A preliminary study of composite action in framed buildings

A Preliminary Study of Composite Action in Framed Buildings [1])

Investigations provisoires sur les influences combinées dans les constructions à étages

Untersuchung des Zusammenwirkens in Stockwerkbauten

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Introduction

In recent years there has been increasing interest shown in the study of the effects of continuity in structural frameworks, leading quite naturally to a study of the composite behaviour of the complete structure, that is to say the combined action of the frame, walls and floors [1]. Tests on completed buildings are difficult to carry out, but have been made in this country and abroad, notably the tests in London on the Cumberland Hotel, Geological Museum, Office and Residential Flats buildings [2], New Government Offices, Whitehall Gardens [3]; and in South Africa on the Dental Hospital, Johannesburg [4]. There is also the additional evidence afforded by wartime damage to buildings, which have sometimes been left standing even when some of the stanchions have been blasted away. It is clear from these tests and controlled laboratory tests that composite action has important effects on the stiffness and strength of structures, and hence there is a need for a systematic study of composite action. Such a study has been started at the Building Research Station.

Essentially the main problem is to determine the real loads which are transmitted from the floors or walls to the supporting frame. The distribution of these interactive forces may be quite different from what is now assumed in design, and in addition changes as plasticity occurs at heavy loads.

This present paper is largely concerned with the interaction between floor slabs and beams, but brief reference is made to some other examples of composite action which will be discussed first.

The Combined Action of Walls and the Frame

There are obvious difficulties in producing an acceptable theory for the structural behaviour of brickwork, and it is hardly surprising that tests on combined walls and beams and columns have so far been of an *ad hoc* nature. Two series of tests made at the Building Research Station have, however, shown the importance of composite action in such construction and these are discussed briefly below.

a) Racking tests on brickwork panels (figs. 1, 2)

Several sideways racking tests on encased steel frames with various wall panel infillings have been carried out by L. G. Simms. Load-displacement curves are given in fig. 1, where it is seen that the maximum racking load sustained by an encased steel frame was raised from 20 tons to 53 tons by the insertion of a $4\frac{1}{2}''$ brick panel [5], together with a considerable increase in stiffness i.e., in the initial resistance to sidesway. The stiffening effect of the

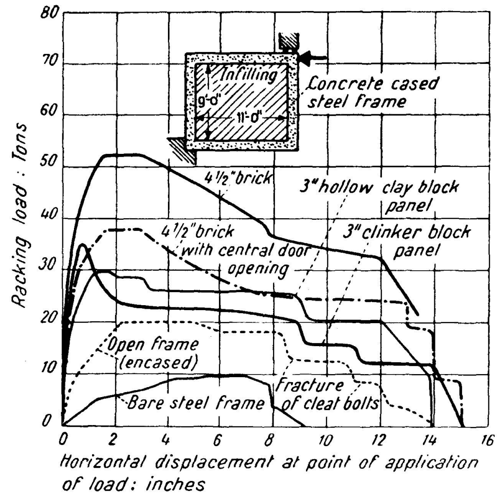

Fig. 1. Effect of infilling panels on concrete-encased steel frame subjected to a racking load.

encasement itself is also a form of composite action, for the first yield of a bare steel frame, without any encasement, took place at about 5 tons and a collapse mechanism developed at about 9 tons. It will be noted from fig. 2 that the mode of collapse has been changed by the brickwork, since plastic hinges can be seen to have formed not far from the centres of the horizontal beams, two corners of the frame remaining intact. As will be seen later, com-

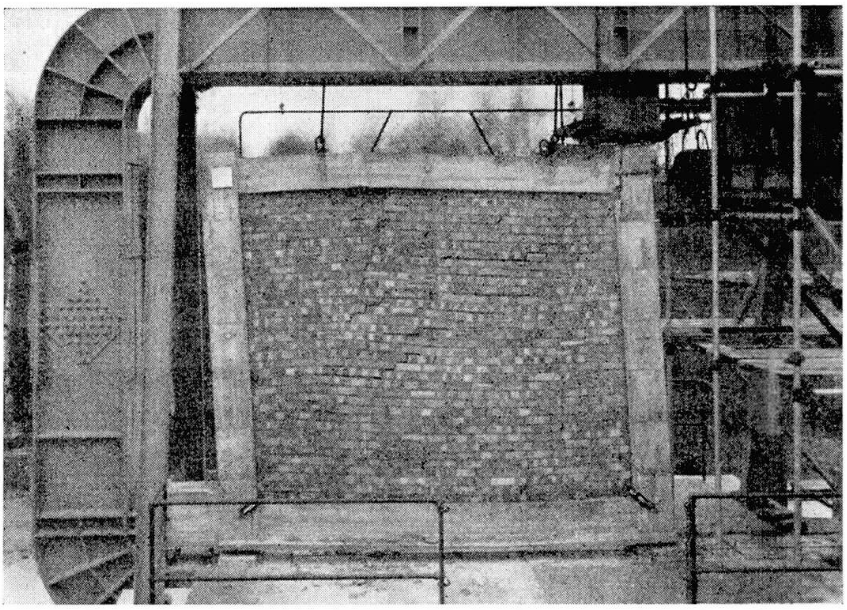

Fig. 2. General view of racking tests.

posite action often results in modes of collapse which are modifications of those associated with bare frames, and may sometimes be quite different.

b) Brick walls on reinforced concrete beams

A series of tests has been carried out [6] on brick walls carried by reinforced concrete beams, distributed load being applied to the top of the brickwork. Fig. 3 shows a brick panel under test with a door opening near to the

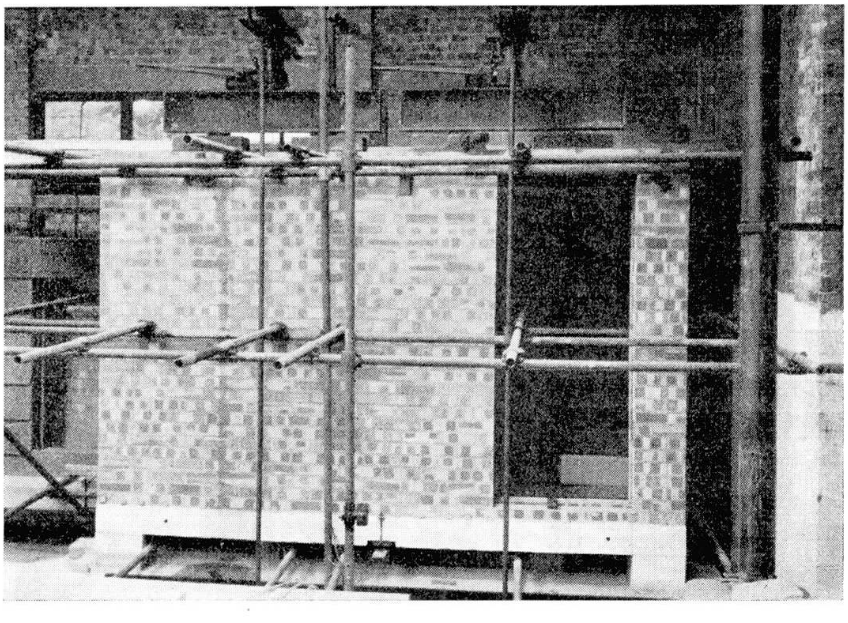

Fig. 3. Vertical loading tests on a cavity brick wall carried by a reinforced concrete beam, with a door opening near the supports.

supports, whilst fig. 4 shows the measured vertical intensity of load on the bottom course of bricks where there is a central door opening. The nature of this reaction shows that considerable arching action has taken place, thus concentrating the load near the supports. The resulting bending moments in the beams were very small indeed, and it was found possible to recommend that (subject to certain conditions) the beams be designed on a basis of $WL/100$ where W is the total load and L is the span, provided that there were no

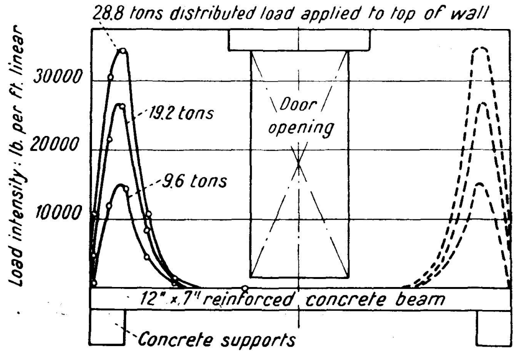

Fig. 4. Brick wall with central door opening supported by a reinforced concrete beam. Vertical load intensities in bottom course of bricks, as measured by roller-mirror extensometers. Distributed loads applied to top of wall. The peak intensities of stress in the brickwork near the supports illustrates the arching action in the composite wall-beam structure.

openings near the supports; and $WL/50$ when door or window openings occurred near the supports. A large number of house walls resting on beams have by now been constructed on this basis. Considerable use could be made of this arching effect in multi-storey buildings where walls are carried on beams, but it would not yet be safe practice to use overhead walls to stiffen beams that carry floor loads unless some form of tensile connectors could be devised.

There appears however to be so much reserve of strength available in brick walls that even an approximately method of allowing for it in design would be of considerable value.

The Composite Action of Reinforced Concrete Floors and Steel (or Reinforced Concrete) Beams

Apart from the immediate effects of the concrete encasement of a steel beam, composite action in a beam and slab system may include the following:

1. Combined bending and twisting of the slab and beams. This is probably the principal type of composite action, present in all systems to a marked degree, and being practically the only type developed when the beam and slab centroids coincide as in fig. 5a.

2. Direct compressive and shear stresses in the slab arising from T-beam action when the beam and slab centroids do not coincide (fig. 5b).
3. Direct tensile and shear stresses in the slab due to stretching of the slab at large deflections. Tensile stresses can also take place by inverted T-beam action when the beams are upstanding [3], but this case is no doubt trivial.

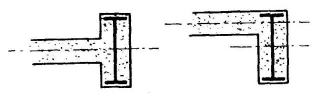

Fig. 5. Boundary conditions for bending alone, and T-beam action.
a) Bending only. b) T-beam action included.

The general boundary conditions between beam and slab have been previously set out in full by the author [1]. In the present paper the study of general composite action will be introduced by reference to (1.) above, neglecting the twist in the beams but not in the slab, and giving evidence of (2.) and (3.) as has so far been observed in laboratory tests. A brief account will be given of the results of tests which have been obtained at both working (elastic) conditions and also at ultimate (plastic) collapse.

a) Governing equations of elasticity for a single square panel carried by beams (fig. 6)

Here we make use of the well-known equation for a slab

$$\nabla^4 w \equiv \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \tag{1}$$

where x and y are rectangular coordinates, w is the deflection, q is the intensity of load on the slab, and D is the flexural rigidity. If p represents the intensity of load per unit length applied directly over the beams, and if we approximate

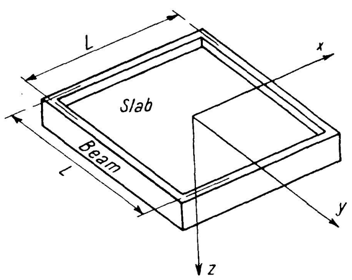

Fig. 6.

by putting Poisson's Ratio equal to zero, then there are two boundary conditions to be satisfied along the edge $y = L/2$, relating to edge bending moments and reactions, respectively:

$$\frac{\partial^2 w}{\partial y^2} = 0 \tag{2}$$

provided that the torsional rigidity of the beams can be neglected; and

$$D\left[\frac{\partial^3 w}{\partial y^3} + 2\,\frac{\partial^3 w}{\partial x^2\,\partial y}\right] + p = E\,I\cdot\frac{\partial^4 w}{\partial x^4} \tag{3}$$

Solutions for a number of examples of square and rectangular panels with various loadings have now been obtained using Finite Different Methods, in terms of the following non-dimensional parameters [1]:

$\lambda = \dfrac{q\,L^3}{D}$, a slab loading term

$\dfrac{p\,L^3}{E\,I}$, a beam loading term

$\gamma = \dfrac{E\,I}{D\,L/2}$, expressing the relative stiffness of the beam

to the stiffness of the adjacent half-width of slab.

b) The reactions on the beams carrying a square floor

The theoretical distribution of load intensity on the beams when the slab is uniformly loaded and all beams are of the same stiffness, can be seen in fig. 7 for a variety of beam/slab stiffness ratios γ, with the commonly assumed

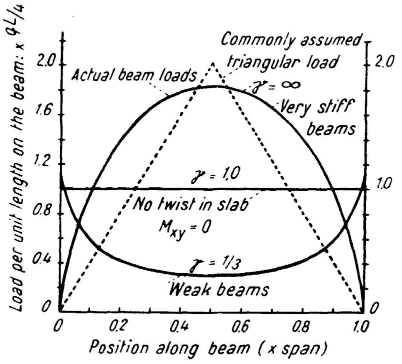

Fig. 7. Theoretical distribution of loads on the beams in relation to the beam/slab stiffness ratios (square slab on beams of equal stiffnesses: uniform load on slab of intensity q).

triangular distribution for comparison. It will be seen that with weak beams the load is reduced at the centre of the span and increased near the supports. This is analogous to the state of affairs with wall/beam systems (fig. 4) where such a reduction in beam loading was the result of arching. In the present case it is due to twisting of the slab. When twist in the slab vanishes with respect to the x and y axes, the reactions on the beams are, to a first approximation, uniform; this is the true case of individual strip action in the slab, every cross-wise strip acting independently of every other strip, taking the same proportion of slab load and repeating the same deflection curve. When opposite pairs of beams are of equal stiffness, it is easy to show that for this state of no-twist to occur we must have

$$\gamma_x \cdot \gamma_y = 1.0 \tag{4}$$

where γ_x and γ_y are the values of γ corresponding to the two pairs of beams.

Thus the condition for this general "twistless" case to occur is that a critical relationship must hold between the slab and edge beam stiffnesses. A rigorous proof is given elsewhere [1], together with a discussion on the essential difference between this case and the well-known Rankine-Grashof rule as modified by MARCUS [7]. Condition (4) holds for rectangular panels and also for continuous floors resting on beams when uniformly loaded. Its importance lies in the fact that a simple analytical solution can be written down for this special case of composite action.

c) *The bending moments in the beams carrying a square floor*

The large variations in beam loads give rise of course to considerable reductions in beam bending moments as the beam stiffness is reduced relative to the slab as shown in fig. 8. As the beam stiffness is made progressively

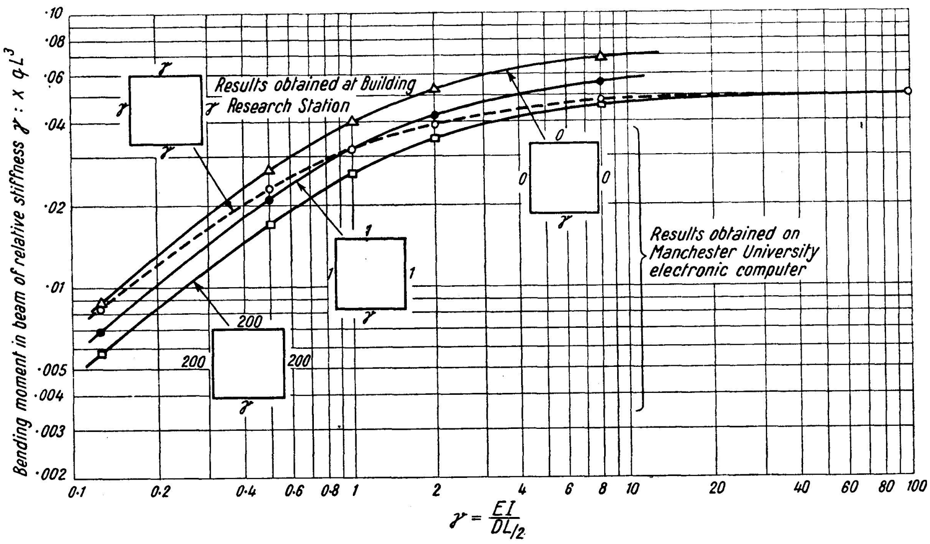

Fig. 8. Bending moments in a beam supporting a square slab carrying a uniform load of intensity q, and the influence of adjacent beams of different stiffness ratios.

smaller a stage is eventually reached (with geometrically similar beam sections) where the reduction in bending moment is equal to the reduction in section modulus, and when that happens there is a tendency for the stress in all such beam sections to remain constant for the same loading. This means that the deflection of the beam, and not the stress in it, may then become the criterion of design. In fig. 8 the bending moments are given for several variations in the relative stiffness of the four supporting beams. When these stiffnesses are unequal, the computations are very lengthy, and these were made therefore with the use of the Manchester University Electronic Computer. Although the relationship between the bending moment in a beam and its stiffness is affected by the stiffnesses of the other beams, it will be seen that there is a band of results indicating that an approximate general relationship might be possible for design purposes.

It now becomes necessary to examine what would happen to the slab.

d) Bending moments in a square slab carried by beams (fig. 9)

When the beams are all infinitely stiff the bending moment across the centre line of the slab is a maximum at the centre, but when the beam stiffnesses are reduced to a value corresponding to $\gamma = 1$ this bending moment is

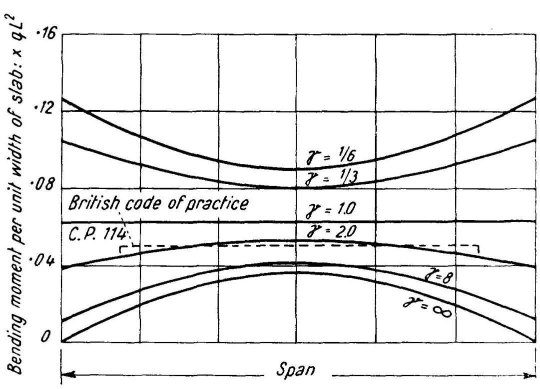

Fig. 9. Square slab supported on beams of equal stiffness. Distribution of bending moments across centre line of slab (Poisson's ratio zero).

uniform for the whole width of the slab, corresponding to the "twistless" case when each strip acts independently. With further reduction in beam stiffness the maximum moment occurs at the edge of the slab — and in laboratory tests the first cracks have occurred at this point. It will be seen that the total bending moment carried by the slab increases rapidly and that to design a slab without reference to the stiffness of the beams and the nature of the composite action can be very misleading. Furthermore the intensity of the local corner reaction is decided by the twist at the corners, and it is clear that the twist in the slab changes sign as the beams are reduced in size, passing through zero twist when, for all beams, $\gamma = 1$. Thus the corners are held down when $\gamma > 1$, and are pushed up when $\gamma < 1$, and torsional reinforcement put into the slab on a basis of non-sinking beams may in some cases be valueless.

e) Enhanced rotational stiffness of beams

Apart from the changes of bending moment in the beams and slabs there is also the effect on adjacent stanchions to be considered. There are two complementary effects of composite action — the increase in rotational stiffness of the beam to applied end couples, and any changes in the Fixed End Moments. With infinitely stiff beams (relative to the slab) the rotational stiffness is the usual value for the beam. When $\gamma = 1$ it has been found that the stiffness is enhanced by a factor of about 1.5 and when $\gamma = \frac{1}{3}$ the enhancement factor is about 2.5. This would have an effect in reducing the moments in an adjacent stanchion, and also cause an increase of stability. The Fixed End Moments for loads applied to the beams are reduced slightly by composite action, leading therefore to a further reduction in the stanchion moments.

f) Continuous square floors on continuous beams

The appropriate beam/slab stiffness ratio in this case would take account of the fact that there is a slab on each side of the beam, so that we choose for continuous systems $\gamma_c = \frac{EI}{2 \cdot DL/2} = \frac{EI}{DL}$. The approximate results of an analysis are given in fig. 10, where the beam support moments, beam centre-span

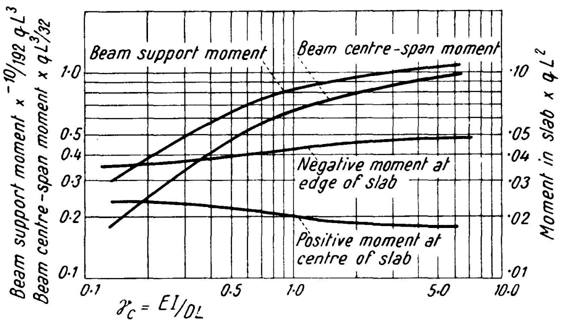

Fig. 10. Beam and slab moments for continuous square panels: all panels carrying a uniform load of intensity q.

moments, maximum positive and negative moments on the slab centre lines are all plotted. The corresponding beam load distribution curves show a similar tendency to those in fig. 7, but are even more sensitive to changes in beam stiffness.

An Introduction to the Study of Collapse of Composite Floor-Beam Structures

This subject will be introduced principally with reference to simple square slabs supported on beams such that T-beam action is absent. Further results are known for rectangular panels [1], and one of the most recent attempts at minimising the total amount of reinforcement without reducing the collapse load of the structure will be indicated.

a) "Limit Analysis" methods applied to composite structures

Whereas in the case of rigid-frame structures the collapse load can usually be determined uniquely [8], with composite floor-beam systems we may have to be satisfied with upper and lower limits or "bounds" for the collapse load. Providing these indications of the collapse load are sufficiently close — as we shall see in many cases they can be made identical — then the actual collapse load will usually be specified conservatively, since we have so far neglected the catenary (tensile) stresses in the slab at large deflections. The "Limit Analysis" procedure, as outlined by PRAGER, DRUCKER, GREENBERG and HODGE [9, 10] identifies the idealised bounds for the collapse load as follows:

Upper Bound (may be unsafe for the given loads)	A collapse mechanism, which is kinematically admissible for the given mechanical conditions of support, where the work done by the external loads is equated to the energy dissipated internally by yielding.
Lower Bound (safe under the given loads)	A stress field which is statically admissible, i. e. it satisfies the equilibrium equation *at every point* together with the boundary conditions of stress, and which nowhere violates the yield criterion.

We may combine the "Fracture Line" methods of JOHANSEN [11] with the plastic design procedure for beams of BAKER [12], and produce a very valuable — and in many cases correct — indication of the collapse load. But this combination only provides an intuitive upper bound for the collapse load. It should really be supplemented by a general examination of other possible collapse mechanisms, and in any case it tells us nothing about the distribution of loads on the supporting beams. It is safer and more illuminating to search for acceptable and economical lower bounds, which means identifying a complete stress system.

b) Upper-bounds for the collapse loads of square floor-beam panels by fracture line methods

The collapse load of a simply supported square slab (fig. 11 a) is well known to be $q\,L^2 = 24\,M_s$, where M_s is the full plastic moment per unit width of slab in all directions, provided the corners are suitably reinforced to prevent the "seesaw" ("Wippe") effect noted by JOHANSEN. But under certain conditions

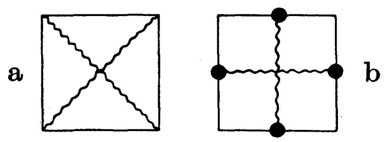

Fig. 11.
a) Diagonal collapse mode (slab only). b) Combined beam and slab collapse.

an alternative mode of collapse could take place, which involves the supporting beams (fig. 11b) which can be easily proved to take place when the total distributed load on the slab is

$$q\,L^2 = 8\,M_s + 16\,\frac{M_b}{L} \tag{5}$$

where M_b is the full plastic moment of each beam.

The criterion for this mode of collapse is that

$$\gamma_p = \frac{M_b}{M_s\,L/2} \leqslant 2 \tag{6}$$

that is, the beams must be below a certain critical strength.

Other combined collapse modes can be readily identified for rectangular panels [1].

c) A lower-bound for the collapse load of a simply supported square reinforced concrete floor

This solution is due to PRAGER [13], and is a suitable starting point for the study of impending collapse of floor-beam systems, and has been discussed in full by the author [1]. Referring to fig. 12 the statically admissible moment

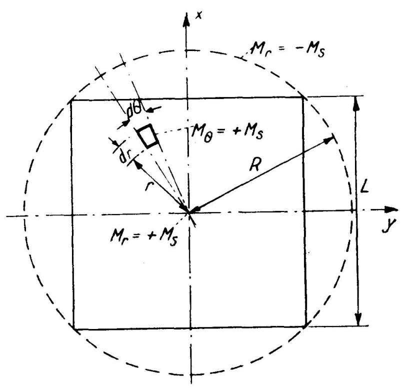

Fig. 12. Illustrating Prager's radial lower-bound solution for collapse of slab.

(i.e. stress) field refers to impending conical collapse of a clamped circumscribing circular plate, where the radial moment $M_r = M_s - q\,r^2/6$ and varies from $+M_s$ at the centre to $-M_s$ round the outer circle, if it is tentatively assumed that $M_s = \frac{q\,R^2}{12} = \frac{q\,L^2}{24}$. The circumferential moment M_θ is everywhere equal to $+M_s$, and the twist $M_{r\theta} = 0$. In order to check that the stress field is indeed statically admissible we note that in rectangular coordinates:

$$\left.\begin{aligned} M_x &= q\,L^2/24 - q\,x^2/6 \\ M_y &= q\,L^2/24 - q\,y^2/6 \\ M_{xy} &= +\,q\,x\,y/6 \end{aligned}\right\} \tag{7}$$

This satisfies everywhere the equation of equilibrium

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} - 2\,\frac{\partial^2 M_{xy}}{\partial x\,\partial y} = -q \tag{8}$$

and also the condition that $M_x = 0$ all along $x = L/2$ and the principal moments do not exceed the yield moment anywhere (namely $\pm M_s$ in any direction). Consequently this stress field corresponds to a valid lower-bound, and gives an identical collapse load to the upper-bound deduced from the fracture-line method.

Now the beam reaction along $x = L/2$ is given by

$$V_x = \left(\frac{\partial M_x}{\partial x} - 2\,\frac{\partial M_{xy}}{\partial y}\right)_{x=L/2} \tag{9}$$

and applying this to eq. (7) we find that $V_x = -q\,L/3$ which indicates a constant upward reaction from the beam, in spite of the fact that there is twist present when referred to rectangular axes. Whereas PRAGER used this radial solution for infinitely stiff supporting beams it would also clearly be valid provided the full plastic moment of each beam were at least equal to $-V_x \cdot L^2/8 = \frac{qL}{3} \cdot \frac{L^2}{8} =$ $= q\,L^3/24$, that is, when $\gamma_p = \frac{M_b}{M_s L/2} = 2$ as before. This solution therefore is particularly appropriate when the modes (a) and (b) of fig. 11 take place simultaneously, since this combined mode of collapse agrees well with PRAGER'S assumption that all radial sections are at full plasticity simultaneously.

d) Lower-bounds for combined collapse of slab and beams

For the general case, consider

$$\left.\begin{aligned} M_x &= M_s - 4\,\frac{M_s \cdot x^2}{L^2} \\ M_y &= M_s - 4\,\frac{M_s \cdot y^2}{L^2} \\ M_{xy} &= 4\,\frac{M_s}{L^2}\,(\gamma_p - 1)\,x\,y \end{aligned}\right\} \tag{10}$$

It is found that this moment field

1. Satisfies $M_x = 0$ when $x = L/2$.
2. Satisfies the equilibrium eq. (8) provided $q\,L^2 = 8\,M_s + 16\,M_b/L$.
3. Gives a constant beam reaction of $4\,\frac{M_s}{L} \cdot \gamma_p$ always leading to the required full plastic moment of M_b.
4. Does not anywhere violate the yield $(\pm M_s)$ criterion.

It follows that eqs. (10) define moment conditions which lead to a valid lower-bound solution. From (2.) above it follows that the collapse load is the same as that deduced from the fracture-line method, eq. (5), which gave an upper bound. Hence, by limit analysis procedure, we have identified the collapse load.

Three cases are of particular interest as set out below:

$\gamma_p = \dfrac{M_b}{M_s L/2} =$	2	1	0
Twist at corners	$+M_s$	0	$-M_s$
Collapse load	$24\,M_s$	$16\,M_s$	$8\,M_s$
Beam reaction (constant)	$q\,L/3$	$q\,L/4$	0
Remarks:	Largest beam before slab collapses independently	Twistless case	Free edge case

We note that the beam reactions are (according to these acceptable stress fields) all of constant intensity. When $\gamma_p = 1.0$ we obtain a "twistless case", i.e. independent strips as in the elastic system when also $\gamma = 1.0$: in fact the two stress distributions are then identical; however, in collapse methods of design the full plastic moment would be allowed for the slab section and this is usually about 6 per cent greater than the moment at which yield first commences in the section.

The general case of rectangular floor panels has been reported elsewhere [1].

e) Reduction of the total reinforcement to a minimum

Whilst the above lower-bound solutions indicate how to reinforce the slab both on the fracture lines and elsewhere, the total amount of reinforcement may be excessive. Paradoxically the system of maximum principal moments

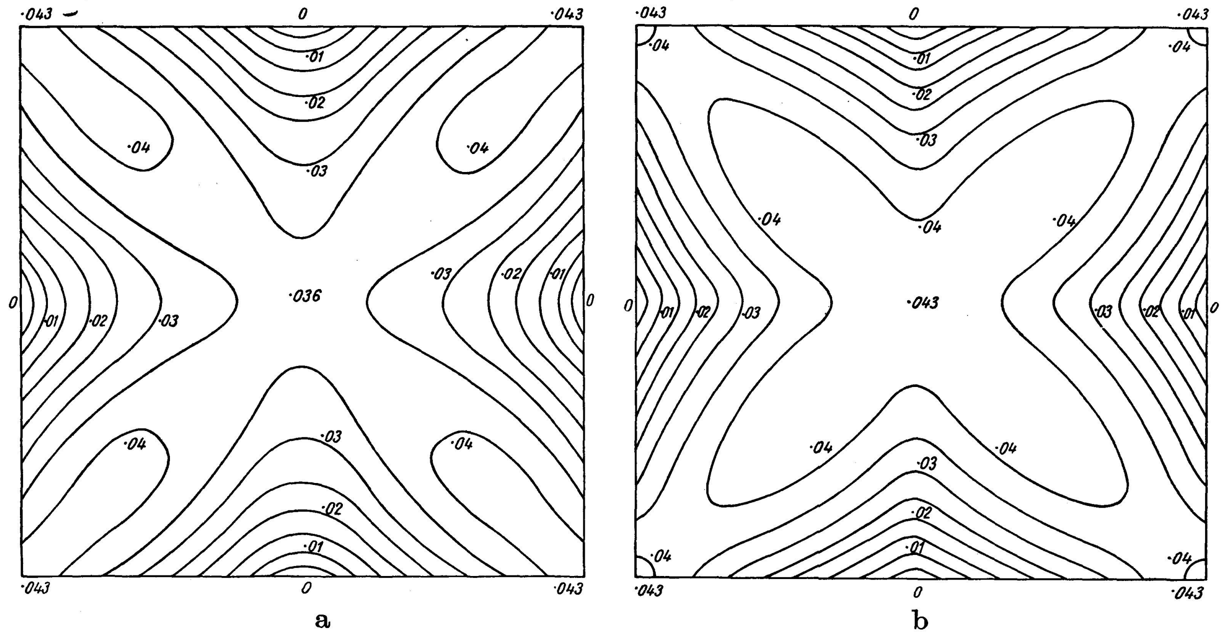

Fig. 13. Contours of maximum principal moments: simply-supported slab uniformly loaded. All moments per unit width: $x\,q\,L^2$.

a) Elastic solution for $\gamma = \infty$ with Poisson's ratio = zero.

b) Lower-bound for collapse: equations 16, 17, 18.

indicated for $\gamma = \infty$ by elastic analysis (fig. 13a), which it must be remembered is a valid lower bound if the reinforcement is arranged to suit the moments, obviously requires less total reinforcement than the radial pattern of PRAGER's solution, so that the question arises — how much reinforcement can we remove without altering the collapse load? Some authorities [14, 15] recommend a "stepped" reinforcement having one constant value in a middle band, and a much smaller value in an edge band. But if there is a sudden reduction in

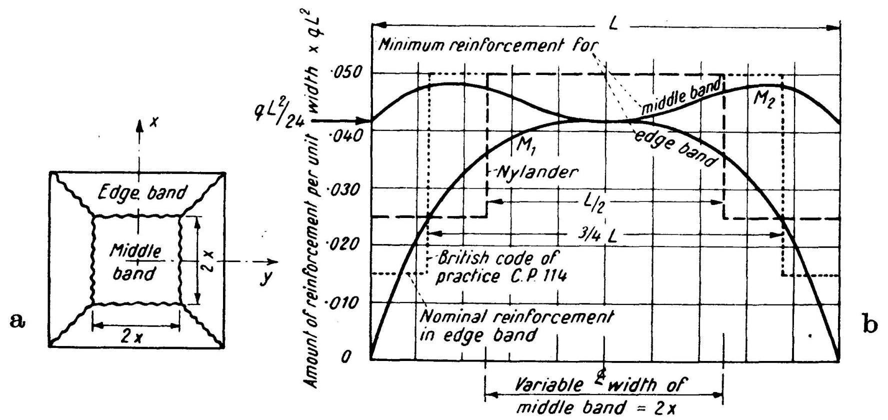

Fig. 14. Collapse of square slabs on stiff beams; required reinforcement according to an upper-bound for the collapse load (equations 11 and 12).

a) Collapse mode which drops out the middle band.

b) Minimum amounts for middle and edge bands of reinforcement.

the full plastic moment per unit width from M_2 to M_1 it is necessary to provide reinforcement not only to resist the usual collapse mode about the diagonals but also to prevent other modes of collapse taking place, of which the most important is likely to be the dropping out of the whole middle band (fig. 14a).

Combining both these requirements it is easy to show that

$$M_2 = \frac{q\,L^2}{24}\left\{1 + \left(\frac{x}{L/2}\right)^2 - \left(\frac{x}{L/2}\right)^3\right\} \tag{11}$$

and

$$M_1 = \frac{q\,L^2}{24}\left\{1 - \left(\frac{x}{L/2}\right)^3\right\} \tag{12}$$

The minimum amounts of reinforcement are shown in fig. 14b, where it is seen that some of the present recommendations are not always even valid upper bounds for collapse[2]). There is no guarantee, without an extensive search, that other modes of failure might not occur, and furthermore an upper-bound analysis gives no statement whatever of the distribution of load on the supporting beams.

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[2]) i. e. according to Limit Analysis. The anchorage length of the middle-band reinforcement would partly alleviate this state of affairs. Tests have also shown that catenary stresses may be, in some cases, of considerable importance.

Accordingly this whole question of minimisation of the reinforcement in composite slab/beam structures is being investigated by lower-bound limit analysis and by laboratory tests at the Building Research Station. One interesting result is now given which considerably reduces the reinforcement when stiff beams are present and provides us with a statement regarding the corresponding minimum size of beam required, and which is still a "safe" solution.

The requirements for an expression for M_x in a square slab on stiff beams would be (see fig. 6):

1. $M_x = 0$ along $x = L/2$ neglecting twist in the beam.
2. M_x may be expected to equal M^*, a maximum yield moment at the centre, when $x = y = 0$.
3. M_x must be symmetrical about the centre line.
4. Although not an absolute requirement we may aim at $M_x = 0$ all along $y = L/2$, since the beams are very stiff, for this is one way to eliminate much circumferential reinforcement.

All these features are satisfied by putting

$$M_x = \frac{16\,M^*}{L^4}\,(x^2 - L^2/4)\,(y^2 - L^2/4) \tag{13}$$

Evidently

$$M_y = M_x \tag{14}$$

To satisfy the equilibrium eq. (8) we find that the twist must be

$$M_{xy} = \frac{q\,x\,y}{2} + \frac{16\,M^*}{L^4}\left(\frac{x^3 y}{3} + \frac{y^3 x}{3} - \frac{L^2}{2}\,x\,y\right) \tag{15}$$

We have not yet chosen M^*. If we choose to make the twist in the corner, $x = y = L/2$, equal to $+M^*$ then we find $M^* = q\,L^2/18.67$. This value is greater than $q\,L^2/24$ but halfway along the diagonals the principal moment falls to $q\,L^2/27.1$. This solution can be further improved by employing the next (higher order) solution of the same type i.e. we consider

$$M_x = \alpha\cdot\frac{16\,M^*}{L^4}\,(x^2 - L^2/4)\,(y^2 - L^2/4) + (1-\alpha)\,\frac{256\,M^*}{L^8}\left(x^4 - \frac{L^4}{16}\right)\left(y^4 - \frac{L^4}{16}\right) \tag{16}$$

$$M_y = M_x \tag{17}$$

so that, to satisfy equilibrium at all points, we require by eq. (8) that the twist should be

$$M_{xy} = \frac{q\,x\,y}{2} + \alpha\cdot\frac{16\,M^*}{L^4}\left(\frac{x^3 y}{3} + \frac{y^3 x}{3} - \frac{L^2}{2}\,x\,y\right) + \\ + (1-\alpha)\,\frac{256\,M^*}{L^8}\left\{\frac{6}{15}\,(x^3 y^5 + y^3 x^5) - \frac{1}{8}\cdot L^4\,(x^3 y + x\,y^3)\right\} \tag{18}$$

We choose α and M^* such that the maximum principal moment is M^* at the corner and also half-way along the diagonals. Thus we find $\alpha = 0.6858$, $(1-\alpha) = .3142$, $M^* = \frac{qL^2}{23.35}$. A plot of the contours of maximum principal moment is shown in fig. 13b which is a sufficiently economical distribution of the reinforcement to introduce this approach to the general subject of minimization of reinforcement. It is seen that the *elastic* solution (Poisson's ratio zero; $\gamma = \infty$, fig. 13a) is an exceptionally good lower-bound for collapse design where the reinforcement is varied to suit the bending moment diagram. This is an intriguing point, well brought out by the study of composite structures, for we have already seen that the elastic and plastic "twistless" stress fields are identical. In fig. 15 the beam load intensities are compared, where it appears

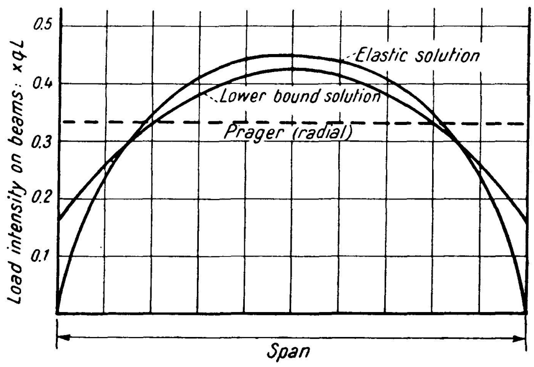

Fig. 15. Comparison of load intensity on beams.

that the distribution of the beam reaction at the point of impending collapse is very much linked up with the placing of the reinforcement. Thus the corresponding bending moment in the beam for this lower-bound solution is $\frac{qL^3}{21.1}\left[\text{c.f. } \frac{qL^3}{24} \text{ (PRAGER: Radial) and } \frac{qL^3}{20} \text{ (Elastic)}\right]$, and it is sufficient if the beam is just strong enough to withstand this bending moment without collapse. Since the total yield moment of half the slab along the centre line is found to be $\frac{qL^3}{66}$, a composite beam-slab mode of collapse will take place (fig. 11b) when $\gamma_p = \frac{66}{21.1} = 3.13$, or less.

At the beginning of this paper it was pointed out that the main problem in a study of composite action was to determine the distribution of forces acting on the beams supporting the slabs. We have seen that in many instances the reaction on the beams may be uniform at the point of collapse but that if the most economical arrangement of reinforcement is desired then non-uniform beam loads are once more encountered. There is evidence that the additional interactive forces which would arise because of membrane stresses in the slab may not necessarily be of the same distribution at working conditions and at collapse. Fig. 16 shows the corresponding deflections of square

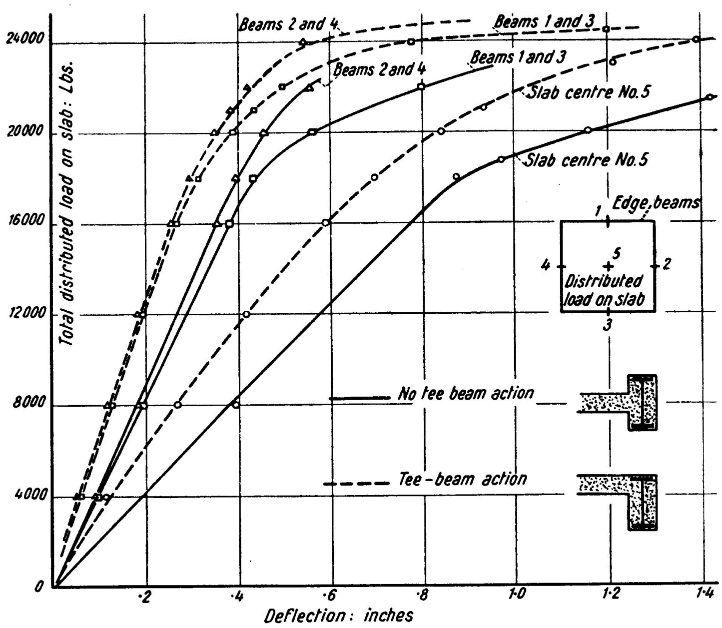

Fig. 16. Measured deflection of two square slabs on beams tested up to collapse, with and without Tee-beam action.

beam/slab systems, determined by laboratory tests, with and without T-beam action, the only difference being the raising of the centroid of the slab. Whereas there was some 30% reduction of beam deflections at working loads because of T-beam action the collapse load showed only a slight increase. Further evidence of membrane stresses has been noticed, since the crushing of the concrete along "fracture lines" often disappears near the centre of a slab and tensile cracks occur instead going right through the slab [3, 1]. It is clear that further developments in limit analysis are required to account for these effects.

Acknowledgements

This paper is published by permission of the Director of Building Research, and forms part of a general study of composite action in buildings. The author wishes to thank Dr. R. K. Livesley and Mr. J. Wright for supplying data obtained by the use of the Manchester University Electronic Computer.

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Summary

The stiffening effects of walls and floors in framed structures is reviewed in the light of recent developments in the theory of composite structures. The main problem appears to be the determination of the actual load distribution on the beams of a supporting frame, which distribution can vary considerably. In this connection there is an analogy between the stiffening effects of walls and floors. The combined behaviour of floors supported on beams is examined at working (elastic) conditions and at impending (plastic) collapse. It is shown that the loads on the beams at collapse may be decided by the placing of the reinforcement. In an effort to minimise the reinforcement by limit-analysis methods it is shown that with reinforced concrete floors an elastic design forms a surprisingly good starting point for the most economical collapse design.

Résumé

L'auteur étudie l'effet de renforcement combiné des parois murales et des plafonds et planchers dans les constructions à étages, en tenant compte de l'évolution récente dans la théorie des ouvrages composés. Le problème principal paraît résider dans la détermination de la répartition effective des charges sur les poutres des cadres, répartition qui peut varier dans des proportions considérables. Dans cet ordre d'idées, il existe une similitude entre l'effet de renforcement exercé par les parois murales et par les plafonds et planchers. L'auteur vérifie le comportement réciproque des poutres et des planchers et plafonds qu'elles supportent, dans les conditions de travail qui correspondent d'une part au domaine élastique et d'autre part au domaine plastique, au voisinage de la rupture; il constate que la charge de la poutre au moment de la rupture est conditionnée par la répartition des armatures. En procédant à des investigations aux limites en vue de réduire l'importance des armatures, l'auteur constate qu'avec des plafonds en béton armé, l'établissement du projet faisant intervenir le comportement élastique fournit un point de départ remarquablement bon pour la résolution du problème du point de vue de la rupture, dans les conditions les plus économiques.

Zusammenfassung

Die versteifende Wirkung von Wänden und Decken in Stockwerkbauten wird unter Berücksichtigung der letzten Entwicklungen in der Theorie zusammengesetzter Bauwerke untersucht. Das Hauptproblem scheint in der Bestimmung der tatsächlichen Lastverteilung auf die Träger der Stützrahmen zu liegen, die ganz beträchtlich wechseln kann. In diesem Zusammenhang liegt eine Ähnlichkeit zwischen der versteifenden Wirkung von Wänden und Decken vor. Das gegenseitige Verhalten von Trägern und darauf abgestützten Decken wird bei Arbeitsbedingungen im elastischen Bereich und bei bevorstehendem Bruch im plastischen Bereich geprüft. Es zeigt sich, daß die Belastung des Balkens beim Bruch durch die Verteilung der Bewehrung bestimmt ist. Bei bewehrten Betondecken ergab sich durch Grenzuntersuchungen, daß ein Entwurf nach elastischem Verhalten einen überraschend guten Ausgangspunkt für die wirtschaftlichste Lösung bezüglich Bruch bildet.

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