

# Between Scylla and Charybdis : Wolfgang Pauli and the transition from the old to the new quantum theory

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# BETWEEN SCYLLA AND CHARYBDIS

## WOLFGANG PAULI AND THE TRANSITION FROM THE *OLD* TO THE *NEW* QUANTUM THEORY

BY

**Karl von MEYENN\***

### INTRODUCTION

The astonishing success of Bohr's atomic theory in the first years after its inception was only short-lived. In the early twentieth more and more difficulties became known and the belief in the far-reaching validity of the new formalism was beginning to wane. The climax of this so called *crisis of the old quantum theory* was reached in the years 1923/24, but could be overcome very quickly. This was due in great measure to the singular opportunities offered to talented young physicists in the years just after *World War I*, when the newly created *Institute for Theoretical Physics* in Copenhagen took its leading role. The new quantum mechanics could be established in a very short time as a self-contained and independent discipline, that did not need anymore the justification by the other branches of theoretical physics. The famous phrase coined by HERTZ, "MAXWELL's theory is nothing else than the system of their equations," could now be applied as well to this new field.

The definite success in this search for a new theory was due mainly to a small group of younger physicists. Without being impeded by the ideals of classical physicists and their old fashioned way of thinking, they were able to abandon the traditional routes of thought and to pursue their own way.

In the center of this revolutionary circle we find, alongside with HEISENBERG, the only twenty years old PAULI, seen here in the picture 1.

PAULI was also the person, who in 1925, shortly after the breakthrough to the new theory, grasped the situation by citing the well known episode from the *Odyssey*, which I chose as the title of my talk. He called upon his new friend KRAMERS, to leave "the SCYLLA of the number mystic school of Munich and the CHARYBDIS of the reactionary, and with zealous excesses propagated assault from Copenhagen," and not to continue delaying the process of "recovery of the Copenhagen physics" started so well by HEISENBERG.

HEISENBERG belonged as well as PAULI to the selected circle of the so called *Wunderschüler* of the famous SOMMERFELD-School, who became – in spite of, or better just because of their youth, – the pioneers of the further developments.

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FIG. 1.

Wolfgang Pauli, ca. 1917, at the Döblinger Gymnasium in Vienna.

Both began their study in Munich after *World War I*, and both took a similar curriculum during their professional formation, as is shown in the diagramm 2.

After a solid basic education with SOMMERFELD they received a more mathematical training with BORN and HILBERT in Göttingen, as well as an initiation to the secrets of Bohr's atomic physics, which, after the War, was en vogue specially in the German speaking countries.

### **Copenhagen is becoming the *Capital of atomic physics***

As a citizen of a country not involved in World War I, BOHR was not inclined to participate in the general actions of boycott taken by most scientists against their colleagues living in the states of the former Central Powers. In April 1920 BOHR came to Berlin and again in June 1922 he gave his widely attended lectures on atomic physics in Göttingen, which considerably contributed to disseminate his ideas.

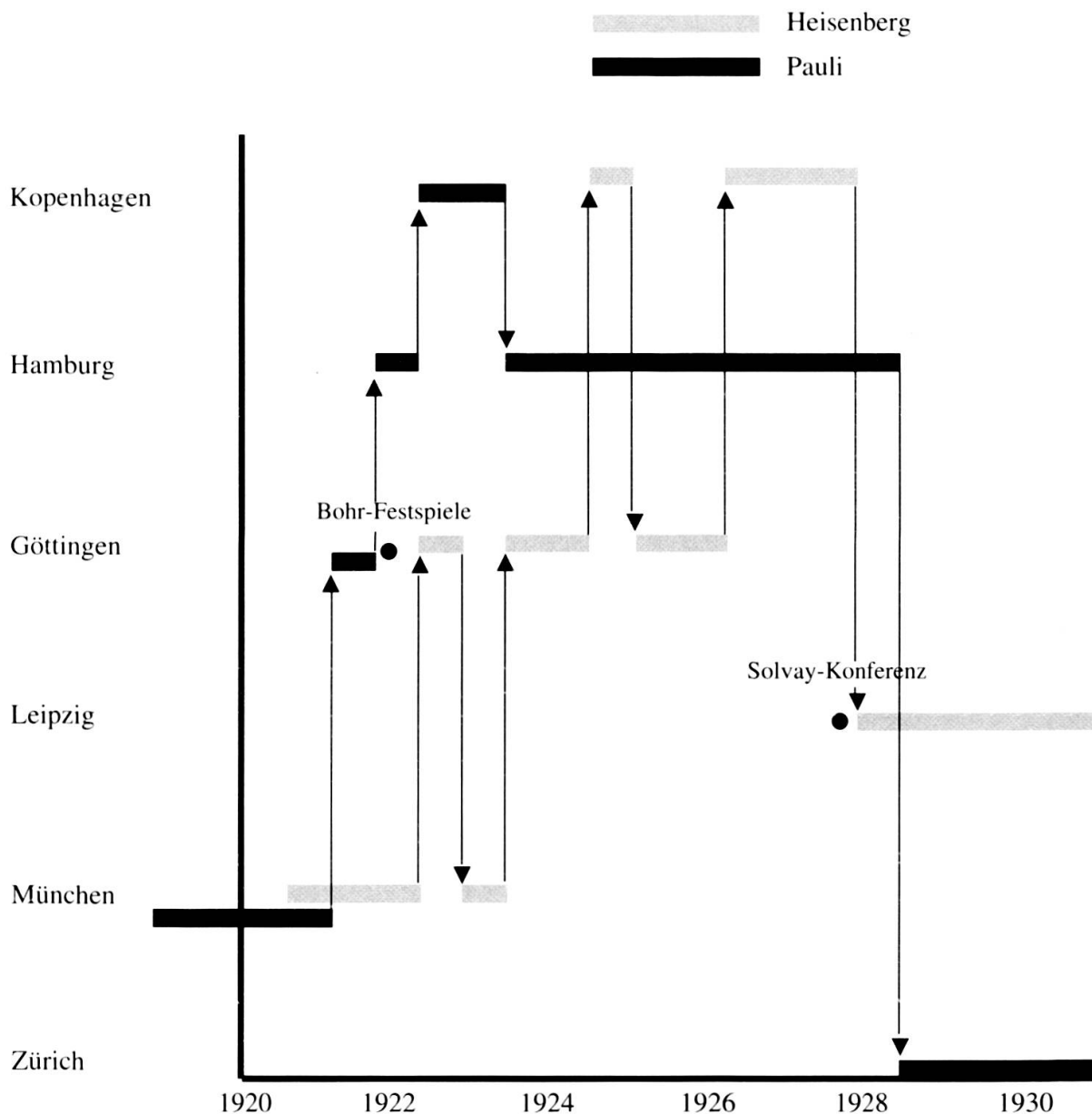


FIG. 2

Pauli and Heisenberg positions in the twenties

On this occasion he also became acquainted with HEISENBERG and PAULI. Both were invited to come to Copenhagen. This was the beginning of a lively scientific exchange with the *Institute for Theoretical Physics* created in 1921 by the Danish authorities specially for BOHR. Copenhagen soon became a kind of *Capital of atomic physics*, frequented by the most prominent quantum physicists from all over the world.

The manifold connections established between Copenhagen and the other centers of physics are shown in the diagram 3 drawn by PAULI's later assistant WEISSKOPF, who, then still learning, was observing attentively these events.

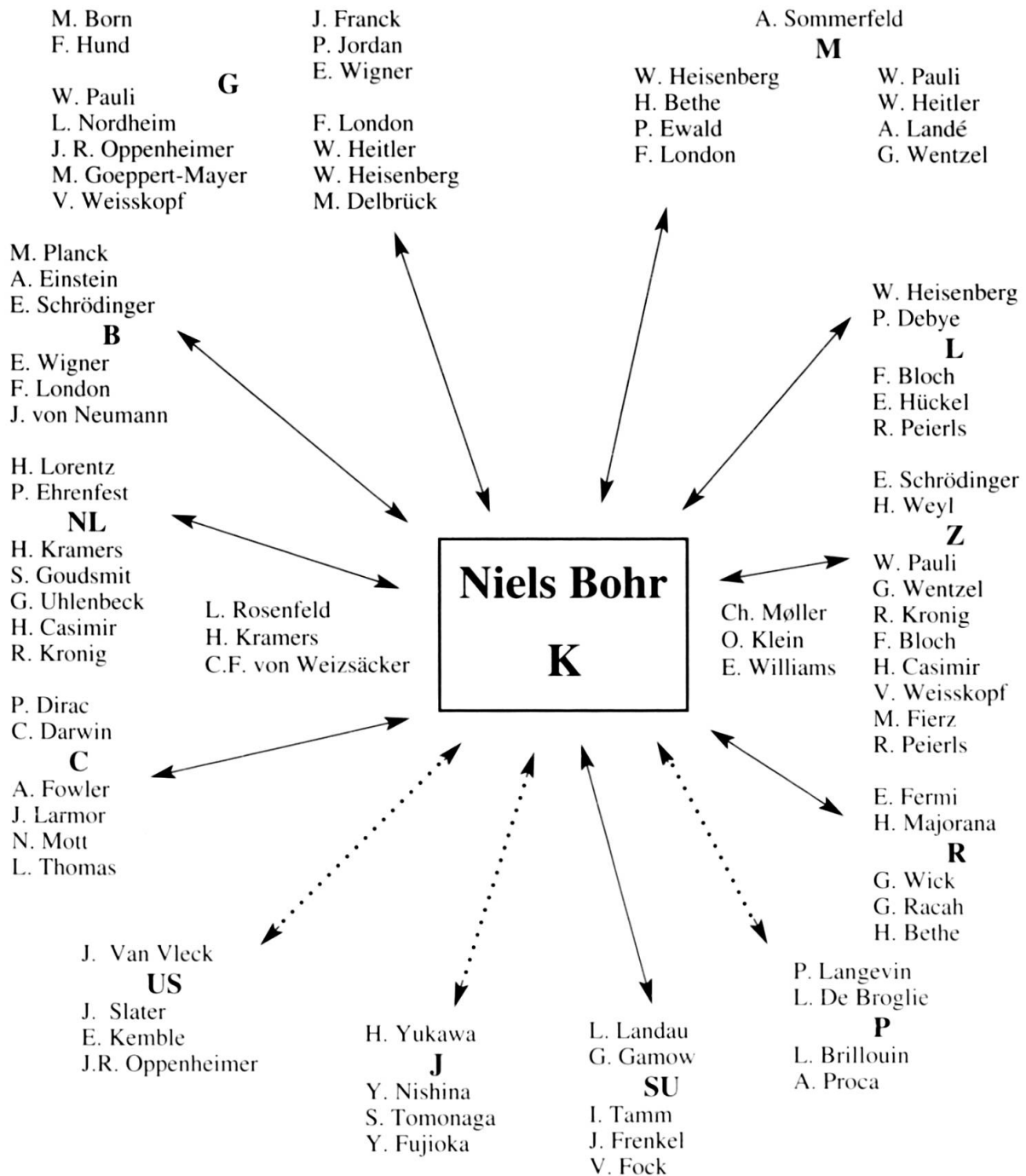


FIG. 3: Early centers of quantum physics.

*Schools of theoretical physics in the years of the foundation of OM*

*Abbreviations:* G - Göttingen, M - Munich, L - Leipzig, Z - Zürich, R - Rome, P - Paris, SU - USSR, J - Japan, US - USA, C - Cambridge (UK) - NL - Holland, B - Berlin, K - Copenhagen.

After the examination of the external circumstances, I will now describe the situation of the quantum theory at the beginning of the twenties. After BOHR's great *trilogy* of 1913 as an outline for a future quantum conception of the atomic world, the next step consisted in an extension of his method to systems with more than one degree of

freedom. SOMMERFELD and his two early pupils EPSTEIN and DEBYE succeeded explaining with the help of BOHR's theory the fine structure of Helium spectra observed by PASCHEN and the splitting of the spectral lines in electric and magnetic fields giving rise to the STARK- and the ZEEMAN-effects. LENARD's disciple KOSSEL, shortly after moving to Munich in 1913, supplied the essential idea about the shell distribution of electrons, which allowed the explanation of the X-ray spectra. All these advances were so overwhelming, that, in his paper submitted for publication on March 29 of 1916, EPSTEIN could speak "of a new, striking demonstration of the correctness of BOHR's atomic model," whose "force of demonstration could not be denied even by the most reserved specialist."

In spite of all these achievements the vulnerability of the new theory could not be overseen. BOHR's two *fundamental postulates* (the existence of *stationary* states und the *frequency condition*) rested only upon their success in the application to atomic processes.

The introduction of probabilities for the description of quantum transitions by EINSTEIN und their fruitful use in the derivation of line intensities with the help of the correspondence principle was accepted only as a temporary substitute for the still missing causal explanation. But there were also many instances, in which the theory provided completely wrong results. Examples of such failures were the non combining term systems of Helium, the existence of an anomalous ZEEMAN effect and the absence of an explanation of the numbers 2, 8, 18, 32, ... of the periods in the system of the chemical elements. A further law of nature was suspected to be hidden behind the amazing regularities of the Zeeman patterns disclosed in 1921 by SOMMERFELD's former student LANDÉ, who obtained a general *g-formula* encompassing nearly the whole abundant spectroscopic material. Because it was impossible to find an appropriate atomic model to explain LANDÉ's formula, SOMMERFELD and his followers began to speak – with allusion to similar recourses by KEPLER – of the number mystery of the ZEEMAN effects. The elucidation of this riddle played a decisive role in the development of quantum mechanics.

### **The Munich institute for number mystics and Sommerfeld's *Wunderschüler***

Already at the beginning of their study, PAULI and HEISENBERG were familiar with most of the foundations and methods of theoretical physics. So SOMMERFELD could immediately share with them also his current research interests. In accordance to PAULI's more analytical skills he delegated him the article about relativity he had promised to write for the mathematical encyclopaedia, whereas the unraveling of the ZEEMAN puzzle was left for HEISENBERG.

Already in school HEISENBERG had shown great skills in problem solving. This time again he showed up with a model, wick reproduced all the phenomena for the simpler case of doublet spectra. What has been impossible to SOMMERFELD's defunct colleague on the basis of a classical oscillator model, HEISENBERG now could explain by a simple mechanism of magnetic coupling between the inner *core* and the orbiting

external electron, that was responsible for the optical spectra. But the success of his *Rumpfmodel* was achieved only at the cost of some very serious violations of the current theoretical principles.

Just in time, STERN and GERLACH had performed their experiments about magnetic deflections of silver atoms leading to the proof of space quantization. During his study in Munich PAULI too had worked already on atomic magnetism and introduced, between other achievements, the *BOHR magneton* as a new atomic unit of magnetism. So he was considered as an expert to be consulted in the discussion of these experiments. As an early partisan of space quantization, PAULI tried also to convince the still sceptical STERN about the outcome of these experiments. This may be one of the reasons why the physicists in Munich were also the firsts to think of oriented electronic orbits ruled by space quantization.

This concept was also at the center of HEISENBERG's attempt. To explain the doublet structure of the alkali spectra, he used mutually inclined and magnetically interacting electronic orbits. But to obtain the right magnitude of the observed effects, he had to introduce also some deviations from the current theory, which – in spite of the general criticism it arose and the many confusions it produced – proved to be of central importance for the development of the atomic theory.

PAULI, who meanwhile had finished his studies in Munich with a dissertation about a quantum model of the  $H_2^+$ -ion and become BORN's assistant in Göttingen, was informed in 19. November 1921 by HEISENBERG about his success. With allusion to the necessary violations he had introduced, HEISENBERG opened his "lecture on atomic mystics of the anomalous ZEEMAN effects" with the provoking "*Motto*: The end justifies the means." Then he presented his own solution of the doublet riddle:

"They are build by a valence electron and a core. In the normal state (s term) each atom has the total impulse 1. This impulse is distributed (now comes the point!) homogeneously in time average between core and electron. That is, mean momentum  $1/2 + 1/2$ . In the excited state: The mean momentum of the core again  $1/2$ , and  $n - 1/2$  for the valence electron. Between both of them exists the magnetic interaction  $H_i$ ."

The orientations of the total angular momentum in an external magnetic field then resulted from space quantization, producing the observed Zeeman patterns by the magnetic interaction. Unfortunately all of PAULI's letters to HEISENBERG from this early period are lost, so that we can learn about his protests only indirectly by studying the letters to other physicists and especially from HEISENBERG's answers.

In view of the excellent experimental agreement, SOMMERFELD was, despite the inconsistent treatment of the angular momenta of core and valence electron and the use of half integral quantum numbers, in favor of a publication. His high opinion for his new disciple was boundless. In a letter to EPSTEIN, who had moved meanwhile to Pasadena, to MILLIKAN's "first class research institution", in 29 of June 1922 he explained:

"From HEISENBERG, who probably is the most gifted of all my pupils, including DEBYE and PAULI, I am expecting something tremendous. His ZEEMAN model en-



counters general objection, specially also by BOHR. But I find the success so enormous, that I postponed all my reservations when publishing. HEISENBERG is in the 4th semester and has 20 years, son of the Byzantine Professor from Munich.”

### **Beginning of the rejection of the pictorial models of the atom**

After his partial misfortune with the calculation of the hydrogen molecule ion PAULI developed together with BORN in Göttingen the perturbation method for the treatment of more complicated atomic systems. As the application of this formalism to the Helium problem again led to wrong results, PAULI lost definitely his confidence in the methods of the old quantum theory and began to criticize the pictorial models of his teacher SOMMERFELD:

“It is not possible to have any doubts about this failure,” he declared in June 1923, “and it seems to be one of the most important results of the last year, that the difficulties in the many-body problem of the atoms are of physical, and not of mathematical nature. (If BORN and HEISENBERG, for example, obtain wrong helium terms, the cause is certainly not the insufficiency of the approximation method.)”

In spite of this, BORN continued to calculate together with HEISENBERG, also “the most general model of excited helium in every detail”, before coming to the definite conclusion, that also here the energy becomes wrong. “The result seems to be very serious for our previous conceptions,” now concluded also HEISENBERG, and “it seems necessary to introduce completely new hypothesis – new quantum conditions or deviations in the mechanics. ... Resuming in short: it is a crying shame.”

BORN now also changed his tactics. He started to look for new possibilities of description appropriate to the quantum processes, which in every instance occur only between the stationary states. For the class of the so called conditionally periodic systems, this could be determined with help of the BOHR-SOMMERFELD quantization rules. However most atomic systems were of a non periodic nature and could not be treated by this procedure. While HEISENBERG was following BORN’s research program, PAULI now began to endorse BOHR’s correspondence-like procedure, trying to solve the intricate ZEEMAN problem.

### **“I am learning Bohr’s physics”**

In October 1922, when taking up his new position in Copenhagen, PAULI wrote to Professor LADENBURG then in the university of Breslau: “I am learning here Bohr’s physics. This is of a complete different order of magnitude than all the remaining physics.” Visiting his parents in Vienna during the Christmas holidays, he also could inform his ambitious colleagues there about the new status of physics in Copenhagen, who had come meanwhile into the reputation of being the center of atomic physics. These news were quickly transmitted by HALPERN to the Polish physicist RUBINOWICZ:

“Well, the BOHR-KRAMERS parahelium calculation is finished; the ionization potential of 3,9 Volt is wrong. *Ergo*: The model is right, the mechanics wrong, because the mechanics already breaks down in elastic electron collisions. Nothing seems to be more



natural than to suppose with BOHR, that for systems with more electrons the mechanics also fails in the same quantum orbits. PAULI completely shares this opinion.”

In view of this general mood of crisis which arose between the quantum physicists, the examination of the anomalous ZEEMAN problem now also was taken up in Copenhagen. Because the contradictions against experience seemed here to be the most obvious, one expected rightly, that this research may supply new insights. Already in December 1922 came the news, that LANDÉ had found “a complete theory of all the multiplets and their Zeeman effects, even with half quantum numbers.” In fact, LANDÉ’s *g-formula*, obtained on the basis of a vector model of the atom, allowed the exact calculation of the magnetic energy and the resulting ZEEMAN splittings. But the quantum theoretical foundation of this formula was unclear and represented a great challenge for the theorists.

BOHR’s strongest objection was LANDÉ’s use of half quantum numbers, which was against the principles of the then existing quantum theory. His opposition remained, also after LANDÉ could show, that their origin was an anomaly of the magnetic moments discovered in 1919 by the Swiss physicist BECK, when redoing the experiment of EINSTEIN and DE HAAS about the gyromagnetic effect of the electrons. Now the anomalous behaviour of the ZEEMAN phenomenon could be seen also as a violation of the laws of classical electrodynamics.

The discovery of the real causes succeeded only step by step, because they were hidden in the use of a wrong atomic model and the limited validity of the old quantum theory. After some desperate attempts to construct a model with whole quantum numbers, BOHR gave up. His failure swiftly was communicated to EPSTEIN in a letter from 5 of May 1923 by SOMMERFELD:

“With respect to the ZEEMAN effect BOHR has capitulated,” he wrote, and also his proposal for the spectra of helium remain “in the darkness, because the resulting terms are incorrect.”

Because of all these misfortunes, PAULI felt temporarily in a deep depression, as he later recalled in his Nobel lecture. But already in March 1923 a first recovery took place. Using a kind of *substitute* model, that attributed the anomalous double magnetism to the core only, PAULI could derive LANDÉ’s *g-formula* also for the case of strong fields. In his publication he cautiously avoided giving any hint of the models he had used in obtaining these results. From that time on most quantum physicists tried to obtain a model-free formulation of the quantum laws.

In summer 1923, after having build his own opinion about the virtues and the faults of BOHR’s atomic theory, PAULI wrote:

“The weakness of the theory is, that it has no explanation for the values 2, 8, 18, 32, ... of the length of the periods, ... and, above all, that, – in view of the failure of classical mechanics also in the stationary states with more than one electron. – it can not supply any sufficient basis for the quantitative calculation of the spectra of such systems.”

He reports further, that BOHR calls the quantum theory of the periodic systems now as “classical quantum theory.” The people in Germany “should not believe, that Bohr

still has any difficult hidden considerations.” EPSTEIN, visiting Europe in August 1923 “to take the pulse of theoretical physics,” estimated then, that “quantum theory has come to a stillstand. Peoples here know nothing, as in our America.”

### Beginning of the discretization of atomic physics

In his effort to explain the anomalous ZEEMAN effect PAULI found support again in HEISENBERG. In October 1923 he received HEISENBERG’s *new theory* from Göttingen. This work represented a still more radical renunciation to the model conceptions and constituted a first step in the direction of BORN’s program, “to discretize atomic physics”. HEISENBERG theory rested upon the following principles:

1. Model conceptions are in principle only of symbolic value; they are the classical analogue to the discrete quantum theory.

2. Until now it was usual, to go from the model conceptions to the real radiation frequencies by taking the energies  $H$  from the symbols and to obtain the quantum frequencies  $\nu_{qu}$  by building the difference (instead of differentiation)  $\nu_{qu} = \Delta H$ .

HEISENBERG illustrated his conception also with an example based on PAULI’s *substitute* and LANDÉ’s *vector model*. As PAULI commented, HEISENBERG had in fact “seriously played with the idea, to associate various mechanical orbits with one given state,” but the whole question seemed to him still too formal and without any new physical content. In February of the next year he modified this judgement. HEISENBERG had just done the right thing in casting doubt on applicability of the orbit concept to the electrons in their stationary states. HEISENBERG considered his proposal also as “a profound modification of our quantum theoretical conceptions.” He published his results only in the summer of 1924, after having asked BOHR for his “papal blessings”.

PAULI had meanwhile returned in autumn to his home university of Hamburg. He continued to regard the “elucidation of the half quantum numbers and the failing of LARMOR’s theorem” as the most important problem in quantum theory. During the Christmas vacation he decided to visit Heisenberg in Munich to be able to scold him properly. However, after meeting him again in February 1924 during a meeting in Braunschweig, PAULI was very impressed and now he began to consider HEISENBERG as somebody “very outstanding, if not a genius” who will advance the science.

The expected advances came very soon. Already in October 1923, BORN had communicated triumphantly to his American colleague VAN VLECK, “whereas you are concerned with the correspondence of high quantum numbers, I am searching instead for strong laws for arbitrary quantum numbers.” Later, in the winter of 1925/26, when BORN was invited to give lectures on atomic theory at the MIT in Cambridge, Mass., he was also the first European who reported about the “results in quantum theory made by my friends, Dr. HEISENBERG and Dr. JORDAN, and myself.”

The Copenhagen physicists, on the other hand, who, since spring 1924, were struggling with the already mentioned radiation theory of BOHR, KRAMERS and SLATER, finally gave up when the experiments of BOTHE and GEIGER became known. But the most important advances occurred with the disentanglement of the Zeeman effect.

### Solving of the Zeeman riddle

LANDÉ and MILLIKAN had discovered in 1924 a very remarkable result. The optical doublets of the alkalis, explained up to now only *magnetically*, could be explained also *relativistically*, if the differently oriented orbits were substituted by orbits of different excentricity. PAULI quickly concluded from this fact, that the origin of the optical doublets "cannot be explained by an interaction of the (differently oriented) atomic core." HEISENBERG instead used the new insight to develop an additional formalism of the Zeeman effects.

More important was PAULI's next contribution from November 1924. He could show, that the ZEEMAN splittings, if really produced by the core, should reveal a relativistic effect caused by the velocity dependence of the mass of the core electrons. As no such effect existed. PAULI could conclude, that the magneto-mechanical anomaly was not related to the core, but only to the valence electron. In a letter to LANDÉ from 24 of November PAULI summed up his result as follows: "In the alkalis the complex structure and the anomalous ZEEMAN effect is produced by the valence electron alone. There can be no question of a contribution of the noble gas-like core. The valence electron is able to run in a mysterious non-mechanical manner in two states (of same  $k$ ) with different impulses."

This was a fundamental insight because it explained a great deal of until then incomprehensible facts. PAULI himself found now also the explanation for the closure of the inner electronic groups searched in vain by BOHR. Whereas BOHR had tried to explain this property in the framework of his correspondence principle as a sort of resonance phenomena between electronic shells, PAULI could now enunciate his exclusion principle: The presence of more than one electron in a state fixed by four quantum numbers is not allowed. As simple as this principle sounds, his exact justification required in those days an extensive and careful examination of the existing spectroscopic material, which mostly were accumulated in the Physical Institute of the University of Tübingen. After his visit to this Institute PAULI submitted his now famous paper in January 1925. He was especially proud, that he was right in his claim against BOHR, "that the correspondence principle in truth had nothing to do with the problem of the closure of the electron groups in the atom."

Once again PAULI was cautious in not attributing any physical meaning, as angular momentum, to the fourth degree of freedom. This was completely in accordance with BOHR's conception of *unmechanical* constraint (Zwang), introduced in 1923 to confer an additional possibility of orientation to the core.

Against such a mechanical interpretation, one could oppose not only serious theoretical arguments, as for example an electron spinning with a velocity faster than the light or an additional factor of two in the energy levels later explained by THOMAS, but also the then general attitude of the quantum theorists hostile to any kind of pictorial representation in their considerations. This was the reason why the spin hypothesis was not proposed by PAULI, but in November 1925 by EHRENFEST's less experienced pupils UHLENBECK and GOUDSMIT.

HEISENBERG justified the expectations PAULI had placed up on him. During the last stage of our story, which began in early 1925 and ended with the formulation of matrix mechanics in June of the same year. Spending a semester in the Winter 1924/25 in Copenhagen HEISENBERG became acquainted with BOHR's correspondence principle. Together with BOHR's most intimate collaborator KRAMERS he developed the quantum theory of dispersion. This collaboration stood at the beginning of a very significant evolution, which questioned the reality of the atomic models with their orbits and frequencies of revolution and proposed to use instead only observable quantities. The new theory based on the concept of virtual oscillators and the use of difference magnitudes as requested by BORN also was the first theory built on the concept of atomic entities performing quantum transitions between stationary states. The leading idea was the correspondence principle, which yielded a connection between the actual behaviour of an atomic system and the behavior of a system, "as expected according to classical electron theory and his structure."

Finally, HEISENBERG tried again to advance the still unsolved ZEEMAN problem. His first attempt was to solve the problem of non mechanical constraints by formal reasoning. For this purpose he brought together the two conceptions mentioned before of magnetic and relativistic coupling in one unified formalism that he considered only as provisional. He also disregarded PAULI's proof of the non existence of the angular momentum of the core, because he could see no reason, why just the "poor" 2 k electrons had to bear all the magnetism. The definite understanding came with the spin hypothesis, accepted by the quantum physicists only in March 1926 after a hard struggle. Just when Heisenberg was returning from Copenhagen the beautiful experiments of BOTHE and GEIGER became known, so that finally the unfortunate paper by BOHR, KRAMERS and SLATER, called an "obstacle to progress" by PAULI, could be laid *ad acta*.

### **From the observables to Heisenberg's *Hexeneinmaleins***

After HEISENBERG returned at the end of April 1925 to Göttingen, he had brought with him the fundamental pieces of knowledge, necessary to "fabricate" quantum mechanics. In his work with KRAMERS he had learned, that the states of atomic electrons can be described by virtual oscillators and their Fourier representations. According to the same procedure he hoped to obtain the quantum mechanics of the hydrogen atom. As he did not succeed, he concentrated his efforts on the anharmonic oscillator. In accordance with the new strategy he tried to use only observable quantities. This meant, that he had to avoid concepts like space coordinates and the frequencies of revolving electrons and had to use instead quantities like the observed spectral frequencies and line intensities. In the context of his oscillator model that meant, that he had in particular to use only the transition frequencies and the products built by the amplitudes of two stationary states of his oscillators.

In June HEISENBERG mentions already a *new formalism* he had discussed with PAULI, in spite of the circumstances that all "is still unclear" to him. He then recognized that the different Fourier components describing any movement can be combined in

such a way, that they really correspond to two stationary states at one time only. After probing this insight in the case of his simple oscillator, he began to understand the method of translation: By comparing the classical expression for the orbital frequencies  $\nu(n, \alpha) = \alpha \frac{1}{h} \frac{dW}{dn}$  with the corresponding quantum frequency condition

$$\nu(n, n - \alpha) = \frac{1}{h} \{ W(n) - W(n - \alpha) \}$$

he obtained the quantum analogue for the amplitudes  $A_\alpha(n) = e^{i\omega(n)\alpha t}$ , considered as complex vectors,  $A(n, n - \alpha) = e^{i\omega(n, n - \alpha)t}$ . With that he obtained already the *quantum representative* of the classical position function. It was then easy to him to find the law of composition for these magnitudes, which were the matrices already known to the mathematicians. After probing his new rules in the case of the oscillator and rotator, he was confident to have found the right formalism. His procedure could now be used to translate every classical expression built by position and momentum variables into quantum language. The proof for the case of the hydrogen atom, which traditionally constituted a kind of quantum theoretical showpiece, was supplied by PAULI only half a year later, because he was working during all this time on his famous article on quantum theory for Springer's Handbook.

To escape from the "flood of erudition of the Göttingen school" and the "imperialism" of the Copenhagen correspondence, the Scylla and Charybdis in PAULI's wording, HEISENBERG retired in the second week of June to the island of Helgoland. He ordered there his ideas before giving the new mechanics its definite form. On his travel back to Göttingen he visited PAULI in Hamburg, who welcomed "with cheer" HEISENBERG's new ideas. Then he wrote the paper, which was submitted by BORN for publication in July 29.

HEISENBERG's paper constituted only an albeit decisive step towards quantum mechanics, which soon was developed to a full blown theory by the common effort of many physicists. In spite of its still unfamiliar matrix formalism, described also as "magic formula" by EINSTEIN, the new theory with its corresponding generalizations became very soon generally accepted as the basis of description of all microphysical phenomena.

Wave mechanics, invented with half a year delay and considered initially as a serious concurrent of HEISENBERG's theory, soon was recognized by PAULI and the same SCHRÖDINGER, as one of the different equivalent representations of the same theory. This theory, together with all its other variants deduced in accordance with JORDAN's and DIRAC's transformation theory, contributed to a deeper comprehension of the new formalism. The still ongoing debate about the interpretation of the quantum theory reveals, how strange these quantum laws are, and what imagination and analytical skills of the young physicists were necessary for their disclosure.