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Large Deflection Effects in Elasto-Plastic Frame Analysis

Grandes déformations des cadres élasto-plastiques

Große Ausbiegungen elasto-plastischer Rahmen

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Introduction

The adoption of high yield stress steels in design of portal frames and multistorey buildings, leads to an economic solution. The higher working stresses permitted with the new steels, however, lead to larger deflections in service, and it is important to examine the practical implications of this in design.

Two separate lines of investigation have recently evolved. The first was based on the study of large deflections leading to a change of initial geometry and its effect on the distribution of forces and deflections throughout the structure [1]. The general conclusion arrived at was that within elastic behaviour change of geometry effects do not produce large departures from the usually assumed elastic behaviour, provided instability effects are included. The second study was concerned with the elasto-plastic behaviour of frames built in high yield stress steel with strain hardening effects and instability both included [2]. This enabled the relative importance of the increase in stiffness due to strain hardening to be examined in realtion to the deterioration of stiffness due to increased instability effects.

The study described in this paper has attempted to combine the two effects in order to examine whether change of geometry in the elasto-plastic range produces any significant changes in the behaviour of structures. It is immediately apparent that this effect is likely to be more significant in the elasto-plastic range where deflections are very much larger than within the purely elastic range, and the study is therefore felt to be of considerable practical importance.

The investigation was carried out by developing fully automatic computer programs based on the stiffness approach capable of simulating the effects of instability, elasto-plastic behaviour, strain hardening and large deflections either separately or in any combination, and applying these to the analysis of several types of frames in high-yield stress steel. The relative importance of large deflections could thus be conveniently investigated.

Basic Assumptions

All the analyses apply only to plane frameworks composed of initially straight uniform members rigidly connected at nodes. The basic element of a framework is a structural member in which strain hardening hinges can form at either or both ends, a modified elastic behaviour being assumed along the lenght of the member. A strain hardening hinge is simulated by a concentrated elastic spring developing an additional moment above the moment causing initial plasticity in the section.

The shape factor of the section is assumed to be unity so that non-elasticity is confined to strain hardening hinges. The stiffness of the hinges, however, includes the effect of spreading plastic zones in an actual member [3].

Axial strains are automatically included in the displacement analysis, and the effect of axial thrust on bending and shearing stiffnesses of individual members, that is the instability effect [4], is also taken into account. Large deflection effects are included by considering the overall displacement of each member from its unloaded position, but the "bowing" effect [5], the difference between actual member length and its chord, is assumed to be small in building frames. It is thus felt that the method includes all the important effects governing the behaviour of steel frames.

Theory of the Method

A longhand analysis of any but the simplest frame is unthinkable, and it was felt that comprehensive computer programs based on the stiffness approach would provide a most powerful tool for the investigation. The stiffness approach has already been used very successfully for instability [4] and elasto-plastic behaviour of frames [6] and grillages [7] and has been modified to include strain hardening effects [2, 8]. These developments have been well documented, and attention will be confined only to the method of treating large deflections.

In small deflection theory equilibrium equations are formulated for the undeformed structure, thus for a structure subjected to loading $[F]$ deforms by $[D]$ where

$$[F] = [S]\,[D], \tag{1}$$

where $[S]$ is the stiffness matrix of the frame. When large deflections are considered, equilibrium must be maintained in the deformed stare, in the new geometry the frame has taken up. Therefore, in general equation (1) will not be satisfied, and for equilibrium a set of external loading $[F_R]$ at joints will have to be applied to maintain equilibrium. Thus:

$$[F] = [S][D] + [F_R], \tag{2}$$

Forces $[F_R]$ can therefore be calculated as the difference of internal member forces $[S][D]$ in the displaced form and the external loading.

Elasto-plastic behaviour commences when the external loading $[F]$ is of sufficient intensity to produce plasticity in some members of the frame. Strain hardening hinges are then brought into play reducing the stiffness of the structure to $[S_p]$ and the external loading by the plastic moments and forces $[F_p]$. The equilibrium conditions can be stated as:

$$[F] = [S_p][D]+[F_p]+[F_R]. \tag{3}$$

The two effects can be combined, and the equation rewritten more conveniently as:

$$[F]-[F_{pR}] = [S_p][D], \tag{4}$$

where $[F_{pR}]$ represents the total reduction of external loading.

Equations for a Single Member

Equations governing the elasto-plastic strain hardening analysis of a typical member of a frame work have been derived fully in the authors' previous publication [2] and only a summary of the results is included here.

A member 1—2 is shown in figure 1 in its undeformed and deformed states. When large deflections are considered, the new length of the member will be defined by the chord length L_c, the member being inclined at an angle α_1 to the x-axis. Since the deformation of the member from its chord is assumed small, the axial force P is assumed to act along the chord, and shearing force S normal to it. Rotations of the two ends of the member relative to its initial position are θ_1 and θ_2.

When plasticity develops at either end of a member, a plastic strain hardening hinge is assumed to form allowing some relative rotation to develop. Considering a hinge at end 1 of the member, the rotation of the elastic portion adjacent to the hinge becomes $\bar{\theta}_1$. Thus the relative hinge rotation is $\theta_1^1 = \theta_1 - \bar{\theta}_1$ and the moment can be expressed by

$$M_{12} = \beta_1 M_p + \frac{E\,I}{k_1 h_1}(\theta_1 - \bar{\theta}_1), \tag{5}$$

where $\beta_1 = \pm 1$ depending on the direction of rotation of the plastic hinge, k_1 and h_1 are the strain hardening factor and equivalent cantilever length respectively for end 1 of the member.

Parameters h_1 and k_1 can be determined directly from the bending moment distribution in the vicinity of the hinge as follows:

$$h_1 = \frac{M_{12} L_c}{M_{12} + M_{21}}, \tag{5a}$$

$$k_1 = 30.345 - 22.066 \frac{M_p}{M_{12}}. \tag{5b}$$

The numerical value of k_1 applies to high yield stress steel to BS 968: 1962, and has been obtained from several tests on steel beams and specimens [3]. The same procedure can be adopted for other steels.

With reference to figure 1, member forces are given by:

$$\left.\begin{aligned}
P_1 &= -P_2 = \frac{EA}{L_0}(q_1 - q_2),\\
M_{12} &= [s\bar{\theta}_1 + sc\,\theta_2 + s(1+c)\,\phi]\frac{EI}{L_c},\\
M_{21} &= [sc\,\bar{\theta}_1 + s\,\theta_2 + s(1+c)\,\phi]\frac{EI}{L_c},\\
S_1 &= -S_2 = \frac{M_{12} + M_{21}}{L_c}.
\end{aligned}\right\} \tag{6}$$

Using the Livesley stability functions [4] the above equations become:

$$\left.\begin{aligned}
M_{12} &= \frac{4EI}{L_c}\phi_3\bar{\theta}_1 + \frac{2EI}{L_c}\phi_4\theta_2 + \frac{6EI}{L_c^2}\phi_2(p_1 - p_2),\\
M_{21} &= \frac{2EI}{L_c}\phi_4\bar{\theta}_1 + \frac{4EI}{L_c}\phi_3\theta_2 + \frac{6EI}{L_c^2}\phi_2(p_1 - p_2),\\
S_1 &= -S_2 = \frac{6EI}{L_c^2}\phi_2\bar{\theta}_1 + \frac{6EI}{L_c^2}\phi_2\theta_2 + \frac{12EI}{L_c^3}\phi_2(p_1 - p_2)
\end{aligned}\right\} \tag{7}$$

respectively. It is important to note that because the axial force acts along the chord of the deformed member and the shear force normal to the chord, there is no $P_1(p_2 - p_1)$ term used in formulating the shear force equation.

Equations (7) define the behaviour of a strain hardening member with instability effects included but are *not* in equilibrium as they have to be modified for the change of geometry. Compatibility conditions will be satisfied by correcting the projected lengths of each member to correspond with the displaced position of the frame. Thus for member 1—2 with deflection components $(x_1 y_1)$, $(x_2 y_2)$ at ends 1 and 2 respectively with respect to x and y axes, the corrected projected lengths are:

$$\begin{aligned}\bar{x} \text{ corrected} &= \bar{x} \text{ initial} - (x_1 - x_2), \\ \bar{y} \text{ corrected} &= \bar{y} \text{ initial} - (y_1 - y_2),\end{aligned} \tag{8}$$

as shown in figure 1 and the corrected chord length becomes

$$L_c = \sqrt{(\bar{x}^2 \text{ corrected} + \bar{y}^2 \text{ corrected})}.$$

In formulating equation (6) it was assumed that the change of length of the member due to axial forces was $(q_1 - q_2)$ while the actual change of length is given by

$$\epsilon = L_0 - L_c. \tag{9}$$

Thus the error which results in using a first order expression is given by $\epsilon_R = \epsilon - (q_1 - q_2)$.

But

$$q_1 - q_2 = (x_1 - x_2)\cos\alpha_1 + (y_1 - y_2)\sin\alpha_1 \tag{10}$$

and from equations (8)

$$L_c \cos\alpha_1 = L_0 \cos\alpha_0 - (x_1 - x_2)$$

and

$$L_c \sin\alpha_1 = L_0 \sin\alpha_0 - (y_1 - y_2).$$

By squaring and adding the above equations and using equation (9)

$$L_0^2 = (L_c + \epsilon)^2 = [L_c \cos\alpha_1 + (x_1 - x_2)]^2 + [L_c \sin\alpha + (y_1 - y_2)]^2,$$

that is

$$\epsilon = \left[\frac{(x_1 - x_2)^2 + (y_1 - y)^2 - \epsilon^2}{2\,L_c}\right] + \cos\alpha_1 (x_1 - x_2) + \sin\alpha_1 (y_1 - y_2)$$

and subtracting equation (10)

$$\epsilon_R = \frac{(x_1 - x_2)^2 + (y_1 - y_2)^2 - \epsilon^2}{2\,L_c}. \tag{11}$$

Hence the axial displacement error, ϵ_R, in a member can be calculated directly from its end deflections and its initial and chord lengths.

Similarly in formulating the slope deflection equations (7), it was assumed that the member sway displacement, ϕ, was given by $p_1 - p_2/L_c$, whereas the actual sway displacement is given by

$$\phi = \alpha_0 - \alpha_1, \tag{12}$$

where α_0 is the initial angle of inclination of the member to the xy axes, and α_1 is the corresponding angle in the deformed geometry as defined in fig. 1. Hence the error, ϕ_R, in the first order approximation to the member sway displacement is

$$\phi_R = \phi - \frac{p_1 - p_2}{L_c}.$$

But

$$p_1 - p_2 = (y_1 - y_2)\cos\alpha_1 - (x_1 - x_2)\sin\alpha_1,$$

therefore

$$\phi_R = \phi - 1/L_c[(y_1 - y_2)\cos\alpha_1 - (x_1 - x_2)\sin\alpha_1]. \tag{13}$$

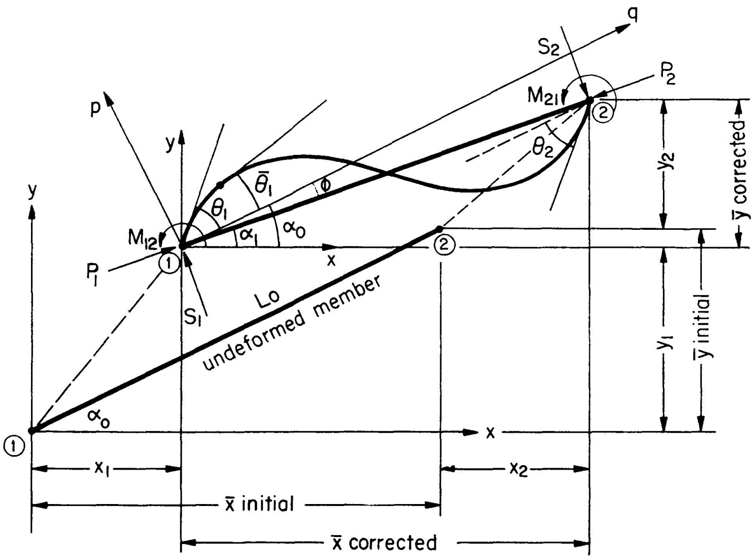

Fig. 1. The deformed member.

From the preceding equations the appropriate restraint forces for the member can be calculated to correct the effects of the displacement errors ϵ_R and ϕ_R and to hold the member in equilibrium.

Thus equations (7) become

$$\begin{aligned}
P_1 &= -P_2 = \frac{EA}{L_0}(q_1-q_2)+\frac{EA}{L_0}\epsilon_R,\\
M_{12} &= \frac{4EI}{L_c}\phi_3\bar{\theta}_1+\frac{2EI}{L_c}\phi_4\theta_2+\frac{6EI}{L_c^2}\phi_2(p_1-p_2)+\frac{6EI}{L_c}\phi_2\phi_R,\\
M_{21} &= \frac{2EI}{L_c}\phi_4\bar{\theta}_1+\frac{4EI}{L_c}\phi_3\theta_2+\frac{6EI}{L_c^2}\phi_2(p_1-p_2)+\frac{6EI}{L_c}\phi_2\phi_R,\\
S_1 &= -S_2 = \frac{6EI}{L_c^2}\phi_2\bar{\theta}_1+\frac{6EI}{L_c^2}\phi_2\theta_2+\frac{12EI}{L_c^3}\phi_2(p_1-p_2)+\frac{12EI}{L_c^2}\phi_2\phi_R.
\end{aligned} \tag{14}$$

By eliminating $\bar{\theta}_1$ from equations (14) using equation (5) and adding contributions from all members meeting at the node the equilibrium equation for the node is obtained. This contains the external loading, forces due to the imposed strain hardening moments and forces on the node, and the external restraints required to hold it in equilibrium.

Organisation of Computer Program

The computer program to carry out the non linear plane frame analysis was written in Algol 60 and run on the English Electric Leo Marconi KDF9 machine.

The required input data is kept to a minimum, and consists of the geometry of the frame, member section properties and the ratio of loads acting on the frame. This ratio is kept constant and loads are automatically increased at every stage of the solution. The analysis is completely automatic and requires no human intervention, the results consisting of a set of displacements and distribution of forces and moments at the formation of every successive strain hardening hinge until collapse.

Due to the non linear response of the structure within any load range, the analysis involves an iterative prediction procedure in determining the load and position at which each strain hardening hinge will form, together with the displacements and member forces in the structure at that load. The stability functions, revised frame geometry, and values of h/L_c and k are calculated from the member axial forces, joint displacements and end moments respectively, predicted to be the condition of the structure when a particular strain hardening hinge will form. These are then used to determine the elements of the matrices $[S_p]$ and $[F_{pR}]$ for the next iteration. The procedure continues until the actual and predicted values have converged to within certain tolerances, showing that the equilibrium position of the deformed structure in its strain hardening state has been found.

The program utilizes the banded nature of the matrix $[S_p]$ for an economic solution. It stores only one half of the band width of non-zero coefficients, the locations of the strain hardening hinges being kept in a separate array within the computer store.

Once matrices $[S_p]$, $[F_{pR}]$ and $[F]$ are established, equations (4) are solved using the Square Root Method to give joint displacements, from which, using the displacement errors, stability functions and values of h and k, predicted from the previous iteration, member forces are calculated.

Using a linear prediction procedure, the load, location, displacements and member forces at which the next strain hardening hinge will form are estimated. From the predicted axial forces, values of P/P_E are obtained and from the predicted bending moment distribution values of h/L_c and k for each strain hardening hinge already present in the structure are calculated using equations (5a) and (5b) respectively. Then using equations (8) corrected projected lengths of each member are calculated from the predicted joint displacements, from which the revised frame geometry is determined. Predicted values for the true axial displacement ϵ, the sway displacement φ, and the displacement errors, ϵ_R and φ_R are then calculated using the predicted values of L_c, α_1 and the joint displacements in equations (9), (12), (11) and (13) respectively. Finally revised stability functions are calculated from the predicted P/P_E values.

These quantities are then used for setting up equations (4) for the next iteration and so the process continues until the predicted and actual quantities satisfy a series of fine tolerances [3]. When the location and condition of the structure at which the next plastic hinge is due to form are established, the results are printed out, the applied load is increased, and the iterative prediction procedure for the next hinge commences. The program terminates when the prediction procedure is incapable of locating the next strain hardening hinge, indicating either a mechanism or a reduction in the load carrying capacity of the frame.

The program contains all the major non linear effects encountered in plane frame analysis, namely the effect of strain hardening, the instability effects of axial forces, and the effect of change in geometry due to the large deflections. The program besides being economical in computer storage and time, is also very versatile, and various effects can be suppressed in the analysis. Thus:

1. The effect of strain hardening can be neglected by putting the value of the strain hardening factor k equal to zero.
2. The analysis can be reverted to a purely elastic analysis by specifying high M_p values for members.
3. The instability effects of axial forces can be suppressed if required.

The program as developed does not make any provision for the possibility of a reduction in the plastic angle of rotation thus restoring elastic behaviour with a certain permanent set in any member. This case was considered during developement of the present program, but was found to be of extremely rare occurrence and of little significance in the accuracy with which load-deformation characteristics for actual frames could be predicted. The penalty in computer effort to check all plastic hinges at every stage and to carry their deformation histories was deemed excessive in an already expensive analysis and this facility was not, therefore, incorporated.

Application to Steel Frames

The computer program was used to analyse three different plane frames shown in figs. 2, 3 and 4, in order to examine the relative important of the effect of large deflections compared with the effects of strain hardening and instability in an elasto-plastic analysis. All the frames were assumed to be fabricated from commercial sections of steel to B.S. 968: 1962 with E = 13,100 t/sq.in. and the yield stress = 28.1 t/sq.in., both values having been obtained from a series of bending and tension tests carried out on the material. In all cases six analyses were carried out, and for easy reference they and their respective failure loads are denoted as follows:

E-P, W_p	simple elasto-plastic analysis
E-P-ST, W_{f1}	elasto-plastic analysis including the effects of instability
E-P-ST-FD, W_{f2}	elasto-plastic analysis including the effects of instability and change of geometry due to large deflections
E-P-SH, W_{f3}	elasto-plastic analysis including the effect of strain hardening
E-P-SH-ST, W_{f4}	elasto-plastic analysis including the effects of strain hardening and instability

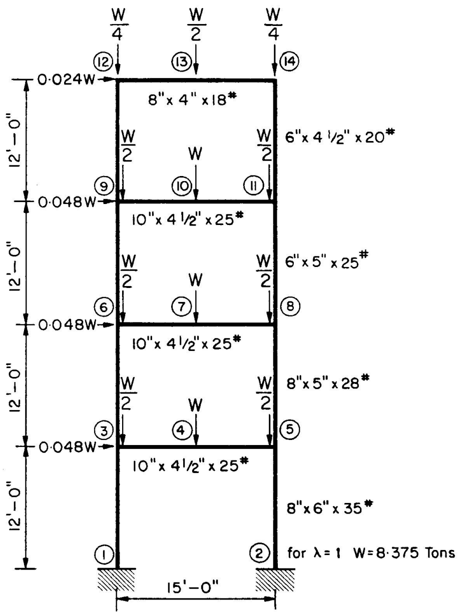

Fig. 2. Wood's four storey single bay frame.

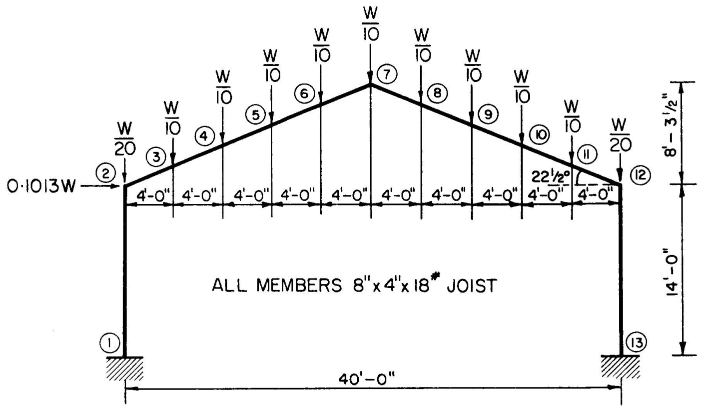

Fig. 3. Heyman's pitched roof portal frame. $\lambda = 1$, $W = 6.96$ tons.

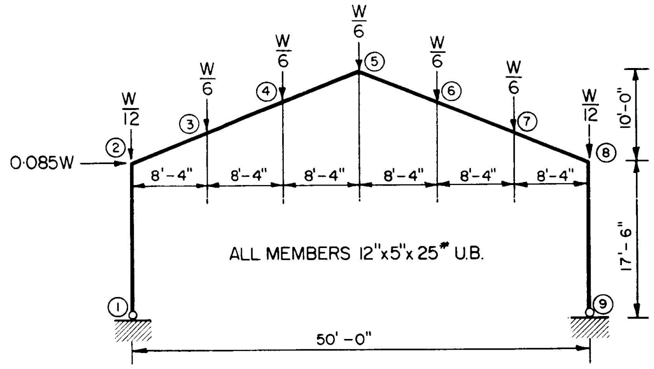

Fig. 4. B.C.S.A. Pitched roof portal frame. $\lambda = 1$, $W = 7.0$ tons.

E-P-SH-ST-FD, W_{f5} elasto-plastic analysis including the effects of strain hardening, instability and change of geometry due to large deflections, that is the analysis described in the paper.

Example 1. Four Storey Single Bay Frame

The frame was first analysed by Wood [9] in mild steel to illustrate the concept of the deteriorated elastic critical load. The results of the analyses are shown in fig 5 and 6 and a summary of the results at the failure load of each analysis is shown in Table 1.

Even within the elastic range instability is an important effect leading to much larger deflections than those given by the elasto-plastic analysis. The instability effects of axial forces reduces the carrying capacity of the structure by 18.2% below the simple plastic collapse load, and the effect of overall change in geometry on this failure load is negligible.

The effect of strain hardening on the analysis including the effects of instability and change of geometry is to increase the failure load to give a reduction in carrying capacity below the simple plastic collapse load of 13.5% and is accompanied by a large increase in deflections in the horizontal direction,

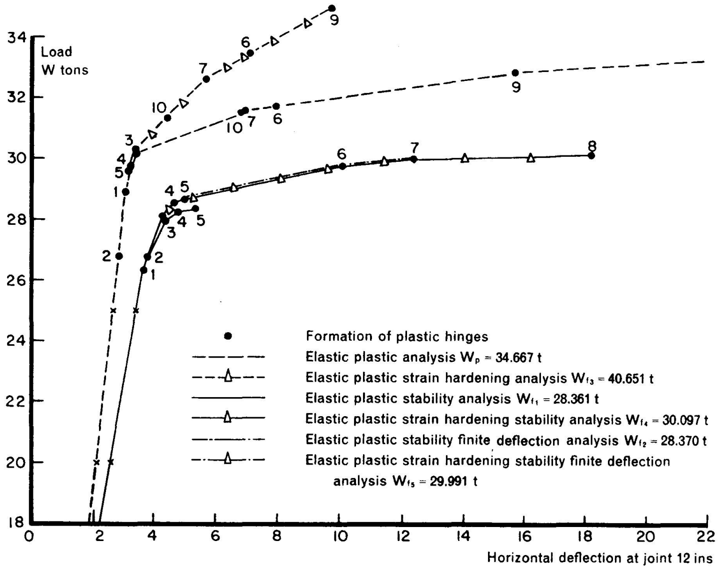

Fig. 6. Elastic plastic strain hardening load deflection curves for Wood's frame. Steel to B.S. 968: 1962.

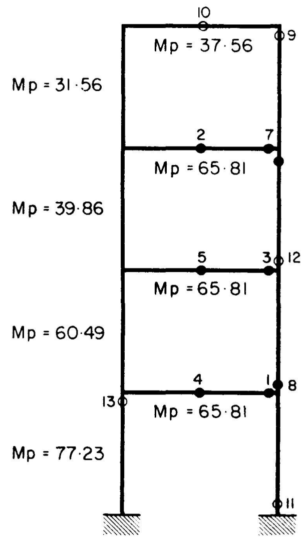

Fig. 5. Order of hinge formation for Wood's frame: Steel to B.S. 968: 1962. Fully plastic moments in t ft., $E = 13{,}100$ t per sq. inch., $fy =$ 28.1 t per sq. inch.

● Plastic hinges. o Additional hinges for simple plastic collapse.

Table 1. Four storey single bay frame

Analysis	W_f tons	No. of hinges	Joint 12 x_{max} ins	Joint 13 y_{max} ins	Percentage changes in ultimate load ($W_f = 100\%$)
E-P	34.667	13	62.278	15.737	0
E-P-ST	28.361	5	5.315	1.408	−18.2
E-P-ST-FD	28.370	5	5.312	1.434	−18.2
E-P-SH	40.651	13	24.380	5.383	+17.3
E-P-SH-ST	30.097	8	18.171	1.408	−13.2
E-P-SH-ST-FD	29.991	7	12.340	1.611	−13.5

as two more strain hardening hinges have formed. This seems to agree with conclusions previously reached [9] that strain hardening has little effect compared with the predominance of instability in the plastic analysis and design of tall buildings. The program required 42 iterations, an average of six per hinge and 5 minutes 22 seconds of computer time to complete the final analysis.

Example 2. 40 ft 0 in Span Pitched Roof Portal Frame

The frame has fixed base connections and is subjected to horizontal and vertical loading and was first used by HEYMAN [10] as a design example to illustrate a method of plastic design for pitched roof portal frames. The results

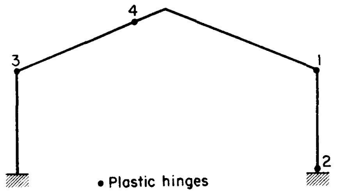

Fig. 7. Order of hinge formation for Heyman's portal. Steel to B.S. 968: 1962. All members

$Mp = 37.56$ t ft.
$E = 13{,}100$ t per sq. inch.
$fy = 28.1$ t per sq. inch.

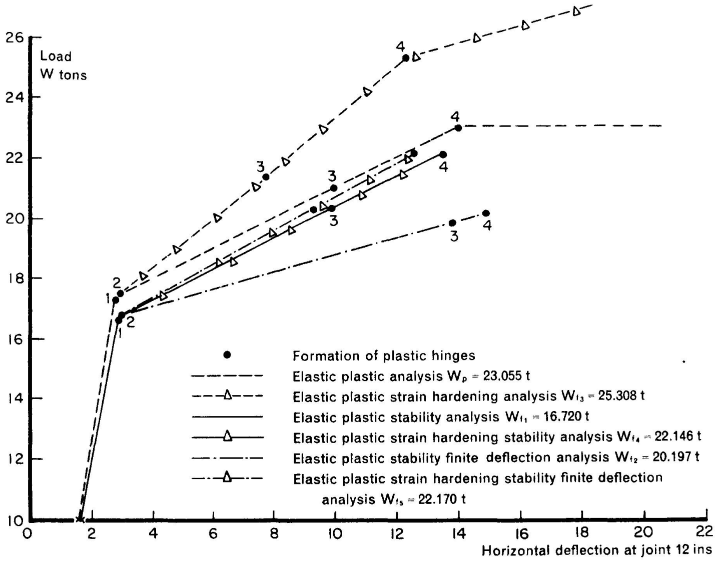

Fig. 8. Elastic plastic strain hardening load deflection curves for Heyman's portal frame. Steel to B.S. 968: 1962.

Table 2. 40 ft 0 in span pitched roof portal frame fixed base connections

Analysis	W_f tons	No. of hinges	Joint 12 x_{max} ins	Joint 7 y_{max} ins	Percentage changes in ultimate load ($W_f = 100\%$)
E-P	23.055	4	14.054	14.468	0
E-P-ST	16.720	2	2.925	4.626	−27.5
E-P-ST-FD	20.197	4	14.928	14.898	−12.4
E-P-SH	25.308	4	12.348	13.979	+ 9.8
E-P-SH-ST	22.146	4	13.571	13.799	− 3.9
E-P-SH-ST-FD	22.170	4	12.640	14.251	− 3.8

of the analyses are shown in fig. 7 and 8 and a summary of the results at the failure load of each analysis is shown in Table 2.

The instability effects of axial forces in an elasto-plastic analysis are smaller in the elastic range than in the four storey frame, but failure follows immediately after the formation of only two hinges by buckling of the right hand column, reducing the carrying capacity of the portal by 27.5% below the simple plastic collapse load. The effect of overall change in geometry on this failure load is to raise it ot 12.4% below the simple plastic collapse load after allowing two more hinges to form thus giving a mechanism and a large increase in deflections.

The effect of strain hardening on the analysis including the effects of instability and change in geometry is to increase the failure load to give a reduction in carrying capacity below the simple plastic colapse load of 3.8%. It is important to note while the effect of overall change in geometry on the elasto-plastic analysis including instability had a pronounced effect, the effect was virtually negligible on the elasto-plastic analysis including instability and strain hardening except for a small variation in deflections. The program required a total of 20 iterations and 1 minute 23 seconds of computer time.

Example 3. 50 ft 0 in Span Pitched Roof Portal Frame

The frame is shown in fig. 4, it has pinned base connections and is subjected to horizontal and vertical loading and was taken from a B.C.S.A. publication [11] where it is elastically designed in mild and high yield steel. The results of the analyses are shown in fig. 9 and 10 and a summary of the results at the failure load of each analysis is shown in Table 3.

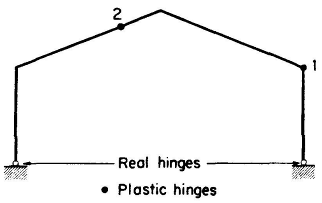

Fig. 9. Order of hinge formation for the B.C.S.A. portal frame. Steel to B.S. 968: 1962. All members

$Mp = 77.14$ t ft.
$E = 13{,}100$ t per sq. inch.
$fy = 28.1$ t per sq. inch.

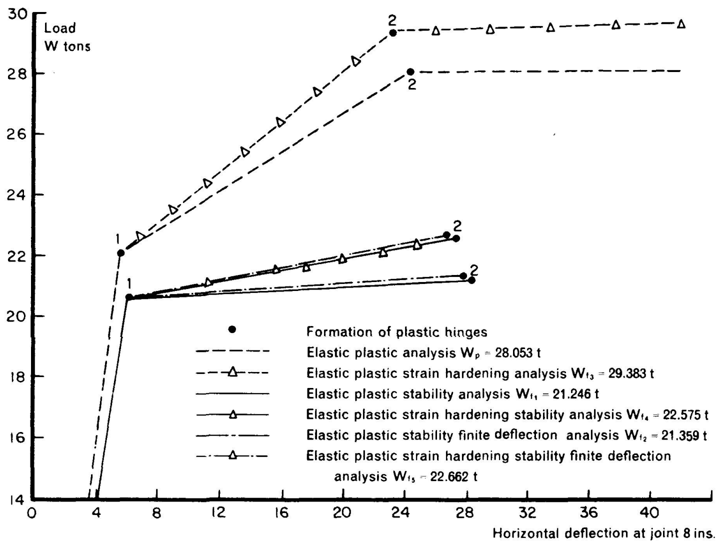

Fig. 10. Elastic plastic strain hardening load deflection curves for the B.C.S.A. frame. Steel to B.S. 968: 1962.

Table 3. 50 ft 0 in span pitched roof portal frame pinned base connections

Analysis	W_f tons	No. of hinges	Joint 8 x_{max} ins	Joint 5 y_{max} ins	Percentage changes in ultimate load ($W_f = 100\%$)
E-P	28.053	2	24.331	14.836	0
E-P-ST	21.246	2	28.383	14.108	−24.3
E-P-ST-FD	21.359	2	27.877	15.548	−23.9
E-P-SH	29.383	2	23.286	14.594	+ 4.7
E-P-SH-ST	22.575	2	27.238	13.817	−19.5
E-P-SH-ST-FD	22.662	2	26.769	15.148	−19.2

As the frame has pinned base connections, only two strain hardening hinges are required to obtain a mechanism, and in all the analyses a mechanism is obtained before local instability. The instability effects of the axial forces reduce the carrying capacity of the structure by 24.3% below the simple plastic collapse load. The effect of overall change in geometry on this failure load is to raise it slightly and is accompanied by a small reduction in the horizontal deflections and a small increase in the vertical deflections.

The effect of strain hardening on the analysis including the effects of instability and change in geometry is to increase the failure load to give a reduction in carrying capacity below the simple plastic collapse load of 19.2%. This shows that for this particular pinned base portal the effects of instability and change of geometry due to large deflections decrease the stiffness of the portal considerably and that strain hardening has only a small effect. The program required a total of 13 iterations and 35 seconds of computer time.

Conclusions

An investigation into the relative effects of strain hardening and instability due to axial forces on plane frames, together with an examination of the concept of the strain hardening hinge has been fully discussed in the authors' previous paper [2]. The purpose of this paper was to investigate the additional effect of change in geometry due to large deflections in an elasto-plastic analysis which includes the effects of instability and strain hardening.

An examination of results of the three examples summarised in Tables 1, 2 and 3, reveals that the effect of change in geometry on the collapse load of multi-storey frame and the two pitched roof portals is very small.

Thus the general conclusion can be drawn that an accurate prediction of the behaviour and collapse load of building frames can be obtained from an elasto-plastic analysis which only considers the effects of strain hardening and instability due to axial forces, and neglects the effect of change in geometry due to large deflections. Such an analysis, discussed in the authors' previous paper is far more economical in computer time and requires fewer iterations for describing the complete behaviour up to collapse of steel frames. In the exceptional case where a more accurate analysis is required this can be performed without difficulty using the program described.

Notation

A	Cross-sectional area
c	Carry over factor
$[D]$	Generalised displacement matrix
E	Young's modulus of elasticity
EI	Flexural rigidity
$[F]$	Generalised force matrix
$[F_R]$	External restraint matrix
$[F_p]$	Plastic force matrix
$[F_{pR}]$	Total reduced external loading matrix $= [F_p]+[F_R]$
h	Equivalent cantilever length

I	Second moment of area of a section
k	The strain hardening factor
L_0	Initial unstrained length of a member
L_c	Chord length of a deformed member
M	Bending moment
M_p	Fully plastic moment
P	Axial force in a member
q, p	Set of member axes
S	Shear force
$[S]$	Stiffness matrix for the entire structure
$[S_p]$	Stiffness matrix for the reduced structure
s	Stiffness factor
W	Load
W_f	Failure load
$x\,y$	Set of general axes
$\bar{x}$	Projected length of a member with respect to the x-axis
$\bar{y}$	Projected length of a member with respect to the y-axis
α_0	Angle of inclination for an undeformed member to the x-axis
α_1	Angle of inclination for a deformed member to the x-axis
β	A unit multiplying factor of ± 1
ϵ	Axial strain in a member
ϵ_R	Axial strain error
θ	Joint rotation
$\bar{\theta}$	Rotation at the end of a member
θ'	Relative hinge rotation
ϕ	Angle between initial member direction and deformed member direction
ϕ_1 to ϕ_5	The Livesley stability functions
ϕ_R	Member sway displacement error

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Summary

High working stresses permitted with high yield stress steels lead to large deflections in the elastic and post elastic states. The paper investigates the effect of large deflections on ultimate load behaviour of structures by analysing three frames using a computer program in which effects of instability, plasticity, strain hardening and large deflections can be included or suppressed. It is concluded that an elasto-plastic analysis which included strain hardening and instability effects is sufficiently accurate in predicting the behaviour of structures up to collapse.

Résumé

Des tensions admissibles élevées, ainsi que les aciers à haute limite d'élasticité provoquent de grandes déformations dans les domaines élastiques et plastiques. Cet article examine l'effet des grandes déformations sur le comportement à la ruine de trois cadres à l'aide d'un programme d'ordinateur où l'on peut prendre en considération ou négliger les influences respectives de l'instabilité, de la plasticité, de l'écrouissage et des grandes déformations. On en arrive à la conclusion qu'un calcul élasto-plastique qui tient compte de l'écrouissage et de l'instabilité est assez exact pour déterminer le comportement à la ruine des structures.

Zusammenfassung

Hohe zulässige Nutzspannungen sowie hohe Streckgrenzen des Stahles führen zu großen Ausbiegungen im elastischen oder nachelastischen Zustand. Dieser Beitrag untersucht den Einfluß der großen Ausbiegungen auf das Traglastverhalten anhand dreier Rahmen, unter Benützung eines Rechenprogramms, das die Einflüsse der Instabilität, Plastizität, Verfestigung und der großen Ausbiegungen entweder berücksichtigt oder nach Wunsch vernachlässigt. Aus den Berechnungen folgt, daß eine elasto-plastische Rechnung, welche die Verfestigung und die Instabilität berücksichtigt, genügend genau ist für die Voraussagung des Verhaltens bis zum Zusammenbruch.