proof stress is more than 3σ away from the characteristic strength and therefore not much improvement in statistical confidence in reliability estimation is expected and (2) if one increases the magnitude of the proof load beyond the optimum value to achieve such improvement, the prohibitive cost is likely to be incurred due to possible loss of the (candidate) structure(s) which is rather expensive (larger value of γ). However, if γ is of the order of 10^{-2} or less, performing the proof-load test appears justified from the point of view of improving (1) the statistical confidence in the reliability estimation (since the points of truncation are at most 2.5σ away from the characteristic value) and (2) the reliability itself. However, the optimum magnitude of the proof-load increases considerably as γ decreases. This may present some difficulty in performing the proof-load test.

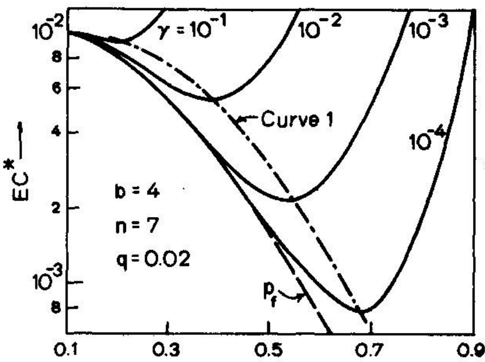

Fig. 1 $\mu \longrightarrow$

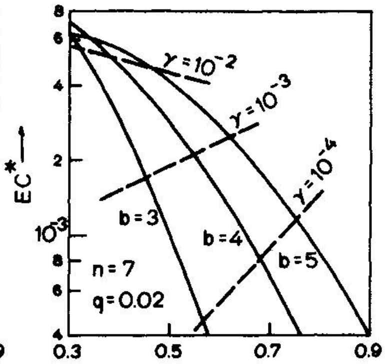

Fig. 2 $\mu \longrightarrow$

Since the preceding observation is based on (1) the computation associated with a particular set of parameters, (2) the particular form of the expected cost and (3) the specific form of strength and load distributions, and sensitivity of these items on the result will be an interesting subject of future study. For example, Fig. 2 shows the loci of the optimum points (such as Curve 1 in Fig. 1) for b = 3, 4 and 5 plotted on the same diagram, indicating the effect of b and γ on the optimum proof-load.

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SUMMARY

The interrelationship among the probability of structural failure, the expected cost of structure and the proof-load testing is established and used for a general reliability analysis. The optimum proof-load is defined for structures designed under a conventional design code and conditions are examined under which the proof-load testing is advantageous economically as well as from the viewpoint of improving both the reliability itself and the statistical confidence in such a reliability estimate.

RESUME

On examine la relation entre la probabilité de ruine, le prix évalué de la construction et les essais de charge. La charge d'essai optimale est définie pour les structures conçues d'après les normes conventionnelles. Puis on examine les conditions sous lesquelles les essais de charge sont aussi bien avantageux économiquement qu'utiles pour la sécurité et pour la certitude de la sécurité évaluée.

ZUSAMMENFASSUNG

Aufgezeigt wird die Beziehung zwischen der Bruchwahrscheinlichkeit, dem Erwartungswert der Kosten sowie der Prüflast und für die Zuverlässigkeitsrechnung verwendet. Die optimale Prüflast wird für nach alten Vorschriften entworfene Bauwerke definiert. Sodann werden die Bedingungen untersucht, für welche das Prüflastverfahren sowohl wirtschaftlich als auch im Hinblick auf die Zuverlässigkeit selbst und das Vertrauen in eine solche Zuverlässigkeitsschätzung vorteilhaft ist.

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