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QUANTUM KINETICS: AN EXTENSION OF QUANTUM STRUCTURE BEYOND MECHANICS

BY

Paul SCHEURER [1]

ABSTRACT

Quantum Kinetics provides an extension of the elementary quantum structure beyond mechanics. This theory is based upon a sharp distinction between the mathematical structure of differentiable manifold (DM-structure) and the structure of physical dimension, or *megethos* in the terminology of Eudoxus (μ-structure).

The DM-structure allows for the construction of a pure geometric quantum algebra (GQA). The GQA of the parameter manifold shows the necessity of coding the physical quantities in terms of operators acting upon the state function and of proceeding to an eigenvalue problem in order to assign numbers to these operators. Then the μ-structure enables the endowment of physical quantities with various megethe according to the choice of the recalibration of the unit by a universal constant so as to obtain a relevant quantum. The conjugation of pairs of physical quantities is exposed by the μ-inversion and coded by the notion of inverse quantities of power K. Thus the Boltzmann and Schrödinger equations are obtained in structural analogy with the relation between information and probability. Some other connections between differentiability and both quantum indeterminism and probability are considered. A discursive representation of the flow of the now as an objective feature of time is given in exact structural analogy with the decrease of temperature in the Big Bang.

This study originated from a typical multidisciplinary (epistemological, historical and critical) inquiry into the representations of motion [1] and, more generally, of change which occurred during the five millenia separating the birth of astronomy in Sumer and Babylon from the present gauge theories of supergravity and superstrings. A convenient label describing this process could be: "From Sumer to Sugra".

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Three centuries after Newton's *Principia* (1687), we still describe motion in terms of differential equations. Thus, in Quantum Kinetics, two structures play an essential role: 1) the mathematical structure of a differentiable manifold (DM-structure); 2) the physical megethos-structure (μ-structure) which assigns an appropriate physical dimension to the variables. Quantum Kinetics asserts two fundamental propositions:

Proposition I: "Given any differentiable manifold (of natural coordinates x_i) a Geometric Quantum Algebra (GQA) can be defined by the commutators:

$$(1) \qquad [\partial/\partial x_i, x_j] = \delta_{ij}; \quad [\partial/\partial x_i, \partial/\partial x_j] = 0; \quad [x_i, x_j] = 0 \,."$$

In fact, the third commutator is trivial for the coordinate functions while the second expresses Schwarz equality of the mixed second derivatives. The first commutator is strongly tied to the geometric duality of tangent space and cotangent space: indeed, *given any differentiable manifold with coordinates* x_i, *the tangent vectors* $\partial/\partial x_i$ and the cotangent vectors dx_j are defined, and both manifest a duality by the contraction $<\partial/\partial x_i, dx_j> \; = \delta_{ij}$. It is introduced here for this merely mathematical reason. But it is also clear that it completes the structural analogy with Poisson's classical and Dirac's quantum brackets [2].

Proposition II: "The μ-structure allows for the use of the *megethos-inversion* (μ-inversion) in GQA:

$$(2) \qquad [K''\partial/\partial x_i, K'x_j] = K\,\delta_{ij} \,."$$

The inverses K' and K'' of power K (i.e. $K = K'\,K''$), where K is a universal constant, are each endowed with a megethos such that both are conjugated in the megethos of K (μ-inversion).

Historically [3], the essentials of Proposition I and only a very special case of Proposition II (no reference is made to the concepts of μ-structure and μ-inversion) are to be found in Schrödinger's 1926 paper [4] on the equivalence of his Wave Mechanics with Heisenberg's Matrix Mechanics. Schrödinger introduced this construction merely as an auxiliary means to demonstrate this equivalence, only to drop it into oblivion upon the publication of Dirac's famous paper on the transformation theory [5]. Instead of the conjugate coordinates p_l and q_l of phase space, he maintains the coordinates q_l of the configuration space and replaces the p_l by means of differential operators. This he does, first by the operators $\partial/\partial q_l$, thereby obtaining the essentials of GQA, and, furthermore, more generally, by the operators $K\partial/\partial q_l$, where K is a universal constant, thereby obtaining $[K\partial/\partial q_l, q_l] = K$. Finally he assigns to K the value $K = h/2\pi i$ in order to recover the quantum conditions (of mechanics, as h is Planck's quantum of action).

The critique of this work reveals a great many structural possibilities that were completely overlooked by Schrödinger (and others as well). To begin with, although he wrote down the essentials of GQA, *he did not notice that the quantum structure* *is determined purely mathematically*, i.e. that the megethos and the calibration of the quantum are entirely irrelevant. Moreover, when touching upon the μ-structure by his use of the universal constant $K = h/2\pi i$, he failed to see that 1) it is possible to select another universal constant endowed with another megethos (e.g. k, Boltzmann's constant of entropy; $c^2/2\pi i$ in kinematics; $e^2/2\pi i$ in an electrodynamics of currents; etc.), 2) it is also possible to take K as a real constant. Finally, the transformation of the identity into the multiplication by K, i.e. the recalibration of the unit, in fact yields the equation $K[\partial/\partial q_l, q_l] = K \cdot 1$, and thus, $[\partial/\partial q_l, Kq_l]$ is a commutator as good as that retained by Schrödinger. The duality which appears here is due tot the μ-inversion and becomes more explicit by using the decomposition of K into the pair (K', K''): $[K''\partial/\partial q_l, K'q_l] = [K'\partial/\partial q_l, K''q_l] = K \cdot 1$.

By such an historical critical analysis one is led almost immediately to Propositions I and II which characterize Quantum Kinetics.

Now, two out of the many a priori possible realizations of this scheme deserve special consideration, viz. 1) Schrödinger's Wave Mechanics ($K = h/2\pi i$), and 2) Boltzmann's Statistical Mechanics ($K = k$) for the light they shed upon the distinction between mechanical reversible motion and thermodynamic irreversible evolution. Both theories are two *physically different* realizations of the *same structural* scheme, in spite of the fact that "the starting point, conception, method and mathematical apparatus appear to be fundamentally different for each theory" [4].

In the following it will be shown how Quantum Kinetics provides the appropriate structural scheme for such equivalence, and more generally, constitutes a suitable frame for a broader examination of the relationships between differentiability and both quantum indeterminism and probability.

Let us begin with a discussion of Proposition I.

As usual in a DM-structure, and completely within Newton's legacy, Dynamics is given by a tangent vector field $\hat{X}$, expressing the dynamical change of a physical system with respect to a change parameter ξ, a real number; by letting $\hat{X}$ operate on the state function $\phi(\xi)$:

$$\hat{X}\phi(\xi) = d\phi(\xi)/d\xi \tag{3}$$

the solution of which, under well known determinate conditions, is the exponential mapping

$$\phi(\xi) = \exp(\xi\hat{X})\phi(o)\,. \tag{4}$$

As is well known, this is fully deterministic.

Now, consider the special case where the number of dimensions of the manifold is reduced to one (the discussion can be restricted to the parameter ξ itself). In this case, only the first commutator of GQA $[d/d\xi, \xi] = 1$ is non-trivial. This is completely analogous to the Born-Wiener commutator [6] and in fact amounts to Leibniz' rule of the derivation of the product of two functions written in terms of operators [7]. Such *transition from functions to operators* in the representation of physical quantities clearly indicates that *an eigenvalue problem of the state function comes to the fore*, because this is a very appropriate way to assign a number to an operator. The concept of state has played an important role at least three times in the history of modern physics. First, in the 17th century, the shift of the status of *motion from change to state* initiated the development of modern Kinematics. Second, in the 19th century, thermodynamics was forced to conceive the physical quantities as *state functions*. Third, in this century, the recognition of the necessity of a *measurement* to assign a value to a physical quantity led to the generalization of the eigenvalue problem (clearly perceived by Schrödinger [8]). The classical situation: *the quantity A has value a*, changed into: *A has value a in the definite state ϕ_a* (which becomes an axiom in Quantum Mechanics): $A\phi_a = a\phi_a$, whereby A becomes an *operator* $\hat{A}$ acting upon the *state eigenfunction* $\phi_a \cdot a$ must be endowed with the same megethos as A (concept of *μ-number*).

Thus, apart from the transformation of Leibniz' rule (and thereby Quantum Kinetics appears as an extension of the quantum structure, albeit only for the restricted case of elementary Quantum Mechanics (of P's and Q's)), the exponential mapping is converted into a plane exponential function; (3) becomes

(5) $\hat{X}\phi(\xi) = X\phi(\xi)$, where X is a fixed number, (real or pure imaginary)

and (4) becomes

$$\phi(\xi) = \phi_o \exp(X\xi)\,. \tag{6}$$

It is seen that the relation of the parameter ξ to the state ϕ becomes structurally identical to the relation of information I to probability P, i.e.:

$$\xi = X^{-1}\log\phi \text{ and } I = -I_o \log P \text{ (up to a constant)}\,. \tag{7}$$

Indeed, the first relation generalizes the second; one has $0 \leqslant P \leqslant 1$, but, if X is real, then $\phi(\xi) \in \mathbf{R}^+$, and if X is imaginary, $\phi(\xi) \in \mathbf{C}$.

The core of the structural equivalence between Boltzmann's and Schrödinger's theories is now meaningfully expressed by the simple purely mathematical *eigenvalue problem for the unit tangent vector* (of norm $+1$ (Euclidean) or -1 (Minkowskian)):

$$d\phi(\xi)/d\xi = \varepsilon\phi(\xi) \qquad \begin{array}{ll} \varepsilon = \pm 1 \text{ (Euclidean)} & \\ \varepsilon = \pm i \text{ (Minkowskian)} & \text{i.e.} \quad \varepsilon^4 = 1\,. \end{array} \tag{8}$$

Hence, the solution:

$$\phi(\xi) = \phi_o \exp(\varepsilon\xi) \quad \text{or} \quad \xi = \varepsilon^{-1}\log\phi + \text{constant} \tag{9}$$

provides an explicit demarcation between reversible motion and irreversible evolution.

Let us introduce the μ-structure and Proposition II (*A* tilda is written over the variables to indicate that they are endowed with a megethos). The discussion essentially bears upon the various ways in which the triplet $(K; K', K'')$ may significantly be realized.

First, with respect to the choice of K, the various recalibrations of the unit into various physical quanta transform the second relation of (9) into the *Boltzmann-Schrödinger Equation* (BSE):

(10) $$(\mathrm{BSE})\ \tilde{S} = \tilde{K}\xi = \varepsilon^{-1}\tilde{K}\log\phi\,.$$

For instance, $\tilde{S} = \tilde{h}\xi$ of $\tilde{k}\xi$ or $\tilde{c}^2\xi$, or $\tilde{e}^2\xi$, etc., which means that $\hat{S}$ is a kind of generalized action. It is easy to see that both the Boltzmann and the Schrödinger equations are two of the possible realizations of this scheme. Here follows a list of the most relevant ones.

information (strict)	$I_o \log P\ (o \leqslant P \leqslant 1)$
information (large)	$\xi = \varepsilon^{-1}\log\phi\ (\varepsilon^4 = 1;\ \phi$ resp. real or complex)
time (kinematics)	$\tau = +i\,\tau_o \log\phi\ (\phi$: complex)
action (mechanics)	$S_a = -ih\log\psi\ (\psi$: amplitude of probability, $\psi\in\mathbf{C}$)
entropy (thermodynamics)	$S_e = k\log W\ (W$: complexion, $W\in\mathbf{R}^+$).

When the different pairings (K', K'') for a given K are introduced, (10) is transformed more generally into

(11) $$(\mathrm{BSE})\ \hat{S} = \tilde{K}\xi = \tilde{K}''(\tilde{K}'\xi) = \tilde{X}\tilde{\xi} = \varepsilon^{-1}\tilde{K}\log\phi\,.$$

As mentioned before, it is always possible to consider dually $\tilde{S} = \tilde{K}'(\tilde{K}''\xi)$.

Furthermore, it is possible to introduce different pairings: $(\tilde{K}'_1, \tilde{K}''_1)$, $(\tilde{K}'_2, \tilde{K}''_2)$, etc.

Some examples follow:

For $\tilde{K} = h$: $h = E_o\tau_o = ET = p\lambda$ (Einstein-de Broglie's relations), where resp. $\tilde{\xi} = \tau_o\xi = \tau$ or $T\xi = t$ or $\lambda\xi = x$;

for $\tilde{K} = k$: $k = -E\theta$ ($\theta = -1$/absolute temperature) where $\tilde{\xi} = \theta\xi$;

for $\tilde{K} = c^2$: $c^2 = vV$ (Brillouin's relation) where $\tilde{\xi} = V\xi = x/T$;

for $\tilde{K} = e^2$: $e^2 = (ev/c)(eV/c)$ where $\tilde{\xi} = eV\xi/c = ex/cT$; etc.

Ultimately, all these simple facts entail, almost trivially, that the following equations obtain:

(12) The case of reversible motion:

$$\begin{aligned}\exp(-i\xi) &= \exp(-i\tau/\tau_o) = \exp[(-i\tau/\tau_o)(ch^2\theta - sh^2\theta)]\\ &= \exp[i(x/\lambda - t/T)] \qquad \text{wave (as } x = c\tau sh\theta,\\ &\qquad\qquad \lambda = c\tau_o/sh\theta \text{ and } t = \tau ch\theta,\ T = \tau_o/ch\theta)\\ &= \exp[i(px - Et)/h] \qquad \text{Schrödinger}\end{aligned}$$

(12b) The case of irreversible motion:

$$\exp(-\xi) = \exp(-\theta/\theta_o) = \exp(-E_o\theta/E_o\theta_o)$$
$$= \exp(-S_e/k) \qquad \text{Boltzmann.}$$

This is but a first relation between differentiability and probability, a topic which has a far greater extension. For instance, there exists a relation between the statistical mean value and a mere derivation, under the following assumptions:

1) There exist several different eigenvalues $\tilde{X}i$ for the same value of $\tilde{\xi}$ (e.g. several energy levels at the same time or temperature);

2) Boltzmann's idea of genius of the *Zustandssumme* can be applied.

Let us introduce the function

$$Z(\xi) = \Sigma_i\phi_{oi}\exp(\varepsilon\tilde{X}_i\tilde{\xi}/\tilde{K})\,. \tag{13}$$

Hence, the simple process of derivation of Z with respect to $\tilde{\xi}$, more ore less directly yields $<\tilde{X}>$. (The second derivation would similarly yield the mean standard deviation).

For the sake of convenience, the order of presentation is reversed:

a) The case of irreversible evolution:

$\varepsilon = \pm 1$, all $\phi i \in \mathbf{R}^+$, all $\phi io = \phi o > 0$; thus Z becomes the *partition function* (Boltzmann's statistics) as $\phi i/Z \equiv Pi$ is a true *probability*. Thus:

$$\tilde{K}/Z\, dZ/d\tilde{\xi} = \Sigma_i\tilde{X}_i\phi_i/Z = \Sigma_i\tilde{X}_iP_i = <\tilde{X}> \tag{14a}$$
$$\text{where} \quad \tilde{\xi} = -1/T, \quad \tilde{X}_i = E_i, \quad \hat{X} = \hat{H}, \quad \tilde{K} = k$$

b) The case of reversible motion:

$\varepsilon = \pm i$, all $\phi_i \in \mathbf{C}$, ϕio various complex coefficients; thus Z transforms into an orthonormal *wave function* Ψ as $\phi i/Z$ is only an *amplitude of probability*. Thus, by the same process, one obtains the axiom of the mean value in Schrödinger's picture of Quantum Mechanics:

$$\bar{\Psi}^{-1}\bar{\Psi}(i\hbar/\Psi)d\Psi/dt = \Sigma_iE_i\phi_i\phi_i/\bar{\Psi}\Psi \equiv <\Psi|\hat{H}|\Psi> = <\tilde{H}> \tag{14b}$$
$$\text{where} \quad \tilde{\xi} = t, \quad \tilde{X}_i = E_i, \quad \hat{X} = \hat{H}, \quad \tilde{K} = h/2\pi = \hbar$$

Three observations can be made. First, it is amazing that Boltzmann's Statistical Mechanics and Schrödinger's Wave Mechanics in fact reveal themselves as two physically different realizations of the same structural scheme. It is also amazing that the core of such scheme viz. the reduction to the elementary eigenvalue problem of the unit tangent vector had not been perceived before. This speaks for the novelty of Quantum Kinetics as a fundamental reinterpretation of the quantum structure.

Secondly, although a linear theory, Quantum Kinetics makes our actual representations of change and motion more coherent. The elementary quantum structure is expanded to all differentiable manifolds. The specification of this structure enables a clarification of the relation between differentiability and the concepts of determinism, quantum indeterminism and probability.

Thirdly, evidently further research is needed in at least two different directions. Quantum Kinetics must be extended to non-linearity, and must integrate more mathematical structure, e.g. such as that of Lie groups en semigroups in the direction indicated by Prigogine and his teams in Brussels and Austin. More profoundly, Quantum Kinetics could be replaced by a more abstract theory repudiating differentiability. This would entail a fundamental change in the interpretation of our concepts of state, change, motion and time.

In conclusion, let us briefly consider two features, the first one structural, the second one discursive in nature.

First, in the spirit of footnote 30 of my paper *Arch. Sc. Genève 35* (1982) p. 197-216, and in accordance with the phenomenon of the Boltzmann-Schrödinger equation, it seems increasingly meaningful to consider the complex action $S = S_e + iS_a$ as an analytic function of the complex time $t = \theta + it$. The Cauchy-Riemann conditions yield:

a) $\partial S_e/\partial\theta = \partial S_a/\partial t = -E$ (energy);

b) $\partial S_e/\partial t = -\partial S_a/\partial\theta \geqslant 0.$

This second condition is remarkable because to the second law of thermodynamics there would correspond a law of decreasing action with decreasing absolute temperature T. This has surely to do with the phenomenon of the expansion of the Universe. But this calls for further investigation.

The second remark concerns the *discursive analogy* between the *expansion of the Big Bang* and the objective phenomenon of the *flow of the now* which can be inferred from the *structural analogy* between the inverse absolute temperature and irreversible time within the framework of the Boltzmann-Schrödinger equations. It is well known that, in the condition of thermal equilibrium, the *particles* of mass below the threshold determined by the given temperature only exist in a *state of virtuality*, as they are unceasingly created and annihilated while the particles with a mass above this threshold uncouple themselves from "the primeval soup" and assume an *actual existence*. Analogically, this image can be applied to *events* occurring above and below a given threshold of time in a temporal equilibrium, but the image must be inverted because time is the action-inverse of mass. The events with greater time subsist in a *state of virtuality*: this is the domain of *Becoming*. The events with a lesser time uncouple themselves and take an *actual existence* in the realm of *Being*. Then, on a level even more elementary than in Prigogine's conception, a convincing physical argument can be provided for the objective existence of a fixed Past severed from an undetermined Future by the flow of the Now (which is itself the passage of the "front wave" of the Big Bang). Contrary to the conception of most physicists, these are not mere illusions of ours, but objective features of the physical world. Clearly, this image too needs further elaboration.

This communication has to be taken for what it is, i.e. a heuristic investigation into new reinterpretations, views, and conceptual analysis in the quantum domain. In particular, it would be misplaced and ill-timed to demand from it the mathematical rigor of a complete axiomatisation. When at last this will take place, it should join with the axiomatisation of Quantum Mechanics due to G. W. Mackey [9]. But until that some way has still to be covered.

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ERRATUM

My paper *Arch. Sc. Genève 37* (1984), p. 229-264 is unfortunately marred by a number of typographical errors. Most of these can easily be corrected by the reader with the exception of the error on p. 249. On the line following the equation (35): instead of "any finite velocity v" the text should read "any θ_v of finite velocity v" and on the following line, instead of "$th(\theta_1 + v)$" the text should read "$th(\theta_1 + \theta_v)$".

An error of mine occurs in the equation (45) on p. 251. Delete and substitute it by: "(45) $(d/d\xi + i)\,(d/d\xi - i)\bar{\psi}\psi = 0$. Thus, if $(d/d\xi + i)\psi = 0$ then $(d/d\xi - i)\bar{\psi} = 0$, as each equation is the ξ-reversal of the other."