Generalised approximate method of assessing the effect of deformations on failure loads

Generalised Approximate Method of Assessing the Effect of Deformations on Failure Loads

Méthodes d'approximation généralisées pour l'évaluation de l'effet des déformations sur les charges de rupture

Allgemeine Näherungsmethoden zur Bestimmung des Einflusses von Verformungen auf die Bruchlasten

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Introduction

Among the many requirements of an engineering structure in relation to strength and stability, three principal items may be distinguished.

1. The stiffness shall be adequate under normal working loads.
2. There shall exist a reserve of strength such that catastrophic collapse would not take place under some degree of overload.
3. The structure shall remain safe under any foreseeable repetition or sequence of loads.

To follow with any accuracy the behaviour of the structure under all conditions of loading is usually impracticable, particularly when local yielding is involved at points of stress concentration. It is therefore common to assess the adequacy of a structure by reference, on the one hand to idealised *load conditions*, and on the other to idealised conceptions of *structural behaviour*.

Load conditions are specified by a working load (load factor $\lambda = 1$), an *ultimate load* (equal to the working load multiplied by a load factor λ_U) and a *load frequency*, giving the load repetitions to be expected, usually at working load level, but possibly at a spectrum of load levels.

Structural behaviour may conveniently be idealised into *elastic behaviour* and *rigid-plastic behaviour*. Elastic behaviour (implying the straight stress-strain relation in Fig. 1a), is used to impose the required deflexion limits at working load level ($\lambda=1$), thus establishing the stiffness requirement. Rigid-plastic behaviour (Fig. 1b) gives the plastic mechanism load factor λ_P, which is an estimate of the failure load, and thus measures the reserve of strength against overloading. Finally, repeated loads are dealt with by using elastic theory to calculate stresses, which are then limited according to the fatigue endurance of the material.

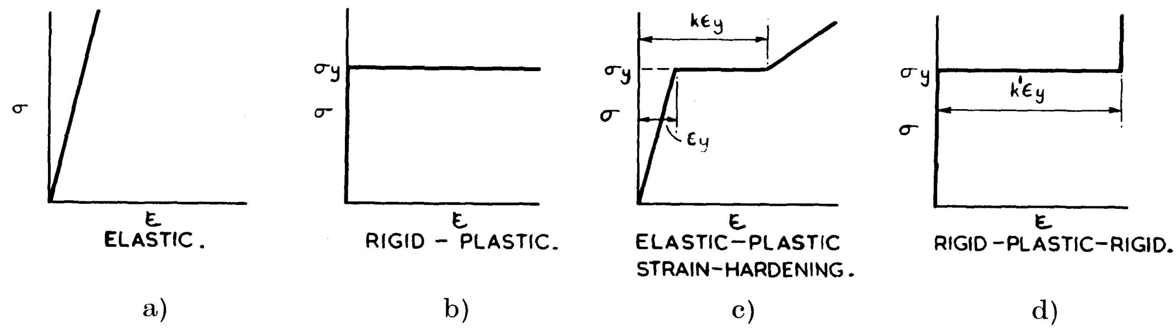


Fig. 1.

In civil engineering structures, the prime factor in design is most frequently the strength criterion, and this has led to the wide acceptance of plastic theory as a valid basis for design, despite the accompanying idealisation of structural behaviour. This has the great advantage of simplicity, since rigid-plastic analysis is easier than elastic analysis, and direct design rather than trial and error procedures may be used. There are however some dangers in using plastic theory in that phenomena associated with elastic and plastic deformations may be ignored. Apart from the question of allowable deflexions at working load level ($\lambda=1$), deformations affect the load-carrying capacity, and the failure load (defined by load factor λ_F) may fall appreciably below the rigid-plastic collapse load (load factor λ_P). To deal with this, it has been found convenient to introduce a second idealisation of structural behaviour defined by the *elastic critical load factor* λ_C. This is the theoretical level of axial loads, induced by external loads, at which the structure would, if it continued to behave elastically, become unstable. On the suggestion of MERCHANT [2], an almost invariably conservative and frequently close estimate of the failure load factor λ_F is then given by the Rankine formula

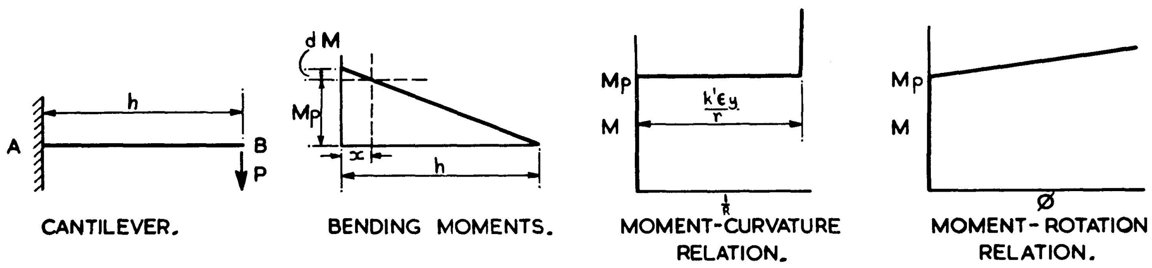
$$\frac{1}{\lambda_F}=\frac{1}{\lambda_P}+\frac{1}{\lambda_C}. \tag{1}$$

Some theoretical justification of the Rankine load has been given by HORNE [3].

Unfortunately, λ_C is appreciably more difficult to estimate than is λ_P, and the attractive simplicity of the plastic theory is in danger of being lost. Since $\lambda_F=\dfrac{\lambda_P}{1+\dfrac{\lambda_P}{\lambda_C}}$ and since for most civil structures $\lambda_C \gg \lambda_P$, only an approximate estimate of λ_C is required. It has been found that a rigid-link mechanism containing pseudo-plastic hinges may be used to derive an approximation to λ_C, this procedure being no more difficult to apply than rigid-plastic theory itself. The idea has arisen from earlier work on the effect of strain-hardening [4], which limits the plastic deformation capacity of a structure (Fig. 1c). The resulting tendency for moments of resistance at plastic hinges to increase with increasing hinge rotations is sufficient in many structures to overcome the reduction of capacity due to change of geometry, and it has been found that this may be assessed by the assumption of "rigid-plastic-rigid" behaviour (Fig. 1d). It is assumed that "strain-locking" occurs at some strain $k' \epsilon_y > k \epsilon_y$, where ϵ_y is the strain at yield and $k \epsilon_y$ is the strain at the beginning of strain-hardening (Fig. 1c).

Rigid-Plastic-Rigid Theory of Strain-Hardening

Consider a cantilever of length h (Fig. 2a), the member being of uniform I-section of depth $2r$ in which the area of the web is negligible compared with



a) b) c) d)

Fig. 2.

that of the flanges, each of area $A/2$. Then the moment-curvature relation, assuming rigid-plastic-rigid behaviour (Fig. 1d), will be as shown in Fig. 2c, the section "locking" at a curvature of $\frac{k' \epsilon_y}{r}$ when the applied moment reaches the full plastic value $M_P = A r \sigma_y$. This locking enables the moment of resistance of the member to rise above M_P, but this is accompanied by a spread of plasticity along the member. Suppose that, under a load P, plasticity has spread a distance x from the fixed end, (Fig. 2b). Then the hinge rotation is $\phi = \frac{k' \epsilon_y}{r} x$ and the maximum moment is

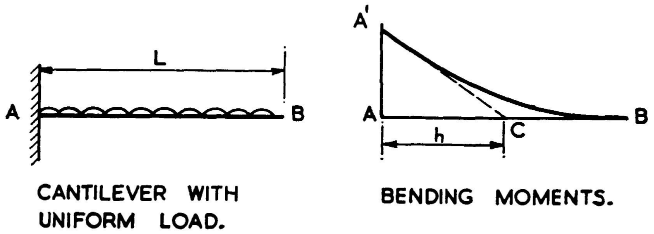
$$M = M_P \frac{h}{h-x} \approx M_P \left(1 + \frac{x}{h}\right) = M_P \left(1 + \frac{r}{k' \epsilon_y} \frac{\phi}{h}\right).$$

Hence the moment-rotation relationship for the rigid-plastic-rigid cantilever is as shown in Fig. 2d.

In the case of a cantilever under a uniformly distributed load (Fig. 3a), the length h in the relation

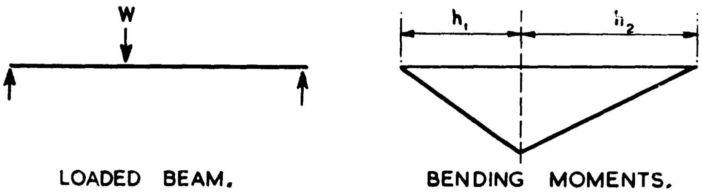
$$M = M_P\left(1 + \frac{r}{k'\,\epsilon_y}\,\frac{\phi}{h}\right)$$

becomes the intercept AC of the tangent $A'C$ to the bending moment diagram at the hinge (Fig. 3b). When plasticity can spread from a hinge in two directions (Fig. 4a), the dimension h is replaced by (h_1+h_2) (Fig. 4b), the sum of the tangent intercepts either side of the hinge.



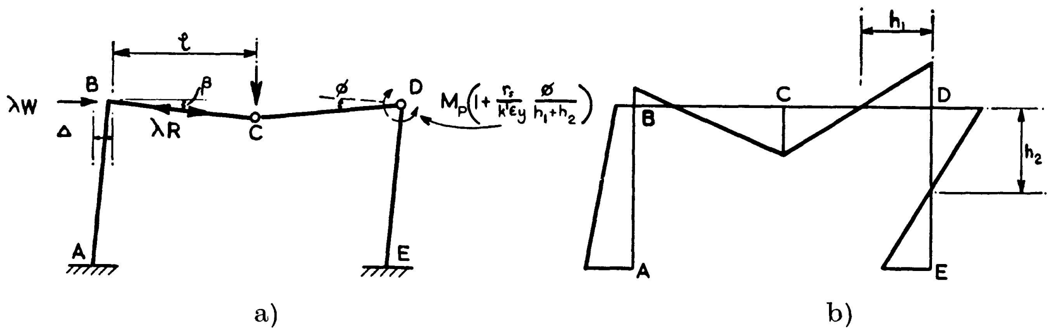
a) b)

Fig. 3.



a) b)

Fig. 4.



a) b)

Fig. 5.

The above strain-hardening characteristics of plastic hinges may be applied to rigid-plastic mechanisms to derive a stability criterion (Fig. 5). Let ϕ represent a typical hinge rotation and Δ the rigid-plastic deflexion corresponding to a typical applied load λW. Let any typical rigid link, such as the left-hand half BC of the beam BD, be of length l, carry an axial thrust λR, and undergo a rotation β. The bending moment diagram, Fig. 5b, establishes the tangent intercepts h_1 and h_2 either side of the plastic hinge at D. If the deformations increase from ϕ, Δ, β to $\phi+d\phi$, $\Delta+d\Delta$, $\beta+d\beta$, the external work is

$$\lambda\{\sum W\,d\Delta + \sum R\,\beta\,l\,d\beta\}$$

and the internal work is

$$\sum M_P \left\{1 + \frac{r}{k' \epsilon_y} \frac{\phi}{h_1 + h_2}\right\} d\phi,$$

both these expressions being correct up to terms of order $\Delta\, d\Delta$, $\phi\, d\phi$ and $\beta\, d\beta$ (see HORNE [5]). Since for small deformations,

$$\frac{d\phi}{\phi} = \frac{d\Delta}{\Delta} = \frac{d\beta}{\beta},$$

it follows that

$$\lambda\{\sum W\Delta + \sum R\beta^2 l\} = \sum M_P\phi + \sum \frac{M_P r}{k' \epsilon_y} \frac{\phi^2}{h_1 + h_2}. \tag{2}$$

The load factor at failure according to rigid-plastic theory is obtained as $\lambda = \lambda_P$ by taking the limiting case as $\phi, \Delta, \beta \to 0$, whence

$$\lambda_P \sum W\Delta = \sum M_P \phi. \tag{3}$$

For small finite values of ϕ, Δ and β the load factor will rise above λ_P if

$$\lambda_P \sum R\beta^2 l < \sum \frac{M_P r}{k' \epsilon_y} \frac{\phi^2}{h_1 + h_2},$$

and will fall if

$$\lambda_P \sum R\beta^2 l > \sum \frac{M_P r}{k' \epsilon_y} \frac{\phi^2}{h_1 + h_2}.$$

A condition of neutral stability will arise if

$$\lambda_P \sum R\beta^2 l = \sum \frac{M_P r}{k' \epsilon_y} \frac{\phi^2}{h_1 + h_2}. \tag{4}$$

Examples of the application of this equation to assess the compensating effect of strain-hardening on frame instability at the plastic-collapse load, together with experimental evidence, have been given elsewhere [4, 6]. As it stands, the equation is applicable only to I-sections with negligible web area. For any section bent about an axis of symmetry, the dimension r is replaced by r_s, the distance from the neutral axis to the "strain centre" of the fibres to one side of the neutral axis. The strain centre is such that the moment of strain to one side of the neutral axis is equal to the moment of the average strain acting at the strain centre, i. e.

$$r_s \sum \frac{z}{R} \delta A = \sum \frac{z^2}{R} \delta A \quad \text{or} \quad r_s = \frac{I}{S},$$

where I is the second moment of area and S is the plastic modulus M_P/σ_y where σ_y is the yield stress. Substituting this value of r_s in Eq. (4) in place of r, and inserting $\epsilon_y = \sigma_y/E$ where E is the elastic modulus, it follows that

$$\lambda_P \sum R\beta^2 l = \sum \frac{EI}{k'} \frac{\phi^2}{h_1 + h_2}. \tag{5}$$

Rigid-Plastic-Rigid Theory as an Approximate Assessment of Elastic Stability

The pure plastic strain, of magnitude $k' \epsilon_y$, controls the degree of stability introduced by strain-hardening when a structure is deforming as a rigid-plastic mechanism. It is interesting to see whether, by introducing a spurious plastic strain $k_E \epsilon_y$ in an otherwise completely rigid material, it is possible to measure elastic stability. This is an entirely artificial device, to be judged entirely by its success in predicting elastic critical loads.

Take first a pin-ended, axially loaded member of length l and flexural rigidity EI (Fig. 6a). This will buckle elastically at the Euler load $P = P_C = \frac{\pi^2 EI}{l^2}$. Consider a rigid-plastic-rigid model of the strut, in which a hinge, full plastic moment M_P, forms at mid-height at the load P_C. It is assumed that a lateral load just sufficient to produce the plastic collapse condition exists, but the axial load is just sufficient to give a state of neutral

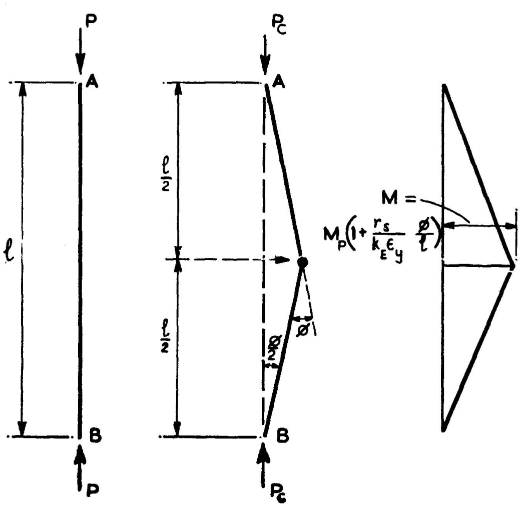


Fig. 6.
Solution for pin-ended strut.

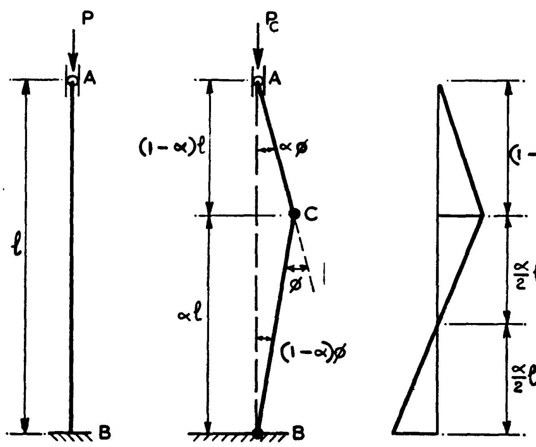


Fig. 7.
Solution for strut pinned at A, fixed at B.

equilibrium under increasing deformation. The bending moment diagram is shown in Fig. 6c, whence in Eq. (5), $h_1 = h_2 = l/2$. Hence, substituting $\lambda_P R = \frac{\pi^2 EI}{l^2}$ in place of $\lambda_P R$ and k_E in place of k', Eq. (5) becomes

$$2\left\{\frac{\pi^2 EI}{l^2}\left(\frac{\phi}{2}\right)^2 \frac{l}{2}\right\} = \frac{EI}{k_E}\frac{\phi^2}{l}$$

giving

$$k_E = \frac{4}{\pi^2}.$$

This same value of k_E will now be used to estimate the elastic critical load P_C of a strut, again of length l, direction fixed at one end and pinned at the other (Fig. 7a). Taking an arbitrary position for the hinge within the length of the member (Fig. 7b), substituting P_C in place of $\lambda_P R$ in Eq. (5) and putting $k' = k_E = 4/\pi^2$, it is found that

$$P_C = \frac{\pi^2 E I}{4} \frac{\dfrac{\phi^2}{(1-\alpha)\,l + \dfrac{\alpha}{2}\,l} + \dfrac{(1-\alpha)^2\,\phi^2}{\dfrac{\alpha}{2}\,l}}{(\alpha\phi)^2\,(1-\alpha)\,l + \{(1-\alpha)\,\phi\}^2\,\alpha\,l},$$

i.e.
$$P_C = \frac{\pi^2 E I}{2\,l^2} \frac{\alpha + (1-\alpha)^2\,(2-\alpha)}{\alpha^2\,(1-\alpha)\,(2-\alpha)}. \tag{6}$$

Eq. (6) gives a minimum P_C when $\alpha = 0.6253$, the minimum P_C being $2.032\,\dfrac{\pi^2 E I}{l^2}$. This compares with the correct value, obtained in the normal way, of $2.046\,\dfrac{\pi^2 E I}{l^2}$.

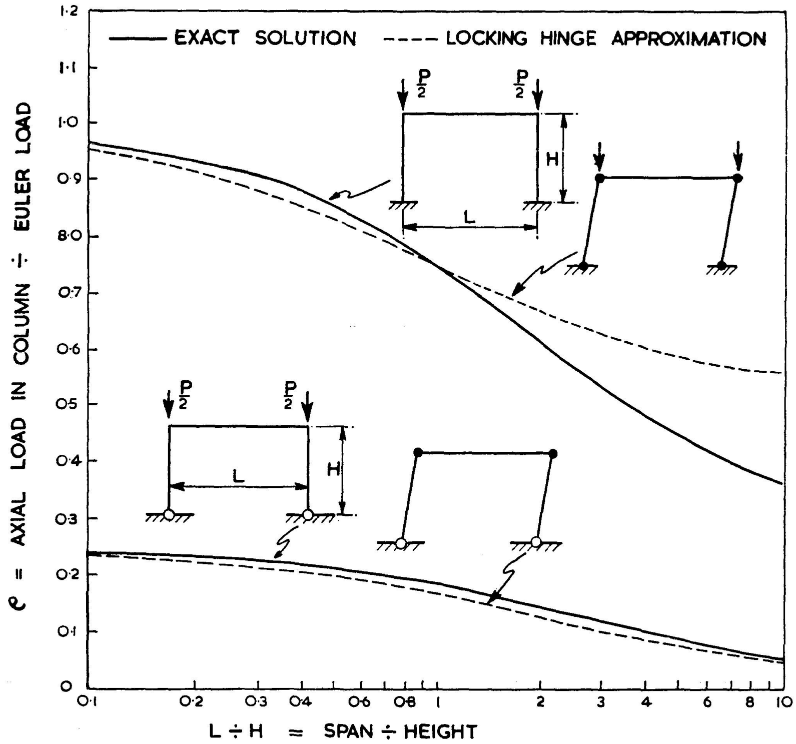


Fig. 8. Comparison of exact and approximate solutions for elastic critical loads of portal frames.

The critical loads obtained by this method ($k_E = 4/\pi^2$) for rectangular portal frames with both hinged and fixed bases are shown in Fig. 8, where they are compared with the accurately calculated values. There is quite a good correlation except at large span to height ratios for the fixed base frames.

Since the aim is to derive a means of obtaining safe (low) estimates of elastic critical loads, it is desirable to adopt a somewhat higher value for k_E than $4/\pi^2$ for general use. Empirically, as a result of investigating quite a number of frames, it is found that a suitable value for k_E is $6/\pi^2$. Hence, replacing λ_P by λ_C in Eq. (5), the approximate expression for the elastic critical load factor becomes

$$\lambda_C = \frac{\pi^2}{6} \frac{\sum \frac{E I \phi^2}{h_1 + h_2}}{\sum R \beta^2 l}, \tag{7}$$

where the axial loads R are those corresponding to unit load factor. Alternately, if $R_P = \lambda_P R$ are the axial loads corresponding to rigid-plastic failure,

$$\frac{\lambda_C}{\lambda_P} = \frac{\pi^2}{6} \frac{\sum \frac{E I \phi^2}{h_1 + h_2}}{\sum R_P \beta^2 l}. \tag{8}$$

It may be noted that Eq. (8) leads to estimates of critical loads of rectangular portal frames (Fig. 8) that are conservative for span to height ratios less than about 8, and the equation is therefore satisfactory for practical frames of this type.

The Estimation of Failure Loads

Comparisons will now be made between the estimates of failure loads obtained from the Rankine formula (using elastic critical loads derived from

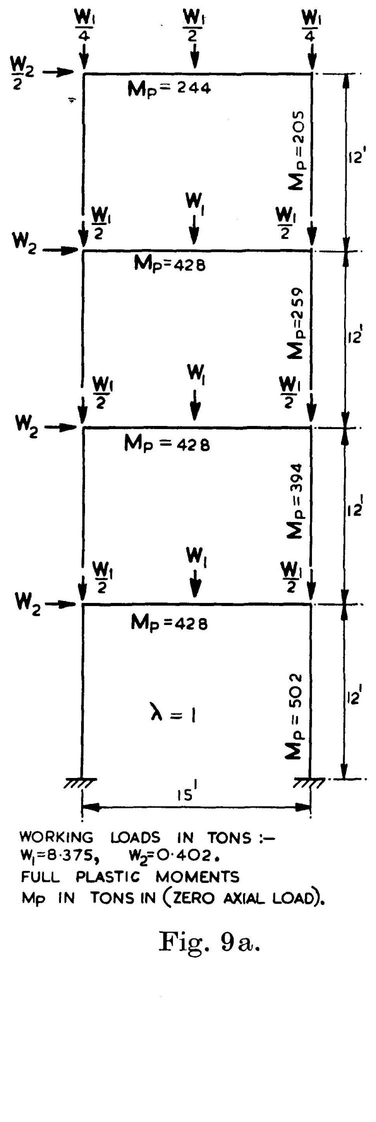


Fig. 9a.

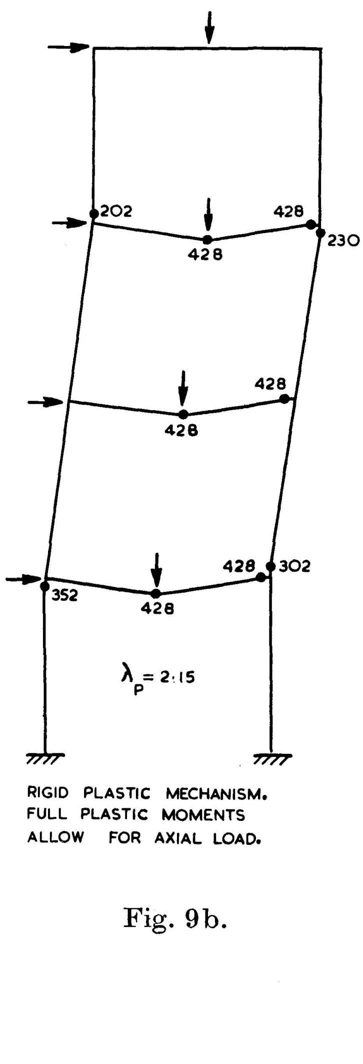


Fig. 9b.

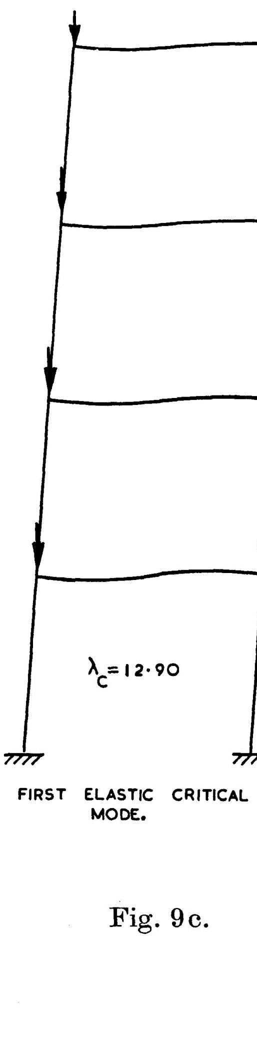


Fig. 9c.

Eq. (8)) and accurately calculated failure loads (using elastic-plastic analysis) for a number of plane frames.

WOOD [1] gives accurate analyses for two four-storey frames, one in which the I-section columns are bent about their major axes and one in which they are bent about their minor axes. The stress-strain relation assumed is elastic-plastic with no strain-hardening. The dimensions and working loads ($\lambda = 1$) of the strong-ways frame are shown in Fig. 9a. Rigid-plastic collapse occurs at $\lambda_P = 2.15$ (Fig. 9b), the accurately calculated elastic critical load factor is $\lambda_C = 12.90$ (Fig. 9c), and WOOD obtains a theoretical failure load factor of 1.90. The Rankine formula (Eq. (1)) gives $\lambda_F = 1.84$, representing an underestimate of 3.2%. The mechanism used for an approximate estimation of the elastic critical load from Eq. (8) is shown in Fig. 9d, and is a sidesway mode, chosen to conform as closely as possible to the rigid-plastic failure mechanism (Fig. 9b). The bending moment diagram for this pseudo-critical load mechanism is shown in Fig. 9e. The axial loads indicated (Fig. 9d) are those corresponding to rigid-plastic failure, and are thus the R_P values in Eq. (8). The full plastic moments allow for the effect of axial loads of this magnitude, this procedure being adopted because the failure load of the frame is not far removed from the rigid-plastic failure load. It may be noted that the beam moments in the second storey are distributed to the column lengths above and below in such a way

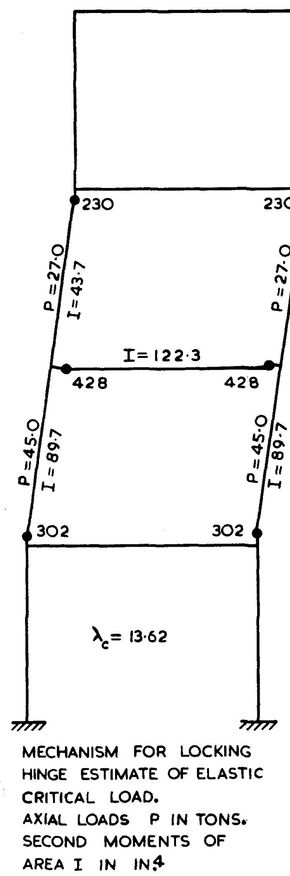


Fig. 9d.

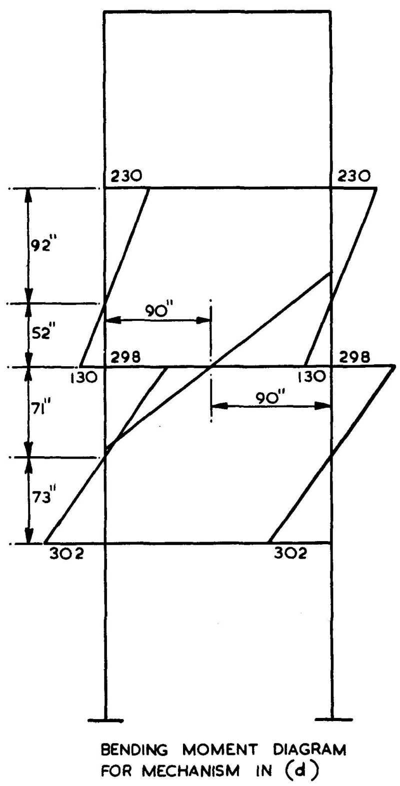
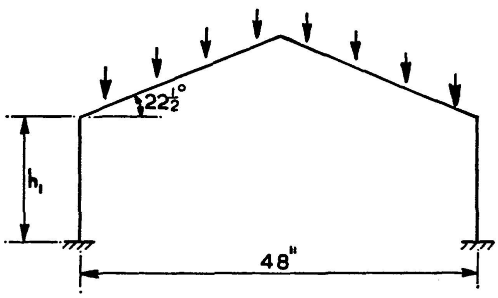


Fig. 9e.

as to make the shear forces in these column lengths proportional to the axial loads they carry. Applying Eq. (8), $\lambda_C = 13.62$, leading to a Rankine estimate of the load factor at failure of $\lambda_F = 1.86$, an underestimate of 2.1%.

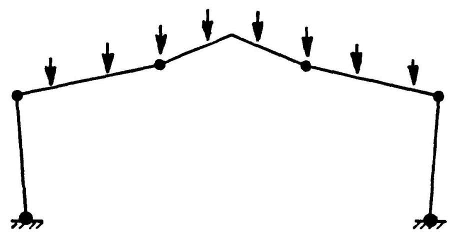
The weak-ways frame analysed by WOOD consists of a four-storey multi-bay frame, and gives a load factor for rigid-plastic failure of $\lambda_P = 2.12$ and an elastic critical load factor of $\lambda_C = 3.47$. This results in a Rankine load factor of 1.32, compared with WOOD's estimated failure load factor of about 1.73. Hence in this case the Rankine load is highly conservative, underestimating the failure load by about 24%. The pseudo-critical load mechanism leads to an estimated critical load factor of 3.80, and hence a Rankine load factor of 1.36, an under-estimate of 21%.

The concept of critical load factors as applied to pitched roof portal frames is ambiguous in that the axial loads in the sloping members are highly sensitive to the state of deformation, and are not simply derivable from the applied loads by inspection, as in the case for multi-storey rectangular frames. It has been argued [3] that, when the elastic critical loads are required for use in the Rankine formula, the appropriate axial loads are those proportional to the axial loads which would be present at rigid-plastic failure. A further ambiguity arises when applying the Rankine formula to frames subjected to vertical loading only, since some frames fail symmetrically while others fail unsymmetrically, and there is a resultant difficulty in deciding which critical elastic mode is applicable. This has been discussed by MAJID [7]. It is found, however, that if the pseudo-mechanism method is used for calculating the



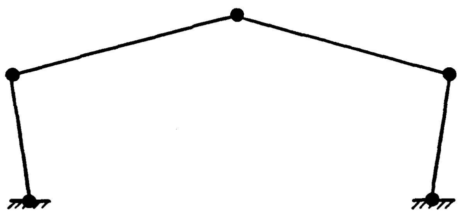
LOADS AND DIMENSIONS

Fig. 10a.



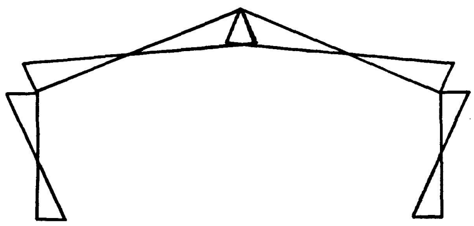
RIGID-PLASTIC COLLAPSE MECHANISM

Fig. 10b.



MECHANISM FOR LOCKING HINGE ESTIMATE OF ELASTIC CRITICAL LOAD

Fig. 10c.



BENDING MOMENT DIAGRAM FOR MECHANISM IN (c)

Fig. 10d.

Table 1.

1	2	3	4	5	6	7	8	9	10	11
Frame No.	Height to eaves h_1 in.	Full plastic moment M_p in.-lb. $10^3 \times$	Flexural rigidity EI lb.-in.^{2} $10^5 \times$	Experimental failure load lb.	Theoretical failure load (exact) lb.	Rigid-plastic failure load lb.	Locking hinge estimate of elastic critical load lb.	Rankine load from (7) and (8)	$(9)-(6)$ %	$(9)-(5)$ %
1	32	1.301	1.447	505	513	561	4540	499	−2.7	−1.2
2	24	1.216	1.266	515	506	560	4060	493	−2.6	−4.5
3	16	1.315	1.398	595	607	685	5170	604	−0.5	1.5
4	12	1.321	1.421	685	664	764	6570	684	3.0	−0.1
5	8	1.316	1.489	845	796	906	11,140	838	5.3	−0.8

elastic critical load, excellent correlation with theoretically calculated failure loads is obtained if attention is confined throughout to symmetrical modes.

Table 1 gives the results for five pitched roof frames tested experimentally and then subjected to accurate analysis by MAJID [7]. The dimensions of the frames are shown in Fig. 10a, the span being in each case 48 ins. and the roof pitch $22\frac{1}{2}°$. The height to eaves h was varied from frame to frame, as shown in column 2 of Table 1. The cross-section of each frame was uniform and approximately $\frac{1}{2}$ in. square, the values of full plastic moment and flexural rigidity being given in columns 3 and 4 of Table 1. The frames were loaded equally at four points on each rafter, and the experimental failure loads are shown in column 5. The loads are the total vertical loads acting. Theoretical failure loads obtained with the aid of an automatic computer are given in column 6. These theoretical failure loads are "exact" in that they allow for the finite sizes of the joints, and follow step-by-step the formation of plastic hinges with full allowance for change of geometry and reduced member stiffness due to axial load. Agreement is excellent except for the frame with the shortest columns, the higher experimental load in the latter case being attributable to the effects of strain-hardening. A complete discussion of these results is given by MAJID.

Rigid-plastic failure, if assumed to be symmetrical, occurs as shown in Fig. 10b, the resulting rigid-plastic loads being given in column 7 of Table 1. The rigid-plastic failure loads make no allowance for the finite sizes of the joints. The mechanism assumed for the calculation of the elastic critical load is shown in Fig. 10c, the bending moment distribution is given in Fig. 10d, and the resulting estimates of elastic critical loads are given in column 8. The axial loads assumed in the calculation (Eq. (8)) are those obtained at rigid-plastic collapse. The Rankine loads, calculated from columns 7 and 8, are given in column 9, while column 10 shows the percentage error compared with the theoretical failure loads in column 6. The percentage errors are in all cases small, and certainly acceptable for all practical purposes. The percentage differences between the estimated failure loads in column 9 and the experimental failure loads in column 5 are given in column 11. Probably fortuitously, the agreement between the approximate treatment and the experimental failure loads is even better than between the approximate treatment and the accurate theoretical loads.

Conclusions

It has been demonstrated that the apparently crude device of postulating a rigid-plastic-rigid mechanism leads to an acceptable estimate of elastic critical loads for use in the Rankine formula. The analysis of a number of other frames for which accurate calculations of theoretical failure loads have been made [3, 8, 9], confirms the trend revealed by the results quoted in detail above. In view of the experimental and theoretical evidence that the Rankine Load is an approximate lower bound to the failure load, it is suggested that the procedure described in this paper may be used to estimate failure loads in those cases where the frame is too slender for the unrestricted application of simple plastic theory.

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Summary

Rigid-plastic theory provides an estimate of the failure loads of structures of elastic-plastic material, but this estimate ignores the effect on the collapse load of finite deformations. The use of the elastic critical load in combination with the rigid-plastic collapse load as a parameter in obtaining a corrected failure load has been suggested by Merchant, using a generalised Rankine formula. While the rigid-plastic load is easily calculated, the elastic critical load is more difficult. It is here shown that the elastic critical load may be estimated with sufficient accuracy by means of a pseudo-elastic energy equation derived from a rigid-plastic mechanism. The application of this procedure to the derivation of elastic-plastic failure loads of various rigid-jointed frames is discussed. Results are compared both with experimental values and with values obtained by exact theoretical analysis, and the agreement is shown to be satisfactory.

Résumé

La théorie rigide-plastique permet d'estimer les charges de rupture d'ossatures constituées de matériaux élastico-plastiques, mais cette estimation ne tient pas compte de l'effet sur la charge de ruine des déformations finies. MERCHANT, utilisant une généralisation de la formule de RANKINE, a proposé de considérer la charge élastique critique conjointement avec la charge rigide-plastique de ruine prise comme paramètre, pour apporter une correction à la charge de rupture. S'il est facile de calculer la charge rigide-plastique, le calcul de la charge élastique critique est plus difficile. Il est montré qu'on peut estimer, avec une précision suffisante, la charge élastique critique en utilisant une équation d'énergie pseudo-élastique, en partant d'un mécanisme rigide-plastique. On traite l'application de cette méthode au calcul des charges élastico-plastiques de rupture de divers types de portiques à joints rigides. On compare les résultats obtenus avec les valeurs expérimentales ainsi qu'avec celles fournies par le calcul exact, et l'on constate que la concordance est satisfaisante.

Zusammenfassung

Die ideal-plastische Theorie erlaubt die Abschätzung der Bruchlast von Tragwerken aus elastisch-plastischem Material, aber sie vernachlässigt den Einfluß endlicher Verformungen auf die Traglast. Die Benützung der elastischen, kritischen Last im Zusammenhang mit der ideal-plastischen Traglast als einen Parameter für die Bestimmung einer korrigierten Bruchlast wurde von MERCHANT vorgeschlagen, unter Benützung der allgemeinen RANKINEschen Formel. Während die ideal-plastische Last einfach zu berechnen ist, so ist dies für die elastische, kritische Last bedeutend schwieriger. Hier wird gezeigt, daß die elastische, kritische Last genügend genau abgeschätzt werden kann bei Anwendung einer pseudo-elastischen Energiegleichung, die von einem ideal-plastischen Mechanismus abgeleitet wird. Die Anwendung dieses Verfahrens für die Ableitung elastisch-plastischer Bruchlasten verschiedener Rahmenformen wird diskutiert. Ein Vergleich dieser Resultate mit Versuchswerten und Werten aus einer genauen theoretischen Berechnung ergab eine zufriedenstellende Übereinstimmung.