Web instability near reinforced rectangular holes

Web Instability near Reinforced Rectangular Holes

Instabilité de l'âme près d'ouvertures rectangulaires, renforcées

Stegbeulen in der Nähe verstärkter rechteckiger Öffnungen

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SUMMARY

Horizontal bar reinforcement for rectangular holes is frequently designed on the basis of ultimate strength procedure. In order that such a procedure is valid, it is necessary to identify cases where buckling of the web near the hole may preclude the full development of ultimate strength at the hole. The following paper deals with an experimental investigation of this possibility of premature buckling for beam sections normally used in the plastic design of structures. Conclusions are summarized in a set of design recommendations, adherence to which will ensure adequacy of this type of reinforcement in preventing such buckling.

RÉSUMÉ

Le renforcement de l'âme près d'ouvertures rectangulaires, au moyen de plaques horizontales est souvent déterminé par un calcul à la rupture. Il est nécessaire, pour que cette méthode soit valide, de déterminer au préalable les cas dans lesquels le voilement de l'âme se produit avant la rupture à proximité de l'ouverture. Une étude expérimentale devait permettre de déterminer le voilement prématuré des sections de poutres normalement considérées dans le calcul plastique des structures. Des recommandations sont présentées pour le calcul; leur application devrait prévenir le voilement, lors de l'emploi de ce type de renforcement.

ZUSAMMENFASSUNG

Die waagrechten Verstärkungsrippen im Bereich rechteckiger Stegöffnungen werden oft durch Traglastuntersuchungen bemessen. Es ist somit notwendig, diejenigen Fälle zu bestimmen, bei denen das Stegbeulen in der Nähe der Öffnung vor Erreichen der Traglast im verstärkten Bereich auftritt. Die Autoren beschreiben experimentelle Untersuchungen betreffend die Gefahr eines vorzeitigen Beulens bei den Trägerquerschnitten, wie sie für nach dem Traglastverfahren bemessene Tragwerke verwendet werden. Die Schlussfolgerungen werden in Form von Empfehlungen zusammengefasst, deren Anwendung das Beulen ausschliessen und daher eine abgewogene Bemessung einer solcher Verstärkung gewährleisten soll.

1. INTRODUCTION

Reinforcement for rectangular holes in structural steel I-beams is frequently provided by flat steel bars welded along the horizontal edges of the hole. The design of such reinforcement when placed symmetrically can be carried out on the basis of ultimate strength considerations [1].

It must be understood, however, that the ultimate strenth method of design of reinforcement rests on the assumption that premature buckling does not occur and the beam is capable of deformation to the extent that full plasticity can be developed at the hole section. To ensure this to be the case, it is necessary to identify those situations where premature buckling, particularly of the web, may govern the design. The state of stress around the hole is complex and particularly so when plastic deformations occur. Under such conditions, it would be extremely difficult to determine the influence of the various factors on the stability of the web solely on the basis of analytical considerations and experimental approach becomes essential.

The purpose of the series of tests was to study the stability of the web area in the vicinity of a reinforced hole, for different hole and beam configurations and under different loading conditions. In particular, the objective of the present tests was to identify those situations where no vertical stiffeners would be necessary and the effectiveness of horizontal reinforcement alone would be adequete in preventing the buckling of the web and in developing the strength predicted on the basis of [1].

Since the type of reinforcement provided makes the beam effectively equivalent to two I-beams over the length of the hole, the possibility of buckling of the webs of these smaller beams is remote and therefore, attention was focused mainly on the stability of the full web near the vertical edges of the hole.

Tests were conducted on 4 specimens. These were selected so as to provide conclusions regarding the effectiveness of this type of reinforcement for holes in webs with slenderness, $\bar{h}=(h_w/t_w)\sqrt{\sigma_{rw}/E}$ varying from 2.28 to 3.43. Results of the tests, and suggestions relating to design are discussed with reference to the web slenderness limits specified in [2] for Class 1 and Class 2 sections. These are defined in terms of web and flange slenderness ratios; both classes are suitable for design on the basis of ultimate strength of the member, but sections of Class 1, unlike those of Class 2, have rotation capacity adequate to permit full moment redistribution in a structure. The relevant web slenderness limits are $\bar{h} \leq 2.47$ for Class 1 sections and $\bar{h} \leq 3.05$ for Class 2, and therefore the range of $\bar{h}$ of the test specimens includes both of these limits.

2. IDENTIFICATION OF SIGNIFICANT PARAMETERS

The significant parameters which may be considered as affecting the stability of the web near a reinforced hole are discussed below with reference to the hole and reinforcement configuration shown in Fig. 1. These parameters are considered to be:

(a) Applied shear V at the hole
(b) Applied moment M or, with (a) above, the M/Vh ratio at the hole
(c) Aspect ratio a/H of the hole
(d) Strength of the reinforcement and its dimensions b_1 and t_1
(e) Anchor length of the reinforcement
(f) Effect of transverse stiffeners not far from the hole
(g) Slenderness, $\bar{H} = (2H/t_w)\sqrt{\sigma_{rw}/E}$ of the web portion between the reinforcement

(h) Slenderness, $\bar{h} = (h_w/t_w)\sqrt{\sigma_{rw}/E}$ of the unperforated web.

It may be noted that while the moment capacity at the hole section may be increased indefinately, the maximum shear force which can be sustained at the hole is limited by the available web depth to $V_{max} = V_{p\ell}(1-2H/h_w)$, and cannot be increased beyond this value by extra amounts of reinforcement of the type under consideration. As is well known from the elastic case [3], the buckling of an unstiffened web is generally associated with high shear force. The tests were therefore designed and the reinforcement was provided with the expectation of loading the beam at the hole with as high a shear force as possible, almost equal to the capacity V_{max}.

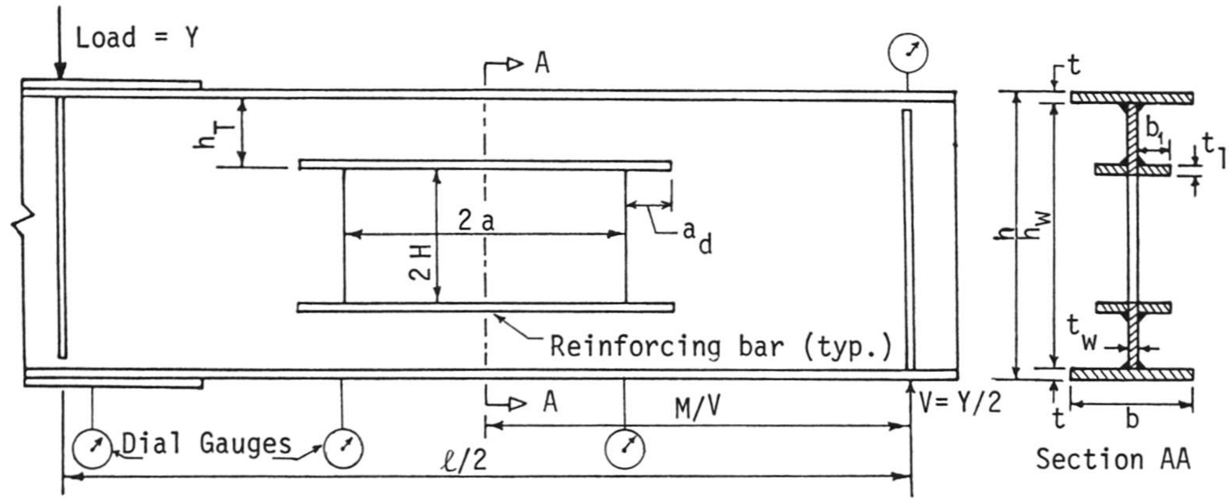

Fig. 1. Notation at the hole.

This requirement of realizing high shear force at the hole section made it necessary to locate the holes near the support, thereby limiting the applied moment M at the hole to about $M_{p\ell}/2$ and the ratio of moment to shear force, M/Vh to about 1.5. A location farther from the support would have increased the magnitude of M but would have then required substantially more reinforcement at the hole and, in addition, extensive cover plates elsewhere on the beam. The tests therefore do not cover the case of high shear coupled with high bending moment, i.e. $M>M_{p\ell}/2$. This case, however, although more critical for the stability of the web, is likely to occur only in rare instances of holes near the interior supports of a heavily loaded continuous beam. It can be shown that even these cases fall within the experimental range of moment if the hole centerline is located at least three times the beam depth from such a support.

The loading cases with low shear $V<V_{p\ell}/3$ and high moment $M>M_{p\ell}/2$ are not considered critical. On the basis of experimental observations and theoretical considerations [3], such cases may be critical only for thinner webs than those considered here.

Regarding the role of aspect ratio a/H as a parameter, it should be noted that for the same shear capacity, a longer hole needs a larger amount of reinforcement. A long hole may result in the web being stiffened by virtue of the large area of reinforcement required, whereas, if a hole is too short, the web may again be stiffened due to the proximity of the hole ends. This effect can be considered as related to the aspect ratio of the web above the hole, $2a/h_T$, and a value of this of about 3 is considered to be optimum in the sense of providing the least favourable effect on the stability of the web. A shorter hole than this is considered to have a stiffening effect on the unperforated web near the hole, while a longer one (within practical limitations) is considered to cause no significant change in the stiffening effect.

The amount of reinforcement for a given beam size and loading, and a chosen hole size, is determined by the ultimate strength procedure of [1]. Interaction curves between the developable moment and corresponding shear capacities at the hole, such as those shown in Fig. 2, may be obtained for different sizes of reinforcement. The minimum reinforcement corresponds to that interaction curve which passes through the point representing the desired moment and shear capacities.

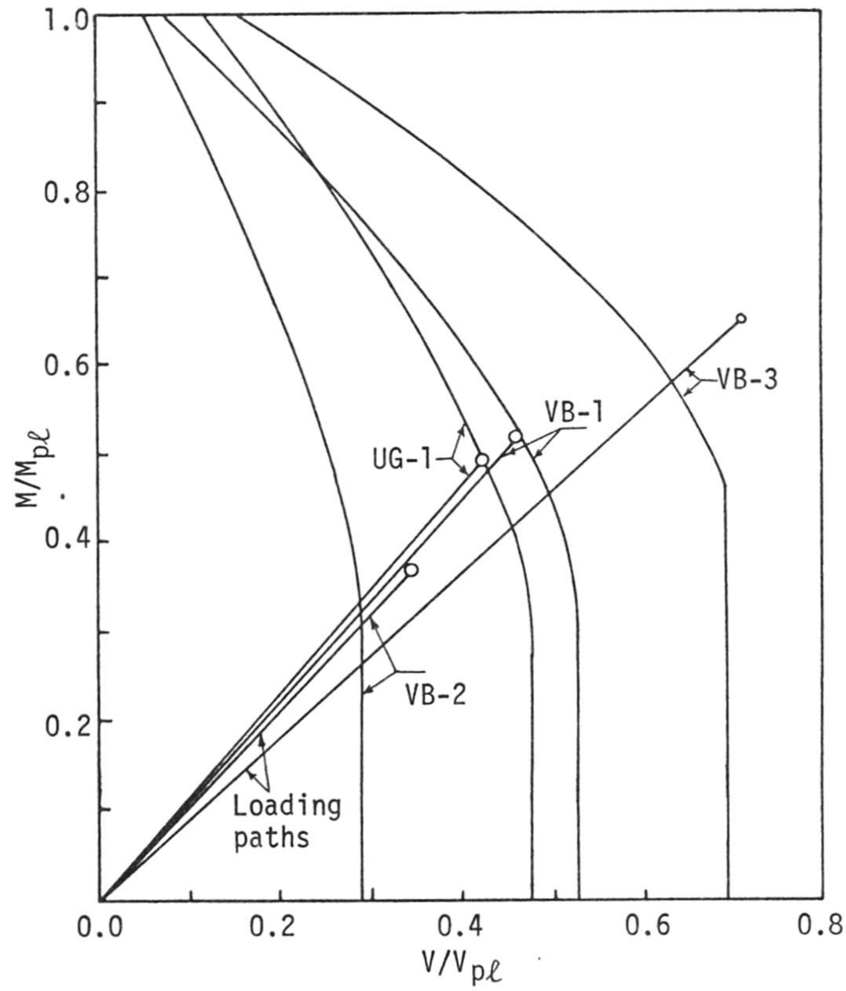

Fig. 2. Interaction diagrams for test beams.

For a particular area of reinforcement, the width b_1 and thickness t_1 of the reinforcing bar may be chosen to satisfy $b_1/t_1=0.376/\sqrt{\sigma_{rb}/E}$. Such proportions ensure safety against local buckling of the reinforcement according to [2]. A thicker plate meeting this restriction, may reduce the susceptibility to buckling by providing greater torsional stiffness of the reinforced section. A thinner plate, on the other hand, may also help stiffen the web by requiring a greater anchor length. Thus, whether a thicker or a thinner plate is chosen, the effect on the stability of the web may be considered the same; it is sufficient to identify the reinforcement area alone as a significant parameter.

For a given area of reinforcement, the minimum development or anchor length can be computed on the basis of transfer of the strength of the reinforcement to the web. This length may depend upon the strength of the web, the weld size, or the thickness of the reinforcing bar. An anchor length larger than the minimum required for the strength could only affect the stability of the web favourably.

In the present series of tests, the area of reinforcement as well as the anchor length were chosen to be nearly equal to the minimum required by the consideration of ultimate strength. In other words, these parameters are not considered independent but are restricted to result in the least favourable conditions for the stability of the web.

Also, in order to eliminate the favourable effect of transverse stiffeners on the stability of the web in the hole region, such stiffeners were placed (if needed) at a distance of not less than the height of the web h_w from the vertical edge of the hole.

Thus with considerations such as the above, the significant parameters for the tests were narrowed down to

(a) Slenderness $\bar{h} = (h_w/t_w)\sqrt{\sigma_{rw}/E}$ of the unperforated web and
(b) Slenderness $\bar{H} = (2H/t_w)\sqrt{\sigma_{rw}/E}$ of the web height between the reinforcement.

All the other parameters were chosen and restricted, (within the limitations mentioned), to result in unfavourable conditions for the stability of the web.

The factors which do not have direct influence on the buckling of the web must be taken care of independently to ensure that the failure does not occur somewhere else before it does at the hole. Such factors are, for example, adequate lateral bracing, buckling strength of the compression flange, stiffeners at concentrated loads, cover plates, etc. These were checked and provided for if necessary to satisfy the requirements [2] for ultimate strength design, with the exception of the flange slenderness. In these tests, the slenderness chosen corresponded approximately to $b/2t=0.376/\sqrt{\sigma_{rf}/E}$, the limit specified for Class 2 sections [2].

3. PREPARATION OF TEST SPECIMENS

The test specimens had average properties listed in Table 1 and were tested in the order of their listing. All were fabricated to order by continuous welding of plate elements. The webs of the first two specimens, UG-1 and VB-1, were intended to be at the limit of Class 2 sections ($\bar{h} \leq 3.05$) but, because of higher yield strength than expected, proved to be more slender, with $\bar{h}$ around 3.52. The last two specimens, VB-2 and VB-3, had slenderness $\bar{h}$ of about 2.35. These latter ones are considered to be representative of Class 1 sections ($\bar{h} \leq 2.47$).

Because of their fabrication, the specimens were not free from imperfections. The flanges were bent towards each other at their edges and sometimes top and bottom flanges had different widths and thicknesses. However, all these imperfections were within the acceptable limits [4], and were therefore considered not significant. Average dimensions as listed in Table 1 were taken to be representative of the sectional properties. Small deviations from a plane surface were also found in the webs. These imperfections were all found to be within the tolerances permitted by [5].

The holes in the beam specimens were cut in the laboratory with an oxy-acetylene torch in such a way that, when edges were ground and finished, the dimensions were close to the specified values, with corners having 5/8 in. radius.

TABLE 1

AVERAGE SECTIONAL PROPERTIES OF TEST SPECIMENS

Specimen		UG-1	VB-1	VB-2	VB-3
Span	L in.	120	120	114	94
Flanges	b in.	7.25	7.33	7.0	7.25
	t in.	0.385	0.385	0.425	0.427
	σ_{rf} ksi	54.0	54.6	40.3	41.65
Web	h in.	20.66	20.63	16.31	17.06
	h_w in.	19.89	19.87	15.46	16.20
	t_w in.	0.257	0.256	0.266	0.279
	σ_{rw} ksi	57.0	58.3	49.0	44.7
$M_{p\ell}$	kip in.	4504.8	4591.1	2683.3	2963.5
*$V_{p\ell}$	kip	168.2	171.1	116.4	116.7
$\bar{h} = (h_w/t_w)\sqrt{\sigma_{rw}/E}$		3.43	3.48	2.39	2.28

* Note that $V_{p\ell} = h_w t_w \sigma_{rw}/\sqrt{3}$, according to von Mises yield criterion.

TABLE 2

PROPERTIES AT HOLE

Specimen		UG-1	VB-1	VB-2	VB-3
Hole	2a in.	20.75	17.91	17.50	15.00
	2H in.	10.37	8.97	11.03	5.00
	M/V in.	31.50	30.00	25.00	23.50
	$2a/h_T$	4.36	3.29	7.85	2.68
	*$V_{max}/V_{p\ell}$	0.478	0.531	0.287	0.691
Reinforcement	b_1 in.	3.00	2.25	2.25	2.25
	t_1 in.	0.375	0.375	0.375	0.375
	a_d in.	5.25	4.5	4.5	5.0
	σ_{rb} ksi	44.0	37.0	37.0	37.3
**Theoretical	$V/V_{p\ell}$	.425	.467	.287	.630
	$M/M_{p\ell}$	.500	.522	.310	.585

* Note that $V_{max}/V_{p\ell} = (1-2H/h_w)$.

** Taken from interaction curves, Fig. 2.

Flat reinforcing bars were then welded flush with the horizontal edges of the hole, with one fillet weld over the length of the hole on the flange side of the reinforcing bar. Anchorage was provided by fillet welds placed on both sides of the bars. Average dimensions and properties at the holes are listed in Table 2. Cover plates and stiffeners were provided if necessary.

Web imperfections were measured again and compared with the initial ones to get an indication of the effect of heat in cutting the hole and welding the reinforcement. It was found that resultant web imperfections were again within the acceptable limits [5].

4. METHOD AND MEANS OF MEASUREMENT

The most meaningful measurements during the tests are the lateral deflections of the web and vertical deflections of the beam at the ends of the hole. Lateral deflections of the web in the vicinity of the hole were measured by a dial gauge instrument along vertical lines marked on the web. For better monitoring of the lateral movements, these lines were spaced closely in the critical area near the edge of the hole. The dial gauge instrument consisted of five dial gauges attached to an aluminium angle and spaced at intervals appropriate to reveal the buckling configuration. For measurement, the reference plane is established by simultaneous contact with the web of three pins protruding from the instrument; two near the bottom flange and one near the top flange. Thus the measurements were recorded with reference to the plane formed by the web junctions at the top and bottom flanges without considering rigid body displacement and rotation of the web. Such rigid body displacements, even if they occur, are not relevant for measuring the buckling of the web.

Vertical deflections of the beam were recorded by dial gauges in contact with the bottom flange and located at the ends of the hole and near the loading point. Fig. 1 shows a typical arrangement of the dial gauges used. Dial gauges were also provided to detect any support movement. All dial gauges employed could be read with an accuracy of ± .001".

For beams VB-2 and VB-3, strain gauges were employed to measure web strains due to lateral and in-plane deformations. As discussed in [1], the reinforcement at the upper corner of the hole at the low moment end is subjected to severe compression. This area was therefore anticipated (and proved) to be the critical area, with large lateral movements of the web in the case of buckling. Accordingly, strain gauges measuring strain in the vertical direction were affixed in this area; one on each side of the web. Divergence of the readings of two strain gauges at one location indicates the degree of lateral bending at that location.

5. TEST SET-UP AND APPLICATION OF LOAD

All the test beams were set up as simply supported at their ends under a 440 kip Baldwin-Tate-Emery hydraulic testing machine. Bearing was on rollers, and 1" thick bearing blocks were used to distribute the load. Lateral bracing of compression flanges at their ends and at the beam centre was provided when necessary.

Loads were applied to the test beams initially in increments of 10 kip or 20 kip. Dial gauge readings and strain gauge readings (if any) were recorded at each increment, while lateral deflections of the web were recorded at alternate increments. As soon as the load deflection curve revealed inelastic behaviour, the load increments were reduced to 5 kip or 2.5 kip and all deflections including the lateral deflections of the web were recorded at such increments. If the deflections continued to increase at constant load (i.e. yielding), sufficient time was allowed for readings to stabilize before they were recorded.

The tests were stopped on visible signs of buckling of the web (and drop in the load) if it occurred. Otherwise, they were continued until it was determined that yield or strain hardening was well developed without buckling. Readings were also taken upon unloading of the test beams.

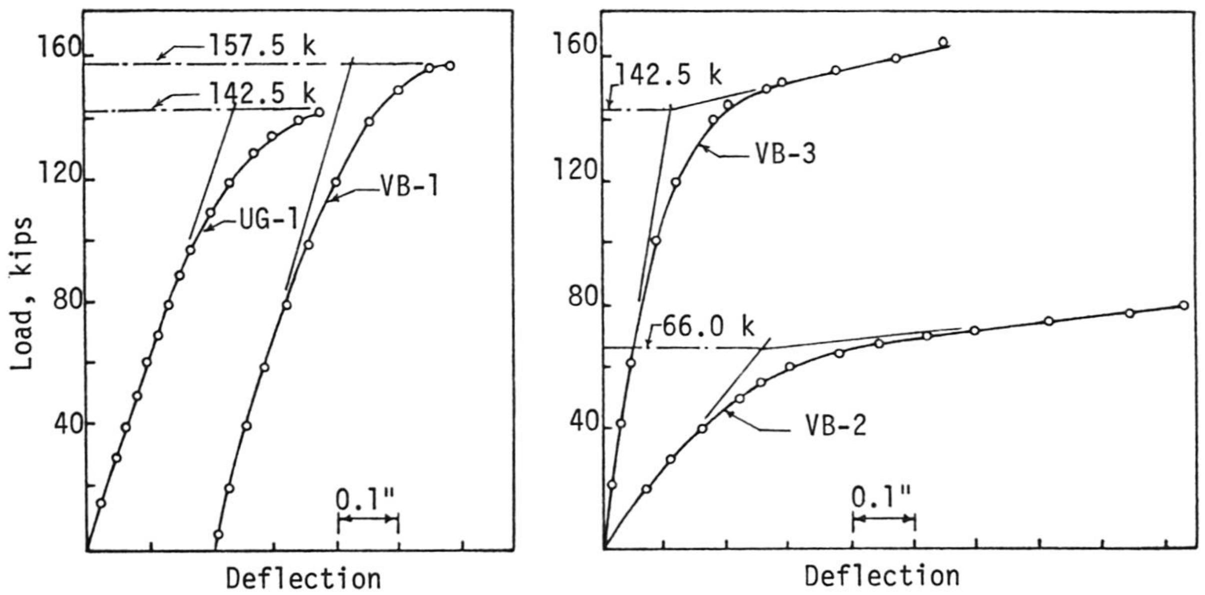

Fig. 3. Relative vertical deflection between hole ends versus load.

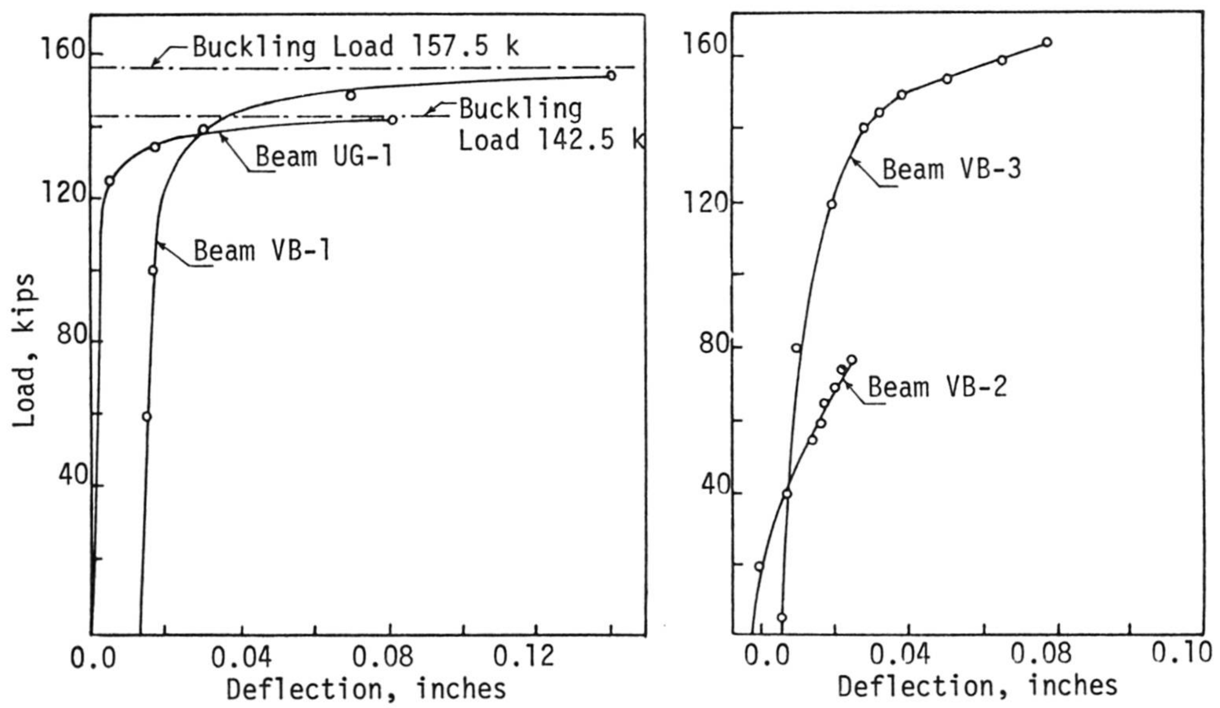

Fig. 4. Lateral deflection of webs at points of maximum movement.

Coupons were cut from each test beam after the completion of the test. At least one coupon each from the bottom flange, the top flange and the web area (removed for the hole) was prepared. Average static yield stress values were obtained for computing the theoretical capacity of the beams.

6. TEST RESULTS

Beam UG-1: For this beam, the web slendernesses were $\bar{h}$ = 3.43 and $\bar{H}$ = 1.79. The load deflection behaviour is exhibited in Figs. 3 and 4. One important observation is that the beam failed by buckling at the hole without undergoing large plastic deformations. Large distortions of the web and flange, seen in Fig. 5, are somewhat misleading because they are mainly due to forced loading of the beam after buckling had become apparent.

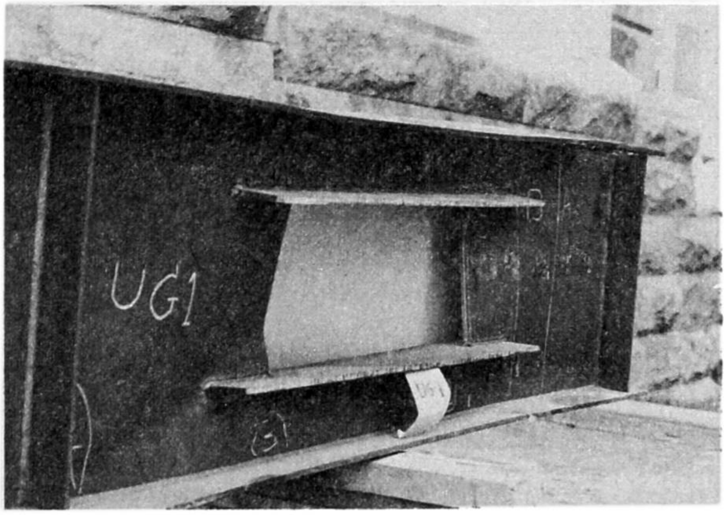

Fig. 5. Beam UG-1

Beam VB-1: The sectional properties were similar to UG-1, but a shallower hole was chosen for this test, see Table 1. It was suspected that UG-1 had failed in buckling probably because of the high slenderness $\bar{H}$ of the web between the reinforcing plates. The hole depth was therefore reduced from 10.375" to 9" in this case, bringing $\bar{H}$ down from 1.79 to 1.57. On the other hand, this test was designed more severely than UG-1 with respect to other considerations. Since the hole was shallower, the shear capacity of the beam was greater and, at the same time, the reinforcement area was also reduced.

The load vs. deflection curves for this test shown in Figs. 3 and 4, provided essentially the same conclusion as that for UG-1; the beam buckled rather suddenly before substantial plastic deformation could occur. A contour plot, Fig. 6, shows the extent and the manner of buckling as disclosed by dial gauge measurements. It should be noted that the beams UG-1 and VB-1 had web slendernesses which exceeded the limit for Class 2 sections.

Beams VB-2 and VB-3: As a result of the experience with UG-1 and VB-1, it became clear that the most significant parameter in the test may not have been $\bar{H}$ but rather $\bar{h}$, the slenderness of the full, unperforated web. To verify this conclusion, test specimens VB-2 and VB-3 with reduced slenderness, $\bar{h}$ equal to 2.39 and 2.28 respectively, were selected. The webs of these beams are stockier and were intended to correspond to Class 1 section limits; such unperforated webs are capable of large plastic deformations without premature buckling.

The two tests corresponded to two extreme cases. For VB-2, a hole size 11" deep and 18" long was chosen, resulting in an $\bar{H}$ of 1.70. On the other hand, a shallow hole, 5" deep and 15" long, resulted in a low $\bar{H}$ equal to 0.70 for VB-3. Thus, while more than 2/3 ($2H = 0.71h_w$) of the web was removed from beam VB-2, less than 1/3 ($2H = 0.31h_w$) was removed from VB-3 at the hole locations. The reinforcement provided was the same in both cases. In the case of VB-2, as can be seen from the interaction curve, Fig. 2, this reinforcement was just sufficient to develop the maximum shear capacity, which is indicated by the vertical part of the diagram.

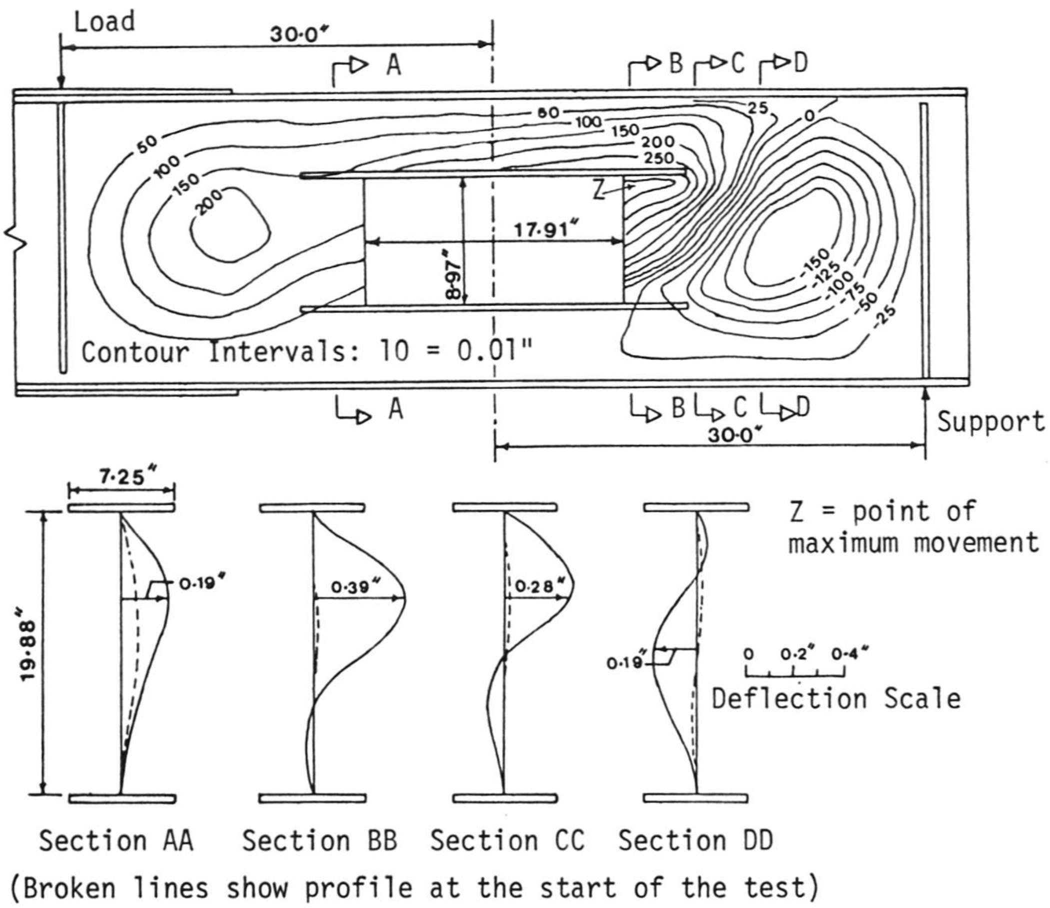

Fig. 6. Buckled web shapes and contours; beam VB-1

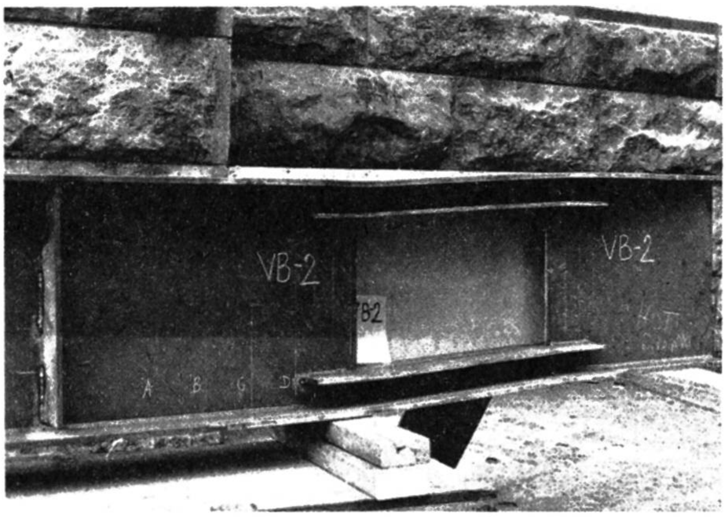

Fig. 7. Beam VB-2

Figs. 3 and 4 show the load deflection behaviour for VB-2. In contrast to UG-1 and VB-1, this beam underwent large deformations well into the strain hardening range. It is remarkable that there were no signs of appreciable lateral movement even at the maximum load despite the fact that no stiffeners at reaction points were provided, see Fig. 7. This observation is also borne out by the load vs. strain curves for the strain gauges, Fig. 8.

As the loading progressed, large relative deflections became evident between the hole ends, confirming the hinge mechanism expected at such loads. With continued loading in the strain hardening range (as indicated by strain gauge readings), the compression flange at the high moment end developed a noticeable kink (local buckling) over the length of the anchorage, starting from the vertical edge of the hole, (see Fig. 7). However, this occurence did not change the gradual nature of the beam deformations and the beam was still yielding without further buckling apparent at the time the test was stopped.

Considering the amount of shear carried and the area of reinforcement provided, the beam VB-3 represented perhaps the most severe of the series of tests. However, in spite of this severity, the beam behaved in a ductile fashion, much like VB-2. This is clearly seen in the load-deflection curve, Fig. 3, and the load-strain curves of the strain gauges, Fig. 8.

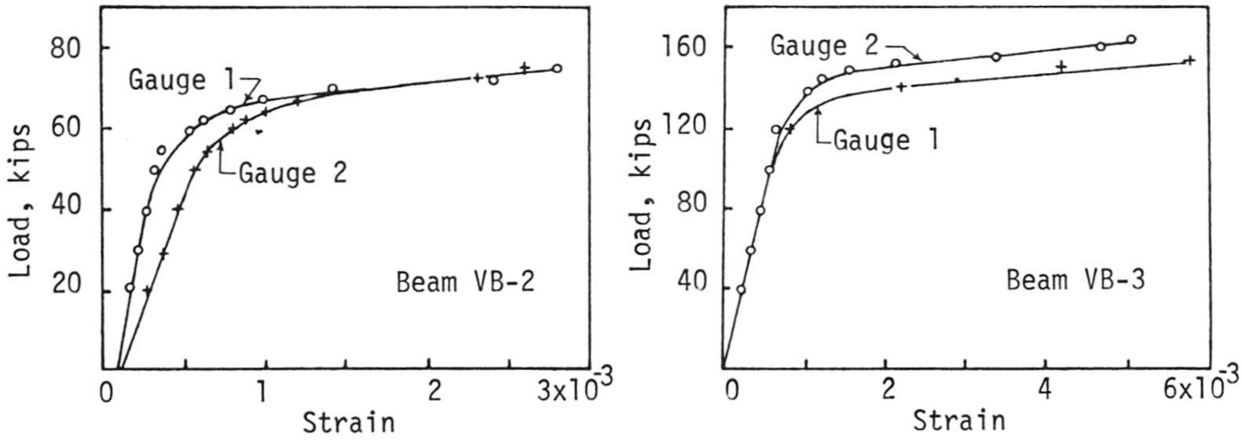

Fig. 8. Variation of compressive strain with load.

Unlike VB-2 however, the lateral movements of the web in VB-3 were progressing appreciably and reached a magnitude of 1/10 in. at the final load applied. The conclusion from the plot of web deflection versus load, Fig. 4, is that the web was approaching its buckling limit when the load was removed. The contour plot of lateral deflections, Fig. 9, shows the manner of impending buckling. Had the loading been continued, the beam would have collapsed by buckling of the web.

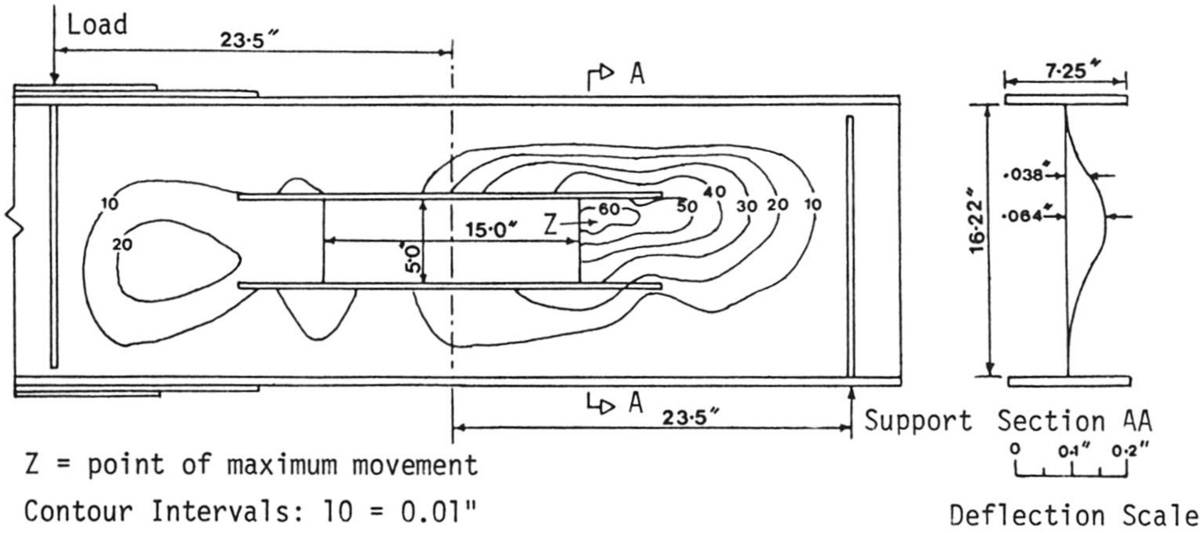

Fig. 9. Web deflection contours at 160 kips; beam VB-3.

However, since the beam had already deformed well into the plastic range, and the load had exceeded the theoretically calculated yield load by 12%, it is reasonable to conclude that the beam (when it was approaching buckling) had already exceeded its ultimate strength defined as its plastic capacity. Like VB-2, this beam also developed local buckling of the compression flange at the high moment end, giving further credence to the conclusion that buckling of the web did not occur until after considerable strain hardening had taken place.

7. DISCUSSION OF THE TEST RESULTS AND CONCLUSIONS

A summary of the test results is given in Table 3. The experimental values of test loads, $Y_{expt.}$, are obtained from the relative deflection load graphs of Fig. 3. It should be noted here that whereas in the cases of tests VB-2 and VB-3 $Y_{expt.}$ is defined by the intersection of lines representing the elastic

TABLE 3

RELEVANT PARAMETERS AND TEST RESULTS

Test	M/Vh	$\bar{H}$	$\bar{h}$	$*V/V_{p\ell}$	$Y_{theo.}$ (=2V) kips	$**Y_{expt.}$ kips	$\frac{Y_{expt.}}{Y_{theo.}}$	Collapse-mode
UG-1	1.52	1.79	3.43	0.425	143.0	142.5	1.00	Buckling of web.
VB-1	1.45	1.57	3.48	0.467	159.8	157.5	0.99	Buckling of web.
VB-2	1.53	1.70	2.39	0.287	66.8	66.5 (80.0)	1.00 (1.20)	Yielding; no sign of buckling.
VB-3	1.38	0.70	2.28	0.630	147.0	142.5 (165.0)	0.97 (1.12)	Yielding and finally approaching buckling.

* $V/V_{p\ell}$ values are taken from the interaction curves according to [1], see Fig. 2. For $M/M_{p\ell}$ values, see Table 1.

** $Y_{expt.}$ is equal to the ultimate load in the cases of buckling only, i.e. for UG-1 and VB-1. For tests VB-2 and VB-3, in which failure occurred by continual yielding, $Y_{expt.}$ is taken at the intersection of lines representing elastic and strain hardening slopes of the relative deflection at hole ends versus load curve, see Fig. 3. Loads at which these tests were stopped are given above in parentheses.

and strain hardening slopes, it is taken simply as the buckling load in the cases of beams UG-1 and VB-1. This procedure is consistent with the fact that the test beams exhibited the same sort of load deflection behaviour as would be expected from their unperforated sections. By definition, Class 1 sections are so proportioned as to be capable of realizing their plastic strength and yet undergo large plastic deformation before collapse. On the other hand, the deformation requirements for Class 2 sections are not so severe; it is sufficient that they are capable of only reaching their plastic strength. The beams UG-1 and VB-1 which had a slenderness ratio $\bar{h} \simeq 3.52$ more than the limit for Class 2 sections ($\bar{h} = 3.05$), failed in buckling just at the point of reaching their theoretical yield strength predicted by [1]. In contrast, the beams VB-2 and VB-3, which are Class 1 sections with $\bar{h} \simeq 2.35$ underwent large plastic deformation without buckling after attaining the predicted loads. In other words, despite the presence of holes, albeit reinforced, the beam sections did not change their characteristic behaviour. The parameter $\bar{h}$ therefore remains an indicator of their deformation characteristic.

The next observation which can be made concerns the effect of the depth of the hole on the stability of the web. For tests on beams VB-2 and VB-3, two extreme hole depths were chosen, these being 71% and 31% respectively of the total web depth. The fact that these two beams behaved in essentially the same fashion (as shown by their load deflection behaviour) leads to the conclusion that in an adequately reinforced hole, the depth of the hole is not a critical factor. This conclusion is supported also by behaviour of the beams UG-1 and VB-1, both of which behaved in a similar fashion, although the variation in $2H/h_w$ for these two beams was not as large.

It seems however that, although the beams were able to develop their shear capacity close to the maximum (based on the remaining web), the susceptibility to buckling does increase with shear at the hole. The beam VB-3, which was loaded to a high shear of more than 0.63 $V_{p\ell}$, did tend to buckle in the final stages of the test. Also the contour plot, Fig. 6, for the beam VB-1 indicates a shape typical of shear type buckling. This beam was loaded with 0.47 $V_{p\ell}$. These observations point out that what is critical is not the depth of the hole or the parameter H per se but the amount of shear transferred at the hole. Therefore, what should be limited is not the depth of the hole but rather the amount of shear. Based on these tests, upper limits of 2/3 $V_{p\ell}$ for Class 1 sections and 1/2 $V_{p\ell}$ for Class 2 sections are suggested. These limits are liberal enough to cover most practical situations.

Since the experiments were conducted with an M/Vh ratio of about 1.5, the moment at the centreline of the hole in no case was more than 0.58 $M_{p\ell}$ (beam VB-3). This means that situations where moments of greater magnitude occur in conjunction with high shear force ($V>V_{p\ell}/2$) are not covered by the experimental data. However, as pointed out earlier, such situations are unusual, occurring mainly near the interior supports of heavily loaded continuous beams. For usual beam sections, this situation can be avoided by requiring the hole centreline to be at least 3 times the depth of the beam away from an interior support.

Moreover, it can be shown that a condition of high moment and high shear at the hole requires such large reinforcement as to be impractical. Thus, if an upper limit on the calculated amount of reinforcement were imposed, it would automatically exclude such considerations by limiting the amount of moment which can be transferred under high shear conditions.

The amount of reinforcement which will limit the developable moment at the high moment edge of the hole to the capacity of the flanges and at the same time develop the full shear capacity V_{max} at the hole can be shown to be $A_r = 2at_w/\sqrt{3}$. For holes as long as the depth of the beam, i.e. with $2a/h \approx 1$, this limit works out to be $A_r \leq A_w/\sqrt{3}$. In view of the fact that the areas of reinforcement provided in the tests were of the order of $A_w/3$ and these were sufficient to develop moment as high as 0.58 $M_{p\ell}$, an upper limit of $A_r \leq A_w/2$ is considered to be not too restrictive and is accordingly suggested.

The above limit on A_r has the additional advantage of restricting the length of the hole, as longer holes generally require more reinforcement. Very long holes are not desirable as they may contribute to the lateral instability of the beam as a whole or cause excessive deflections over the hole length.

In the case of high moment and small or no shear, the reinforcement may be very small or may not even be necessary. In such cases, the web of the section above the hole is under high compression and concern should be to prevent its instability. If it is determined that reinforcement is required, even the smallest amount will greatly improve the stability of the web. The case of unreinforced webs has been considered elsewhere.

It is concluded that the anchor lengths performed well in the above tests without any visible signs of distress in the web or in the weld. The extent to which the stability of the web is affected by the anchor length is not fully understood, but from the results of these and other tests, a length provided on the basis of development of the strength of the reinforcement is considered adequate for the type of sections tested. However, a minimum anchor length of 3" is deemed necessary in any case.

8. SPECIFIC RECOMMENDATIONS

The following set of recommendations summarizes the conclusions reached in the above discussion of the test results.

The strength method of [1] is adequate in predicting the strength of symmetrically reinforced holes in beams of Class 1 sections ($\bar{h} \leq 2.47$) and Class 2 sections ($\bar{h} \leq 3.05$). Such a procedure can be used in designing the reinforcement for these sections under the following restrictions:

(1) The total shear at the hole should not exceed 2/3 $V_{p\ell}$ for Class 1 sections and 1/2 $V_{p\ell}$ for Class 2 sections.
(2) The amount of calculated reinforcement A_r should not exceed $A_w/2$ or A_f, whichever is less.
(3) The anchor length should be at least equal to that required for development of the strength of the reinforcement, but in no case less than 3 inch.

With the above restrictions, the reinforcement and its anchor length will provide sufficient stiffness to prevent premature buckling and maintain deformation characteristics of the section. No vertical stiffeners at the hole need be provided in such cases.

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NOTATIONS

A	Area
A_f	Area of one flange = b x t
A_r	Area of reinforcement = $2b_1 \times t_1$
A_w	Area of web = $h_w \times t_w$
a	Half the length of the hole
a_d	Development length of reinforcement
b	Width of flange
b_1	Width of one reinforcing bar
E	Modulus of elasticity for steel = 29000 ksi
H	Half the depth of the hole
$\bar{H}$	Slenderness related to the hole depth = $(2H/t_w)\sqrt{\sigma_{rw}/E}$
h	Depth of beam
h_T	Half the depth of the web remaining after the hole

h_w Total depth of web
$\bar{h}$ Slenderness related to the total depth = $(h_w/t_w)\sqrt{\sigma_{rw}/E}$
ℓ Span
M Applied moment at the centreline of the hole
$M_{p\ell}$ Plastic moment capacity of the full section
t Thickness of flange
t_w Thickness of web
t_1 Thickness of reinforcing bar
V Applied shear at the centreline of the hole
V_{max} Max. shear capacity at the hole = $V_{p\ell}(1-2H/h_w)$
$V_{p\ell}$ Plastic shear capacity of the full section
Y Applied load
$Y_{expt.}$ Load determined from load deflection curve
$Y_{theo.}$ Load predicated by strength procedure of [1]
σ_r Static yield strength
σ_{rb} Average static yield strength of the reinforcement
σ_{rf} Average static yield strength of the flanges
σ_{rw} Average static yield strength of the web

Note that, according to [2], the web slenderness is defined in the form $\bar{h}_1 = (h_w/t_w)\sqrt{\sigma_{rw}}$. Therefore, the slenderness limits $\bar{h}_1 \leq 420$ and $\bar{h}_1 \leq 520$, specified respectively for Class 1 and Class 2 sections in [2], correspond to $\bar{h} \leq 2.47$ and $\bar{h} \leq 3.05$ of the present non-dimensional notation.

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